



**VARIANCE OF TIME TO RECRUITMENT WITH DIFFERENT  
RENEWAL PROCESS FOR EXITS UNDER GEOMETRIC  
PROCESS FOR LOSS OF MANPOWER DUE TO EXITS AND  
BREAKING DECISIONS FORMS A MODIFIED  
RENEWAL PROCESS**

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**Abstract:**

A stochastic model is considered in this paper to determine the mean and variance of time to recruitment for a single grade manpower system. It is assumed that the depletion of manpower in the organization is classified into voluntary, involuntary exit of personnel and frequent breaks taken by the existing workers in the organization. The loss of manpower due to voluntary exits are assumed to forms geometric process. By assuming that inter-voluntary exit times and inter-involuntary exit times forms different renewal process and the inter-breaking decision times forms modified renewal process, variance of time to recruitment is determined for three different cases of inter-voluntary exit times using a univariate CUM policy of recruitment. The results are numerically illustrated by assuming specific distributions. The influence of the nodal parameters on the mean and variance of time to recruitment are studied and relevant conclusions are presented.

**Key Words:** Single Grade Manpower System, Geometric Process, Inter-Voluntary Exit Times, Inter-Involuntary exit Times, Inter-Breaking Decision Times, Different Renewal Process, Modified Renewal Process & Univariate CUM Policy

**Introduction:**

Resignation, retirement and death are the common phenomenon of loss of manpower in the organization. The exodus is possible whenever the organization announces revised policies regarding sales target, revision of wages, incentives perquisites etc. This leads to reduction in the total strength of marketing personnel and will adversely affect the sales turnover of the organization, if recruitment is not planned. Frequent recruitment may also be expensive due to the cost of recruitment and training. As the loss of manpower is unpredictable, suitable recruitment policy has to be designed to overcome this loss. A univariate recruitment policy, usually known as CUM policy of recruitment [2] and [6] are based on the replacement policy associated with the shock model approach in reliability theory is stated as follows: Recruitment is made whenever the cumulative loss of man hours exceeds a breakdown threshold. Several stochastic models of manpower system have been proposed and studied by many authors [2], [3] and [4] extensively in the past. More specifically [7] have initiated the study on finding the expected time to recruitment for a single grade manpower system using shock model approach in reliability theory. Recently [1] have studied the problem of time to recruitment when the loss of manpower due to frequent breaks taken by the existing workers forms a modified renewal process. In this paper exits of personnel is classified into voluntary and involuntary exits which forms different renewal process and the inter-breaking decisions forms a modified renewal process. By assuming that the loss of manpower due to voluntary exits forms geometric process, a stochastic model is constructed and the inter-voluntary exit times are in the following cases: (Case-i) as a sequence of independent and identically distributed exponential random variables, (Case-ii) as a sequence of exchangeable and constantly correlated exponential random variables and (Case-iii) as a geometric process. Variance of time to recruitment is obtained using an univariate CUM policy of recruitment by assuming specific distribution for the loss of manpower and thresholds. The results are illustrated and specific conclusions are given on the influence of parameters over the performance measures.

**Model Description:**

Consider an organization with single grade in which exit of personnel takes place either by voluntary and involuntary exit of personnel from the organization or due to breaks taken by the existing workers in the organization. Let  $X_{I_q}$ ,  $q = 1, 2, 3, \dots$  be a sequence of exponential random variables denoting the loss of manpower due to the  $q^{\text{th}}$  involuntary exit of personnel from the organization with Laplace transform  $\bar{f}_{X_{I_q}}(\cdot)$  with parameter  $\alpha_1, \alpha_1 > 0$ . Let  $X_{V_r}$ ,  $r = 1, 2, 3, \dots$  be a sequence of exponential random variables denoting the loss of manpower due to the  $r^{\text{th}}$  voluntary exit of personnel from the organization with Laplace transform  $\bar{f}_{X_{V_r}}(\cdot)$  forms a geometric process with parameter  $\alpha_2, \alpha_2 > 0$ , and a positive constant  $a' > 0$ . Let  $Y_l$ ,  $l = 1, 2, 3, \dots$  be a sequence of

exponential random variables denoting the loss of manpower due to the 1<sup>th</sup> break taken by the existing personnel from the organization with Laplace transform  $\bar{g}_{\gamma_1}(\cdot)$  with parameter  $\gamma, \gamma > 0$ . The loss of manpower are assumed to be linear and cumulative. Let  $\bar{X}_{I_q}$  be the cumulative loss of manpower in the first q involuntary exits of personnel from the organization. Let  $\bar{X}_{V_r}$  be the cumulative loss of manpower in the first r voluntary exits of personnel from the organization. Let  $\bar{Y}_l$  be the cumulative loss of manpower in the first l breaks. Let  $U_q, q = 1, 2, 3, \dots$  be independent and identically distributed exponential random variables representing the time between q-1<sup>th</sup> and q<sup>th</sup> involuntary exit of personnel from the organization with mean  $\frac{1}{\lambda_1}, \lambda_1 > 0$ . Let  $U_r, r = 1, 2, 3, \dots$  be independent exponential random variables representing the time between r-1<sup>th</sup> and r<sup>th</sup> voluntary exit of personnel from the organization with mean  $\frac{1}{\lambda_2}, \lambda_2 > 0$ . Let  $V_l, l = 1, 2, 3, \dots$  be independent and identically distributed exponential random variables representing the time between l-1<sup>th</sup> and l<sup>th</sup> break with mean  $\frac{1}{\beta_1}, \beta_1 > 0$  and forms a modified renewal process with parameter  $\beta_2, (\beta_2 = p\beta_1), \beta_2 > 0$  and  $0 < p < 1$  in the sense that, before there is any loss in manpower, every breaking decision has a fixed probability p of causing loss in manpower in the organization. After the first occurrence of loss in manpower, p changes to 1. Let  $N_{11}(t)$  be the number of involuntary exit decisions in  $(0, t]$ ,  $N_{12}(t)$  be the number of voluntary exit decisions in  $(0, t]$  and  $N_2(t)$  be the number of breaking decisions in  $(0, t]$ . Let  $Z_{11}$  be the exponential threshold for the cumulative loss of manpower due to involuntary exits of personnel from the organization with parameter  $\theta_{11} > 0$ ,  $Z_{12}$  be the exponential threshold for the cumulative loss of manpower due to voluntary exits of personnel from the organization with parameter  $\theta_{12} > 0$  and  $Z_2$  be the exponential threshold for the cumulative loss of manpower due to breaks with parameter  $\theta_2 > 0$ . Let  $Z_{11} + Z_{12} + Z_2$  be the breakdown threshold for the organization. Let W be the time to recruitment for the organization with the distribution function  $L(\cdot)$ , density function  $l(\cdot)$  and the Laplace transform  $\bar{L}(\cdot)$ . It is assumed that the loss of man hours due to involuntary exits, voluntary exits and breaks, inter-involuntary and voluntary exit times, inter-breaking decision times and breakdown threshold are stochastically independent.

**Analytical Results:**

Let the event  $\{W > t\}$  be the time to recruitment which occurs beyond the time t, and  $\{X_{I_{N_{11}(t)}} + X_{V_{N_{12}(t)}} + Y_{N_2(t)} < Z_{11} + Z_{12} + Z_2\}$  represent the event that the cumulative loss of man hours due to the three types of decisions does not cross the breakdown threshold upto the time t. It is proved that the occurrence of these two events are equal. Hence

$$\{W > t\} \Leftrightarrow \{X_{I_{N_{11}(t)}} + X_{V_{N_{12}(t)}} + Y_{N_2(t)} < Z_{11} + Z_{12} + Z_2\}$$

Conditioning upon  $N_{11}(t), N_{12}(t)$  and  $N_2(t)$  by using the result of renewal theory [5], the distribution function and density function for time to recruitment are derived. Hence the r<sup>th</sup> moment for the time to recruitment is determined by taking the r<sup>th</sup> derivative with respect to s for the Laplace transform of the time to recruitment at  $s = 0$ .

**Theorem 1:**

Let  $Z_i, i = 1, 2, 3, \dots, k$  be a sequence of exchangeable and constantly correlated exponential random variables with the correlation  $\rho, \rho \in (-1, 1)$ , mean u and the relation  $v = u(1-\rho)$ . If the probability density function of  $Z_i; i = 1, 2, 3, \dots, k$  is  $\frac{1}{u} e^{-\frac{t}{u}}, u > 0, 0 < t < \infty$  and the k-fold convolution of the distribution of these random variables is

$$Z_k(t) = \frac{1-\rho}{1-\rho+k\rho} \sum_{i=0}^{\infty} \left( \frac{1-\rho}{1-\rho+k\rho} \right)^i \left[ 1 - \sum_{j=0}^{k+i-1} e^{-\frac{t}{u}} \frac{\int_0^{\frac{t}{u}} (e^{-z} z^{k+i-1}) dz}{(k+i-1)!} \right]$$

then the Laplace Stieltje's transform for

$$Z_k(t) \text{ is } \bar{Z}_k(s) = \frac{(1-\rho)(vs+1)^{1-k}}{(1-\rho)(vs+1) + k\rho vs}$$

**Proof:**

By taking the Laplace transform and using the relation of Laplace transform and the Laplace Stieltje's transform,

$$\bar{Z}_K(s) = \frac{s(1-\rho)}{1-\rho+k\rho} \sum_{i=0}^{\infty} \left( \frac{1-\rho}{1-\rho+k\rho} \right)^i \left( \frac{1}{s} - \sum_{j=0}^{k+i-1} \frac{(k+i-j-1)!}{v^{k+i-j-1} \left( s + \left( \frac{1}{v} \right)^{k+i-j} \right) (k+i-j-1)!} \right)$$

Simplifying the above expression, then the Laplace Stieltje's transform for  $Z_k(t)$  is reduced to

$$\bar{Z}_K(s) = \frac{(1-\rho)}{1-\rho+k\rho} \sum_{i=0}^{\infty} \left( \frac{1-\rho}{1-\rho+k\rho} \right)^i (1+vs)^{-(k+i)}$$

Doing further simplifications the Laplace Stieltje's transform is proved. Henceforth the moments are derived by taking the  $r^{\text{th}}$  derivative with respect to  $s$  for the Laplace Stieltje's transform of  $Z_k(t)$  at  $s=0$ .

**Theorem 2:**

Let  $Z_k$  be a sequence of non-negative random variables and  $a$  be a positive constant. If a normalized stochastic process  $\{V_k\}_{k=1}^{\infty}$ , where  $V_k = a^{n-1} Z_k$ ,  $k=1,2,3,\dots$  is a geometric process with a parameter  $a$ , then the

Laplace transform for  $V_k$  is 
$$\bar{V}_K(s) = \prod_{r=1}^k \bar{f}\left(\frac{s}{a^{r-1}}\right)$$

**Proof:**

The distribution function  $V_k(t) = F(a^{k-1}t)$ ;  $k = 1,2,3,\dots$  is determined from the definition of geometric process and by differentiating the distribution function with respect to  $t$ , the density function of  $v_k(t) = a^{k-1}f(a^{k-1}t)$ ;  $k = 1, 2, 3, \dots$  is derived.

Now using the property, Laplace transform of the convolution of random variables is the product of their Laplace transforms, the Laplace transform for  $v_k(t)$  is derived. Hence the theorem is proved.

**Corollary:**

If  $a = 1$ , then the sequence of random variables  $V_k$ ,  $k = 1, 2, 3; \dots$  forms an ordinary renewal process.

**Remark:**

If  $a > 1$ ,  $\{V_k\}_{k=1}^{\infty}$  is stochastically decreasing and when  $0 < a < 1$ ,  $\{V_k\}_{k=1}^{\infty}$  forms a stochastically increasing sequence.

**Case (i):**

Inter voluntary-exit times are assumed to form a sequence of independent and identically distributed exponential random variables. Taking derivative for the Laplace transform of the random variable  $W$  with respect to  $s$  and at  $s = 0$ , the mean time to recruitment for the present case is derived.

$$E(W) = C_1 D_1 - C_2 D_2 + C_3 D_3 \tag{1}$$

Where

$$D_1 = \frac{((1-A_3)h_1 - \mu_1\mu_2)}{m_1\mu_2\mu_1} \sum_{r=0}^{\infty} (\bar{f}_{U_V}(m_1)^{r+1} - \bar{f}_{U_V}(m_1)^r) \bar{f}_{X_V}(\theta_{11}) + \frac{(1-A_3)(\mu_1h_2 - h_1)}{m_2(\mu_2 - \mu_1)\mu_1} \sum_{r=0}^{\infty} (\bar{f}_{U_V}(m_2)^{r+1} - \bar{f}_{U_V}(m_2)^r) \bar{f}_{X_V}(\theta_{11})$$

$$+ \frac{(1-A_3)(h_1 - \mu_2h_2)}{m_3(\mu_2 - \mu_1)\mu_2} \sum_{r=0}^{\infty} (\bar{f}_{U_V}(m_3)^{r+1} - \bar{f}_{U_V}(m_3)^r) \bar{f}_{X_V}(\theta_{11})$$

$$D_2 = \frac{((1-A_6)h_1 - \mu_1\mu_2)}{m_4\mu_2\mu_1} \sum_{r=0}^{\infty} (\bar{f}_{U_V}(m_4)^{r+1} - \bar{f}_{U_V}(m_4)^r) \bar{f}_{X_V}(\theta_{12}) + \frac{(1-A_6)(\mu_1h_2 - h_1)}{m_5(\mu_2 - \mu_1)\mu_1} \sum_{r=0}^{\infty} (\bar{f}_{U_V}(m_5)^{r+1} - \bar{f}_{U_V}(m_5)^r) \bar{f}_{X_V}(\theta_{12})$$

$$+ \frac{(1-A_6)(h_1 - \mu_2h_2)}{m_6(\mu_2 - \mu_1)\mu_2} \sum_{r=0}^{\infty} (\bar{f}_{U_V}(m_6)^{r+1} - \bar{f}_{U_V}(m_6)^r) \bar{f}_{X_V}(\theta_{12})$$

$$D_3 = \frac{((1-A_9)h_1 - \mu_1\mu_2)}{m_7\mu_2\mu_1} \sum_{r=0}^{\infty} (\bar{f}_{U_V}(m_7)^{r+1} - \bar{f}_{U_V}(m_7)^r) \bar{f}_{X_V}(\theta_2) + \frac{(1-A_9)(\mu_1h_2 - h_1)}{m_8(\mu_2 - \mu_1)\mu_1} \sum_{r=0}^{\infty} (\bar{f}_{U_V}(m_8)^{r+1} - \bar{f}_{U_V}(m_8)^r) \bar{f}_{X_V}(\theta_2)$$

$$+ \frac{(1-A_9)(h_1 - \mu_2h_2)}{m_9(\mu_2 - \mu_1)\mu_2} \sum_{r=0}^{\infty} (\bar{f}_{U_V}(m_9)^{r+1} - \bar{f}_{U_V}(m_9)^r) \bar{f}_{X_V}(\theta_2)$$

The second moment of the random variable  $W$  is derived by differentiating twice the Laplace transform of  $W$  with respect to  $s$  and at  $s = 0$ . From these results the variance of time to recruitment for the present case is determined.

**Case (ii):**

In case-II inter voluntary-exit times are assumed to form a sequence of exchangeable and constantly correlated exponential random variables with mean  $\lambda_2$ , correlation  $\rho$  with the relation  $v = \lambda_2(1-\rho)$

Using the Theorem-1 taking derivative with respect to  $s$  for the Laplace transform of  $W$  at  $s=0$  gives the mean time to recruitment for the present case.

$$E(W) = C_1D_4 - C_2D_5 + C_3D_6 \quad (2)$$

Where

$$D_4 = \frac{((1-A_3)h_1 - \mu_1\mu_2)}{m_1\mu_2\mu_1} \sum_{r=0}^{\infty} (\overline{f_{U_{Vr+1}}}(m_1) - \overline{f_{U_{Vr}}}(m_1)) \overline{f_{X_{Vr}}}(\theta_{11}) + \frac{(1-A_3)(\mu_1h_2 - h_1)}{m_2(\mu_2 - \mu_1)\mu_1} \sum_{r=0}^{\infty} (\overline{f_{U_{Vr+1}}}(m_2) - \overline{f_{U_{Vr}}}(m_2)) \overline{f_{X_{Vr}}}(\theta_{11})$$

$$+ \frac{(1-A_3)(h_1 - \mu_2h_2)}{m_3(\mu_2 - \mu_1)\mu_2} \sum_{r=0}^{\infty} (\overline{f_{U_{Vr+1}}}(m_3) - \overline{f_{U_{Vr}}}(m_3)) \overline{f_{X_{Vr}}}(\theta_{11})$$

$$D_5 = \frac{((1-A_6)h_1 - \mu_1\mu_2)}{m_4\mu_2\mu_1} \sum_{r=0}^{\infty} (\overline{f_{U_{Vr+1}}}(m_4) - \overline{f_{U_{Vr}}}(m_4)) \overline{f_{X_{Vr}}}(\theta_{12}) + \frac{(1-A_6)(\mu_1h_2 - h_1)}{m_5(\mu_2 - \mu_1)\mu_1} \sum_{r=0}^{\infty} (\overline{f_{U_{Vr+1}}}(m_5) - \overline{f_{U_{Vr}}}(m_5)) \overline{f_{X_{Vr}}}(\theta_{12})$$

$$+ \frac{(1-A_6)(h_1 - \mu_2h_2)}{m_6(\mu_2 - \mu_1)\mu_2} \sum_{r=0}^{\infty} (\overline{f_{U_{Vr+1}}}(m_6) - \overline{f_{U_{Vr}}}(m_6)) \overline{f_{X_{Vr}}}(\theta_{12})$$

$$D_6 = \frac{((1-A_9)h_1 - \mu_1\mu_2)}{m_7\mu_2\mu_1} \sum_{r=0}^{\infty} (\overline{f_{U_{Vr+1}}}(m_7) - \overline{f_{U_{Vr}}}(m_7)) \overline{f_{X_{Vr}}}(\theta_2) + \frac{(1-A_9)(\mu_1h_2 - h_1)}{m_8(\mu_2 - \mu_1)\mu_1} \sum_{r=0}^{\infty} (\overline{f_{U_{Vr+1}}}(m_8) - \overline{f_{U_{Vr}}}(m_8)) \overline{f_{X_{Vr}}}(\theta_2)$$

$$+ \frac{(1-A_9)(h_1 - \mu_2h_2)}{m_9(\mu_2 - \mu_1)\mu_2} \sum_{r=0}^{\infty} (\overline{f_{U_{Vr+1}}}(m_9) - \overline{f_{U_{Vr}}}(m_9)) \overline{f_{X_{Vr}}}(\theta_2)$$

Now differentiating twice the Laplace transform of W with respect to s and at s = 0, the second moment of W is determined. From these results the variance of time to recruitment for the present case is determined.

**Case (iii):**

In this case inter voluntary-exit times are assumed to form a geometric process with a parameter  $a > 0$ . Using Theorem 2, the mean time to recruitment for Case-III is determined by taking first derivative with respect to s for the Laplace transform of W at s=0.

$$E(W) = C_1D_7 - C_2D_8 + C_3D_9 \quad (3)$$

$$D_7 = \frac{((1-A_3)h_1 - \mu_1\mu_2)}{m_1\mu_2\mu_1} \sum_{r=0}^{\infty} (\overline{f_{U_{Vr+1}}}(m_1) - \overline{f_{U_{Vr}}}(m_1)) \overline{f_{X_{Vr}}}(\theta_{11}) + \frac{(1-A_3)(\mu_1h_2 - h_1)}{m_2(\mu_2 - \mu_1)\mu_1} \sum_{r=0}^{\infty} (\overline{f_{U_{Vr+1}}}(m_2) - \overline{f_{U_{Vr}}}(m_2)) \overline{f_{X_{Vr}}}(\theta_{11})$$

$$+ \frac{(1-A_3)(h_1 - \mu_2h_2)}{m_3(\mu_2 - \mu_1)\mu_2} \sum_{r=0}^{\infty} (\overline{f_{U_{Vr+1}}}(m_3) - \overline{f_{U_{Vr}}}(m_3)) \overline{f_{X_{Vr}}}(\theta_{11})$$

$$D_8 = \frac{((1-A_6)h_1 - \mu_1\mu_2)}{m_4\mu_2\mu_1} \sum_{r=0}^{\infty} (\overline{f_{U_{Vr+1}}}(m_4) - \overline{f_{U_{Vr}}}(m_4)) \overline{f_{X_{Vr}}}(\theta_{12}) + \frac{(1-A_6)(\mu_1h_2 - h_1)}{m_5(\mu_2 - \mu_1)\mu_1} \sum_{r=0}^{\infty} (\overline{f_{U_{Vr+1}}}(m_5) - \overline{f_{U_{Vr}}}(m_5)) \overline{f_{X_{Vr}}}(\theta_{12})$$

$$+ \frac{(1-A_6)(h_1 - \mu_2h_2)}{m_6(\mu_2 - \mu_1)\mu_2} \sum_{r=0}^{\infty} (\overline{f_{U_{Vr+1}}}(m_6) - \overline{f_{U_{Vr}}}(m_6)) \overline{f_{X_{Vr}}}(\theta_{12})$$

$$D_9 = \frac{((1-A_9)h_1 - \mu_1\mu_2)}{m_7\mu_2\mu_1} \sum_{r=0}^{\infty} (\overline{f_{U_{Vr+1}}}(m_7) - \overline{f_{U_{Vr}}}(m_7)) \overline{f_{X_{Vr}}}(\theta_2) + \frac{(1-A_9)(\mu_1h_2 - h_1)}{m_8(\mu_2 - \mu_1)\mu_1} \sum_{r=0}^{\infty} (\overline{f_{U_{Vr+1}}}(m_8) - \overline{f_{U_{Vr}}}(m_8)) \overline{f_{X_{Vr}}}(\theta_2)$$

$$+ \frac{(1-A_9)(h_1 - \mu_2h_2)}{m_9(\mu_2 - \mu_1)\mu_2} \sum_{r=0}^{\infty} (\overline{f_{U_{Vr+1}}}(m_9) - \overline{f_{U_{Vr}}}(m_9)) \overline{f_{X_{Vr}}}(\theta_2)$$

Taking second derivative for the Laplace transform of W with respect to s and at s = 0, the second moment for W is derived. From these two results the variance of time to recruitment is determined for the present case. The expressions for the notations used in the equations (1), (2) and (3) are given below.

$$A_1 = \frac{\alpha_1}{\alpha_1 + \theta_{11}}, A_2 = \frac{\alpha_2}{\alpha_2 + \theta_{11}}, A_3 = \frac{\gamma}{\gamma + \theta_{11}}, A_4 = \frac{\alpha_1}{\alpha_1 + \theta_{12}}, A_5 = \frac{\alpha_2}{\alpha_2 + \theta_{12}}, A_6 = \frac{\gamma}{\gamma + \theta_{12}},$$

$$A_7 = \frac{\alpha_1}{\alpha_1 + \theta_2}, A_8 = \frac{\alpha_2}{\alpha_2 + \theta_2}, A_9 = \frac{\gamma}{\gamma + \theta_2}; h_1 = \beta_1\beta_2; h_2 = \beta_2; h_3 = \beta_1 + \beta_2 - \beta_1A_3; h_4 = \beta_1\beta_2 - \beta_1\beta_2A_3;$$

$$m_1 = \lambda_1(1 - A_1) + \lambda_2(1 - A_2); m_2 = \lambda_1(1 - A_1) + \lambda_2(1 - A_2) + \mu_1; m_3 = \lambda_1(1 - A_1) + \lambda_2(1 - A_2) + \mu_2;$$

$$m_4 = \lambda_1(1 - A_4) + \lambda_2(1 - A_5); m_5 = \lambda_1(1 - A_4) + \lambda_2(1 - A_5) + \mu_1; m_6 = \lambda_1(1 - A_4) + \lambda_2(1 - A_5) + \mu_2;$$

$$m_7 = \lambda_1(1 - A_7) + \lambda_2(1 - A_8); m_8 = \lambda_1(1 - A_7) + \lambda_2(1 - A_8) + \mu_1; m_9 = \lambda_1(1 - A_7) + \lambda_2(1 - A_8) + \mu_2;$$

$$C_1 = \frac{\theta_{12}\theta_2}{(\theta_{12} - \theta_{11})(\theta_2 - \theta_{11})}; C_2 = \frac{\theta_{11}\theta_2}{(\theta_{12} - \theta_{11})(\theta_2 - \theta_{12})}; C_3 = \frac{\theta_{12}\theta_{11}}{(\theta_2 - \theta_{11})(\theta_2 - \theta_{12})};$$

and  $\theta_{11} \neq \theta_{12} \neq \theta_2, A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9 \neq 1, \mu_1 \neq \mu_2, \beta_1 \neq \beta_2$ .

**Numerical Illustration:**

The influence of nodal parameters on the performance measures namely mean and variance of the time to recruitment is studied numerically using MATLAB. The performance measures are calculated for all the three cases.

**Case (i):** Effect of  $\lambda_1, \lambda_2, \alpha_1, \alpha_2, \gamma, a'$  for the mean and variance of time to recruitment is studied by fixing the value of the parameters  $\beta_1 = 0.2; p = 0.01; \theta_{11} = 0.7; \theta_{12} = 0.8, \theta_2 = 0.9, r = 3$ .

$\lambda_1$	$\lambda_2$	$\alpha_1$	$\alpha_2$	$\gamma$	$a'$	E(W)	V(W)
0.8	0.6	0.1	0.1	0.2	0.3	0.1499	0.9973
0.8	0.6	0.1	0.1	0.2	0.6	0.1342	1.2263
0.8	0.6	0.1	0.1	0.2	2	0.1032	1.6902
0.8	0.6	0.1	0.1	0.2	3	0.0971	1.7745
0.8	0.6	0.1	0.1	0.3	0.3	0.1469	1.0074
0.8	0.6	0.1	0.1	0.4	0.3	0.1447	1.0156
0.8	0.6	0.1	0.1	0.5	0.3	0.1431	1.0223
0.8	0.6	0.1	0.2	0.2	0.3	0.1719	0.7231
0.8	0.6	0.1	0.3	0.2	0.3	0.1895	0.5130
0.8	0.6	0.1	0.4	0.2	0.3	0.2036	0.3488
0.8	0.6	0.2	0.1	0.2	0.3	0.1801	0.6255
0.8	0.6	0.3	0.1	0.2	0.3	0.2052	0.3317
0.8	0.6	0.4	0.1	0.2	0.3	0.2266	0.0956
0.8	0.45	0.1	0.1	0.2	0.3	0.1545	0.2732
0.8	0.5	0.1	0.1	0.2	0.3	0.1532	0.5371
0.8	0.55	0.1	0.1	0.2	0.3	0.1517	0.7760
0.5	0.6	0.1	0.1	0.2	0.3	0.2091	0.2867
0.6	0.6	0.1	0.1	0.2	0.3	0.1855	0.5456
0.7	0.6	0.1	0.1	0.2	0.3	0.1661	0.7815

**Case (ii):** Effect of  $\lambda_1, \lambda_2, \alpha_1, \alpha_2, \gamma, \rho, a'$  for the mean and variance of time to recruitment is studied by fixing the value of the parameters  $\beta_1 = 0.2; p = 0.01; \theta_{11} = 0.7; \theta_{12} = 0.8, \theta_2 = 0.9, r = 3$ .

$\lambda_1$	$\lambda_2$	$\alpha_1$	$\alpha_2$	$\gamma$	$a'$	$\rho$	E(W)	V(W)
0.7	0.4	0.4	0.1	0.2	0.3	0.2	0.7083	1.5514
0.7	0.4	0.4	0.1	0.2	0.3	0.5	0.7432	1.4181
0.7	0.4	0.4	0.1	0.2	0.3	-0.5	0.7564	1.5194
0.7	0.5	0.4	0.1	0.2	0.3	-0.2	0.7090	1.5998
0.7	0.5	0.4	0.1	0.2	0.4	0.2	0.7029	1.5989
0.7	0.5	0.4	0.1	0.2	0.5	0.2	0.6896	1.6557
0.7	0.5	0.4	0.1	0.2	2	0.2	0.2018	2.4387
0.7	0.5	0.4	0.1	0.2	2.5	0.2	0.0831	2.5197
0.7	0.5	0.4	0.1	0.3	0.3	0.2	0.5375	1.7504
0.7	0.5	0.4	0.1	0.4	0.3	0.2	0.3645	1.8955
0.7	0.5	0.4	0.1	0.5	0.3	0.2	0.1898	1.9835
0.7	0.5	0.4	0.2	0.2	0.3	0.2	0.8735	1.6260
0.7	0.5	0.4	0.3	0.2	0.3	0.2	1.0449	1.5868
0.7	0.5	0.4	0.4	0.2	0.3	0.2	1.2144	1.4553
0.7	0.5	0.45	0.1	0.2	0.3	0.2	0.7726	1.5525
0.7	0.5	0.5	0.1	0.2	0.3	0.2	0.8385	1.5372
0.7	0.5	0.55	0.1	0.2	0.3	0.2	0.9053	1.5064
0.7	0.6	0.4	0.1	0.2	0.3	0.2	0.5236	1.7444
0.7	0.7	0.4	0.1	0.2	0.3	0.2	0.6063	1.5549
0.7	0.8	0.4	0.1	0.2	0.3	0.2	0.9004	0.9839
0.55	0.5	0.4	0.1	0.2	0.3	0.2	1.1228	1.1194
0.6	0.5	0.4	0.1	0.2	0.3	0.2	0.9622	1.3367
0.65	0.5	0.4	0.1	0.2	0.3	0.2	0.8247	1.4376

**Case (iii):** Effect of  $\lambda_1, \lambda_2, \alpha_1, \alpha_2, \gamma, a', a$  for the mean and variance of time to recruitment is studied by fixing the value of the parameters  $\beta_1 = 0.4; p = 0.01; \theta_{11} = 0.7; \theta_{12} = 0.8, \theta_2 = 0.9, r = 3$ .

$\lambda_1$	$\lambda_2$	$\alpha_1$	$\alpha_2$	$\gamma$	$a'$	$a$	E(W)	V(W)
0.7	0.5	0.4	0.1	0.1	0.4	0.6	0.3906	2.5810

0.7	0.5	0.4	0.1	0.1	0.4	0.7	0.4111	2.5043
0.7	0.5	0.4	0.1	0.1	0.4	2	0.7649	1.5802
0.7	0.5	0.4	0.1	0.1	0.4	3	0.9387	1.1225
0.7	0.5	0.4	0.1	0.1	0.5	0.6	0.3941	2.6063
0.7	0.5	0.4	0.1	0.1	0.6	0.6	0.3971	2.6281
0.7	0.5	0.4	0.1	0.1	2	0.6	0.4146	2.7693
0.7	0.5	0.4	0.1	0.1	3	0.6	0.4182	2.8045
0.7	0.5	0.4	0.1	0.12	0.4	0.6	0.2952	2.6314
0.7	0.5	0.4	0.1	0.14	0.4	0.6	0.1995	2.6648
0.7	0.5	0.4	0.1	0.16	0.4	0.6	0.1035	2.6810
0.7	0.5	0.4	0.15	0.1	0.4	0.6	0.2781	2.9006
0.7	0.5	0.4	0.2	0.1	0.4	0.6	0.1764	3.1728
0.7	0.5	0.4	0.25	0.1	0.4	0.6	0.0842	3.4060
0.7	0.5	0.45	0.1	0.1	0.4	0.6	0.3093	2.7714
0.7	0.5	0.5	0.1	0.1	0.4	0.6	0.2338	2.9378
0.7	0.5	0.55	0.1	0.1	0.4	0.6	0.1635	3.0834
0.7	0.55	0.4	0.1	0.1	0.4	0.6	0.5729	2.1595
0.7	0.6	0.4	0.1	0.1	0.4	0.6	0.7339	1.7359
0.7	0.65	0.4	0.1	0.1	0.4	0.6	0.8770	1.3191
0.75	0.5	0.4	0.1	0.1	0.4	0.6	0.5169	2.3406
0.8	0.5	0.4	0.1	0.1	0.4	0.6	0.6393	2.0814
0.85	0.5	0.4	0.1	0.1	0.4	0.6	0.7579	1.8056

**Conclusions:**

In Case (i), increasing  $\alpha_2$  ( $\alpha_1$ ) by fixing the other parameters the mean time to recruitment increases and variance of time to recruitment decreases. For ( $a > 1, a < 1$ ), increasing the geometric process parameter  $a'(\gamma, \lambda_2, \lambda_1)$  by fixing the other parameters the mean time to recruitment decreases and variance of time to recruitment increases. In Case (ii), increasing the negative value of  $\rho(\gamma, a', \lambda_1)$  by fixing the other parameters the mean time to recruitment decreases and variance of time to recruitment increases. When increasing the positive value of  $\rho(\alpha_1, \lambda_2)$  by fixing the other parameters the mean time to recruitment increases and variance of time to recruitment decreases. Increasing  $\alpha_2$  by fixing the other parameters the mean and variance of time to recruitment increases.

In Case (iii), for ( $a > 1; a < 1$ ) increasing the geometric process parameter for inter-voluntary exit times  $a(\lambda_1, \lambda_2)$  by fixing the other parameters the mean time to recruitment increases and variance of time to recruitment decreases. While increasing  $\alpha_1$  ( $\alpha_2, \gamma$ ) by fixing the other parameters the mean time to recruitment decreases and variance of time to recruitment increases. For ( $a' > 1, a' < 1$ ) increasing the geometric process parameter for the loss of man hours due to voluntary exits  $a'$  by fixing the other parameters the mean and variance of time to recruitment increases.

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