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# MORE ON THE DIOPHANTINE EQUATION $2^x + 87^y = z^2$

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**Abstract:**

In this paper, we show that (3, 0, 3) is a unique non-negative integer solution for the Diophantine solution $2^x + 87^y = z^2$, where x,y and z are non-negative integers.

**1. Introduction:**

In 2007, Acu [1] proved that (3, 0, 3) and (2, 1, 3) are only two solutions in non-negative integers of the Diophantine equation $2^x+5^y=z^2$. In 2013, Sroysang [2] proved that more on the Diophantine equation $2^x + 32^y = z^2$ has non-negative integer (3, 0, 3) is a unique non- negative integer solution. In this paper we show that (3, 0, 3) is a unique non- negative integer solution for the Diophantine equation $2^x + 87^y = z^2$ where x, y and z are non-negative integers.

**2. Preliminaries:**

In 1844, Catalan [3] conjectures that the Diophantine equation $a^x - b^y=1$ has a unique integer solution with min{a, b, x, y}>1. The solution (a, b, x, y) is (3, 2, 2, 3). This conjecture was proven by Mihailescu [4] in 2004

**Preposition 2.1:**

([5]). (3, 2, 2, 3) is a unique solution (a, b, x, y) of the Diophantine equation $a^x - b^y=1$, where a, b, x and y are integers with min{a, b, x, y}>1

**Lemma 2.2:**

[1] (3, 3) is a unique solution of (x, z) for the Diophantine equation $2^x + 1=z^2$, Where x and z are non-negative integers.

**Lemma 2.3:**

The Diophantine equation $1+87^y=z^2$ has no non-negative integer solution where y and z are non-negative integers.

**Proof:**

Suppose that there are non-negative integers y and z such that $1+87^y=z^2$. If y=0, then $z^2=2$ which is impossible. Then $y\geq1$. Thus, $z^2=87^y+1\geq87^1+1=88$, then z>9. Now we consider on the equation $z^2-87^y=1$. By preposition 2.1, we have y=1. Then $z^2=88$. This is a contradiction. Hence, the equation $1+87^y=z^2$ has no non negative integer solution.

**3. Results:**

**Theorem 3.1:**

(3, 0, 3) is a unique solution (x, y, z) for the Diophantine equation $2^x + 87^y = z^2$ where x, y and z non-negative integers.

**Proof**:

Let x, y and z be non-negative integers such that $2^x + 87^y = z^2$. By lemma 2.3, we have $x\geq1$. Thus z is odd then there is a non-negative integer t such that z=2t+1. We obtain that $2^x+87^y=4(t^2+t)+1$. Then $87^y\equiv1 \pmod 4$. Thus y is even. Then there is a non-negative integer k such that y=2k. We divide the number y into two cases.

Case y=0. By lemma 2.2, we have x=3 and z=3.

Case $y\geq2$. Then $k\geq1$. Then $z^2-87^{2k}=2^x$. Then $(z-87^k)(z+87^k)=2^x$. We obtain that $z-87^k=2^\alpha$, where α is a non-negative integer. Then $z+87^k=2^{x-\alpha}$. it follows that $2(87^k)=2^{x-\alpha}-2^\alpha=2^\alpha(2^{x-2\alpha}-1)$. We divide the number α into two sub cases.

Sub case α=0. Then $z-87^k=1$. Then z is even. This is a contradiction.

Sub case α=1. Then $2^{x-2}-1=87^k$. It follows that $2^{x-2}=87^k+1\geq87+1=88$. Thus $x\geq8$.More over $2^{x-2}-87^k=1$. By preposition 2.1, we have k=1, then $2^{x-2}=88$. This is impossible.

Therefore, (3, 0, 3) is a unique solution (x, y, z) for the equation $2^x+87^y=z^2$

**Corollary 3.2:**

The Diophantine equation $2^x+87^y=w^4$ has no non-negative integer solution. Where x, y and w are non-negative integers.

**Proof:**

Suppose that there are non-negative integers x, y and w such that $2^x+87^y=w^4$. Let $z=w^2$. Then $2^x+87^y=z^2$. By lemma 3.1, we have (x, y, z) = (3, 0, 3). Then $w^2=z=3$. This is a contradiction.

**Corollary 3.3:**

(1, 0, 3) is a unique solution of (x, y, z) for the Diophantine equation $8^\alpha+87^y=z^2$, where y, α and z are non-negative integers.

**Proof:**

Let x, y and z are non-negative integers such that $8^\alpha+87^y=z^2$. Let x=3α. Then $2^x+87^y=z^2$. By theorem 3.1 we have (x, y, z) = (3, 0, 3). Then x=3α=3. Thus α=1. Therefore, (1, 0, 3) is a unique solution (x, α, z) for the equation $8^\alpha+87^y=z^2$.

**Corollary 3.4:**

The Diophantine equation $32^\alpha+87^y=z^2$ has no non-negative integer solution. Where α, y and z are non-negative integers.

**Proof:**

Suppose that there are non-negative integers α, y and z such that $32^\alpha+87^y=z^2$. Let x=5α. Then $2^x + 87^y = z^2$. By theorem 3.1, we have x=5α=3. This is contradiction.

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