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TRIPLE CONNECTED DOMINATION NUMBER FOR SOME SPECIAL GRAPHS

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Abstract:

A subset S of V of a nontrivial connected graph G is said to be a triple connected dominating set (tcd-set), if S is a dominating set and the induced subgraph $<S>$is triple connected. The minimum cardinality taken over all triple connected dominating sets is called the triple connected domination number of G and is denoted by $\gamma_{tc}(G)$. Any triple connected dominating set with γ_{tc} vertices is called a γ_{tc}-set of G. In this paper we obtain triple connected domination number for triangular snake graphs and double triangular snake graphs.

Key Words: Triangular Snake Graphs, Double Triangular Snake Graphs, Domination Number, Triple Connected Graph & Triple Connected Domination Number

1. Introduction:

The study of dominating sets in graph theory began around 1960. In 1958, domination was formalized as a theoretical area in graph theory by Berge. He referred to the domination number as a coefficient of external stability. In 1962, Ore was the first to use the term dominating set and domination number for undirected graphs. In 1977 Cockayneand Hedetniemi made an interesting and extensive survey of the results about dominating sets in graphs. Hedetniemi and Laskar published their bibliography on domination in graphs and some basic definitions of domination parameters. The concept of triple connected graphs has been introduced by Joseph et.al. . They have studied the properties of triple connected graphs and established many results on them. The concept of triple connected domination number of a graph has been introduced by Mahadevan et.al., Paulson and Lilly highlighted applications of domination in graphs in several fields and the importance of graph theoretical ideas in various areas of science and engineering. Consider a finite, simple connected and undirected graph $G(V, E)$, where V and E denote the number of vertices and number of edges of a graph respectively[3].

2. Preliminaries

Definition 2.1[2]: A graph G is said to be triple connected if any three vertices lie on a path in G.

Definition 2.2[2]: A subset S of V is called a dominating set of G if every vertex in $V - S$ is adjacent to at least one vertex in S. The domination number $\gamma(G)$ of G is the minimum cardinality taken over all dominating sets in G.

Definition 2.3 [4]: The triangular snake graph T_n is obtained from the path P_n by replacing each edge of the path by a triangle C_3.

Example:

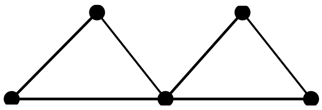

Figure 2.1: Triangular snake graph T_3

Definition 2.4 [4]: A double triangular snake graph $D(T_n)$ consists of two triangular snakes that have a common path.

Example:

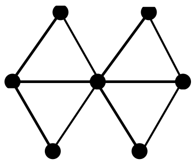

Figure 2.2: double triangular snake $D(T_3)$

Definition 2.5 [2]: A subset S of V of a nontrivial connected graph G is said to be a triple connected dominating set (tcd-set), if S is a dominating set and the induced subgraph$<S>$is triple connected.

The minimum cardinality taken over all triple connected dominating sets is called the triple connected domination number of G and is denoted by $\gamma_{tc}(G)$. Any triple connected dominating set with γ_{tc} vertices is called a γ_{tc}-set of G.

Example:

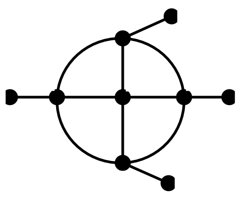

Figure 2.3: Helm graph H_5

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For the graph H_5in figure 2.3,
S = {1, 2, 3, 4} is a tcd set.
S_1 = {1, 2, 3, 4, 9} is a tcd set.
Here,S = {1, 2, 3, 4} is a minimum tcd set. Therefore, $\gamma_{tc}(H_5)= 4$.

3. Triple Connected Domination Number for Some Special Graphs:

3.2.1. We obtain the algorithm andtheorem for triple connected domination number for triangular snake graph T_n.

Algorithm:

Step 1: Consider the Triangular snake (T_n) graph. T_n contains $2n-1$ vertices and $n-1$ triangles, where $n = 2,3, ...$

Step 2: Label the upper vertices which starts from $v_1, v_2,\ldots, v_{n-1}$ and the lower vertices which starts from $v_n, v_{n+1},\ldots,v_{2n-1}$.

Step 3: Collect the subset which satisfies the conditions of dominating set and its induced subgraph is triple connected.

Step 4: Choose the minimum cardinality subset and the resulting cardinality will be the triple connected domination number of T_n.

Theorem 3.2.2:

For any triangular snake graph of order $p\geq 3$, $\gamma_{tc}(T_n) = \begin{cases} 3 & if\ n \leq 4 \\ n-2 & if\ n \geq 5 \end{cases}$

Proof:

The graph T_n contains $2n-1$ vertices and $n-1$ triangles. The upper vertices labelled from v_1 to v_{n-1} and the lower vertices labelled from v_n to v_{2n-1}

Take the vertex v_{n+1}, it is adjacent to v_1, v_2, v_n and v_{n+2}. Then take the vertex v_{n+3}, it is adjacent to v_3, v_4, v_{n+2} and v_{n+4}. In a similar manner we will take v_{2n-2}, it is adjacent to v_{n-2}, v_{n-1}, v_{2n-3} and v_{2n-1}. The set $\{v_{n+1}, v_{n+3}, \ldots, v_{2n-2}\}$ is a minimum dominating set. But its induced subgraph is not triple connected. So consider all v_i's lying in the dominating set such that v_i's are adjacent to v_j's in the dominating set, where i lies between $v_{n+1}$and v_{2n-2}. Thus, the set $S=\{v_{n+1}, v_{n+2}, \ldots,v_{2n-3},v_{2n-2}\}$is a dominating set and$\langle S\rangle$is triple connected. So it forms tcd set. Also it is a minimum tcd set.

Therefore $\gamma_{tc}(T_n)= n-2\ if\ n \geq 5$.

Trivially $\gamma_{tc}(T_n)= 3\ if\ n \leq 4$.

Thus for any triangular snake graph of order $p\geq 3$, $\gamma_{tc}(T_n) = \begin{cases} 3 & if\ n \leq 4 \\ n-2 & if\ n \geq 5 \end{cases}$.

For example,
Consider the graph T_4.

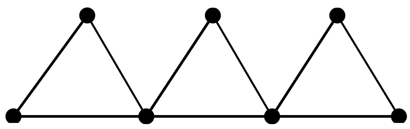

Figure 3.2.1: Triangular snake graph T_4

For the graph T_4in Figure 3.2.1,
$S = \{v_2, v_5, v_6\}$ is a minimum tcd set. Therefore, $\gamma_{tc}(T_4)= 3$.
Consider the graph T_9.

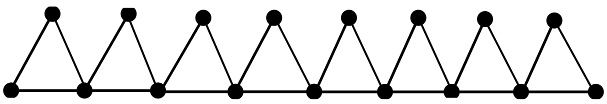

Figure 3.2.2: Triangular snake graph T_9

For the graph T_9 in Figure 3.2.2,
$S = \{v_{10},v_{11}, v_{12}, v_{13}, v_{14}, v_{15}, v_{16}\}$ is a minimum tcd set. Therefore, $\gamma_{tc}(T_9)= 7 = n-2$.

3.2.3. We obtain the algorithm and theorem for triple connected domination number for double triangular snake graph $D(T_n)$.

Algorithm:

Step 1: Consider the double triangular snake $D(T_n)$ graph. $D(T_n)$ contains $3n-2$ vertices, where $n = 2,3, ...$

Step 2: Label the upper vertices which starts from $v_1, v_2,\ldots, v_{n-1}$, middle vertices which starts from $v_n, v_{n+1},\ldots,v_{2n-1}$ and the lower vertices which is starts from $v_{2n}, v_{2n+1},\ldots,v_{3n-2}$

Step 3: Collect the subset which satisfies the conditions of dominating set and its induced subgraph is triple connected.

Step 4: Choose the minimum cardinality subset and the resulting cardinality will be the triple connected domination number of D(T_n).

Theorem 3.2.4:

For any double triangular snake of order $p\geq 3$, $\gamma_{tc}(D(T_n))= \begin{cases} 3 & if\ n \leq 4 \\ n-2 & if\ n \geq 5 \end{cases}$

Proof:

The graph $D(T_n)$contains $3n-2$ vertices. The upper vertices labelled from v_1 to v_{n-1} and the middle vertices labelled from $v_n, v_{n+1},\ldots,v_{2n-1}$ and the lower vertices labelled from $v_{2n}, v_{2n+1},\ldots,v_{3n-2}$

Take the vertex v_{n+1}, it is adjacent to v_1, v_2, v_n, v_{n+2}, v_{2n} and v_{2n+1}. Then take the vertex v_{n+3}, it is adjacent to v_3, v_4, v_{n+2}, v_{n+4}, v_{2n+2} and v_{2n+3}. In a similar manner we will take v_{2n-2}, it is adjacent to v_{n-2}, v_{n-1}, v_{2n-3}, v_{2n-1}, v_{3n-3} and v_{3n-2}. The set $\{v_{n+1}, v_{n+3}, \ldots, v_{2n-2}\}$ is a minimum dominating set. But its induced subgraph is not triple connected. So consider all v_i's lying in the dominating set such that v_i's are adjacent to v_j's in the dominating set, where i lies between v_{n+1} and v_{2n-2}. Thus, the set $S=\{v_{n+1}, v_{n+2}, \ldots, v_{2n-3}, v_{2n-2}\}$ is a dominating set and $\langle S \rangle$ is triple connected. So it forms tcd set. Also it is a minimum tcd set.

Therefore $\gamma_{tc}(D(T_n)) = n-2 \text{ if } n \geq 5$.

Trivially $\gamma_{tc}(D(T_n)) = 3 \text{ if } n \leq 4$.

Thus for any double triangular snake of order $p \geq 3$, $\gamma_{tc}(D(T_n)) = \begin{cases} 3 & \text{if } n \leq 4 \\ n-2 & \text{if } n \geq 5 \end{cases}$

For example, Consider the graph $D(T_4)$.

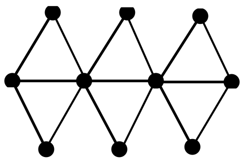

Figure 3.2.3: Double triangular snake graph $D(T_4)$

For the graph $D(T_4)$,

$S = \{v_2, v_5, v_6\}$ is a minimum tcd set. Therefore, $\gamma_{tc}(D(T_4)) = 3$.

Consider the graph $D(T_9)$.

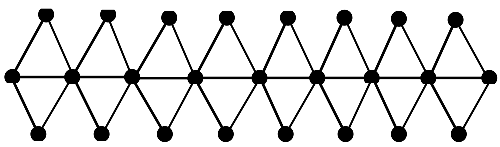

Figure 3.2.4: Double triangular snake graph $D(T_9)$

For the graph $D(T_9)$,

$S = \{v_{10}, v_{11}, v_{12}, v_{13}, v_{14}, v_{15}, v_{16}\}$ is a minimum tcd set. Therefore, $\gamma_{tc}(D(T_9)) = 7 = n-2$.

4. Conclusion:

In this paper, we have discussed about triple connected domination number for triangular snake graphs and double triangular snake graphs. We will find triple connected domination number for some special graphs such as alternate triangular snake graphs, double alternate triangular snake graphs, quadrilateral snake graph, etcin our future work [1].

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