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A MULTICRITERIA GROUP DECISION MAKING USING BIPOLAR HESITANT FUZZY SOFT A-IDEALS IN BCK/BCI ALGEBRAS

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Abstract:

In this paper, we introduce the notion of hesitant set in bipolar fuzzy soft set and studied its relations and properties. Also the notion of bipolar hesitant fuzzy soft subalgebras and soft a-deals is introduced. The relation between the concepts of bipolar hesitant fuzzy set function and soft a-ideals in BCK/BCI algebras are considered. A multicriteria group decision making approach using bipolar hesitant fuzzy soft a-ideal are also proposed.

Key Words: Soft Sets, Fuzzy Soft Sets, Hesitant Fuzzy Sets, Hesitant Fuzzy Soft Sets & Bipolar Hesitant Fuzzy Soft A-Ideal

1. Introduction:

Since the fuzzy set (FS) was proposed by Zadeh in 1965 [1], it has been widely studied, developed, and successfully applied in various fields. Fuzzy. In real MCDM cases, due to the fuzziness and uncertainty of decision making problems, the criteria's weights and evaluation values of alternatives can be inaccurate, uncertain, or incomplete. For the problems like those, FSs, especially fuzzy numbers, can provide good solutions. However, in FSs the membership degree of the element is represented by a single value between zero and one, and a major drawback of FSs is that single values cannot convey information precisely. Maji et al. [6] described the application of soft set theory in a decision making problem. However, in some cases, the membership degree of an element is neither a single value nor an interval, but a set of possible values. To manage such situations where decision makers are hesitant in expressing their preferences over alternatives, hesitant fuzzy sets (HFSs), another extension of traditional FSs, provide a useful reference. HFSs are first introduced by Torra [7, 8] and permit the membership degree of an element to be a set of several possible values between 0 and 1. HFSs are tremendously useful in handling the situations where people have hesitancy in providing their preferences over objects in a decision making process.

Using the notion of bipolar-valued fuzzy sets, Lee [13] discussed bipolar fuzzy subalgebra and ideals of BCK/BCI -algebras. Also, Lee and Jun [13] discuss bipolar fuzzy a -ideals in BCK/BCI--algebras. Jun [11, 12, 13] applied the notion of soft sets to the theory of BCK\BCI-algebras and d-algebras, and introduced the notion of soft BCK/BCI -algebras, soft subalgebras and soft d-algebras, and then described their basic properties. Jun et al. [13] introduced the notion of soft p-ideals and p-ideas of soft BCI -algebras and developed their basic properties. The algebraic structure of set theories dealing with uncertainties has been introduced by some authors. Aktas and Cagman [9] studied the basic concepts of soft set theory, and compared soft sets to fuzzy and rough sets, providing examples to clarify their differences and they defined the notion of fuzzy soft groups. Applications of soft set theory in real life problems are now given new momentum due to the general nature parametrization expressed by soft set. Recently, to the best of our knowledge net works are available on bipolar fuzzy soft sets in BCK/BCI -algebras. For this reason we are motivated to developed the theories on bipolar fuzzy soft sets in BCK/BCI -algebras.

In this paper, we introduced the concept of bipolar hesitant fuzzy soft a-ideals and corresponding operations are given. Its properties are used in analyzing the multicriteria decision making approach.

2. Preliminaries:

Throughout this paper X always denotes a BCI-algebra without any specification.

An Algebra (X,*, 0) of type (2, 0) is said to be a BCI-algebra if it satisfies the following conditions:

(BCI-1): $(\forall\, x,y,z \in X)\left(\left((x*y)*(x*z)\right)*(z*y)=0\right)$,

(BCI-2): $(\forall\, x,y \in X)\left(\left(x*(x*y)\right)*y=0\right)$,

(BCI-3): $(\forall x \in X)(x*x=0)$,

(BCI-4): $(x,y \in X)(x*y=0, y*x=0 \Rightarrow x=y)$.

If a BCI-algebra X Satifies the following identity: $(\forall\in X)(0*x=0)$, then X is called a BCK-algebra.

Proposition 2.1: In a BCI-algebra, the following are true:

1.$(\forall x \in X)(x*0=x)$,

2.$(\forall x,y,z \in X)(x \le y \Rightarrow x*z \le y*z, z*y \le z*x)$,

3.$(\forall x,y,z \in X)\left((x*y)*z\right)=\left(x*(y*z)\right)$.

Definition 2.2: A non-empty subset I of a BCI-algebra X is said to be an ideal of X if for any x ∈ X

(I1) 0 ∈ I

(I2) x*y ∈ I and y ∈ I implies x ∈ I

Definition 2.3: A fuzzy set μ of a universe X is a function from X into the unit closed interval [0, 1].

Definition 2.4: A fuzzy set μ of a BCI-algebra X is said to be a fuzzy ideal of X if it satisfies $\mu(x*y) > \min(\mu(x),\mu(y))$, for all x, y ∈X.

Definition 2.5: A fuzzy set μ of a BCI-algebra X is said to be a fuzzy subalgebra of X if it satisfies (F1) and (F2), where (F1) $\mu(0) > \mu(x)$

(F2) $\mu(x) > \min(\mu(x*y), \mu(y))$, for all x, y ∈X.

Definition 2.6: A bipolar fuzzy set μ of a BCI-algebra X is defined as $\mu = \{(x, \mu^P(x), \mu^N(x)) : x\epsilon X\}$ where $\mu^P : X \to [0,1]$ and $\mu^N : X \to [-1,0]$ are the mappings. The positive membership function $\mu^P(x)$ denote the satisfaction degree of the element x to the property corresponding to the fuzzy set $\mu = \{(x, \mu^P(x), \mu^N(x)) : x\epsilon X\}$ and the negative membership degree $\mu^N(x)$ denotes the satisfaction degree of an element x to some implicit counter part of $\mu = \{(x, \mu^P(x), \mu^N(x)) : x\epsilon X\}$. If $\mu^P(x) \neq 0, \mu^N(x) = 0$, in this regard we have only positive satisfaction degree of $\mu = \{(x, \mu^P(x), \mu^N(x)) : x\epsilon X\}$. If $\mu^P(x) = 0, \mu^N(x) \neq 0$, in this regard we have only the negative satisfaction degree of $\mu = \{(x, \mu^P(x), \mu^N(x)) : x\epsilon X\}$. We use the symbol $\mu = (\mu^P, \mu^N)$ for the bipolar fuzzy set $\mu = \{(x, \mu^P(x), \mu^N(x)) : x\epsilon X\}$.

Definition 2.7: A bipolar fuzzy set $\mu = (\mu^P, \mu^N)$ of a BCI-algebra X is said to be a bipolar fuzzy ideal of X if it satisfies $\mu^P(x * y) \geq \min(\mu x, \mu(y))$ and $\mu N x * y \leq \max(\mu x, \mu(y))$, for all x, y ∈X.

Definition 2.8: A hesitant fuzzy set on a reference set (or an initial universe set) U is defined in terms of a function that when applied to U returns a subset of [0, 1], which can be viewed as the mathematical representation: H={(u, h_H(u) / u ∈U}, where h_H : U→ P([0, 1])

Denote by HF (U) the set of all hesitant fuzzy sets on the reference set (or on a universe set) U.

Definition 2.9: A pair $(\widetilde{H}, A)$ is said to be a hesitant fuzzy soft set over a reference set U, where $\widetilde{H}$ is a mapping given by

$$\widetilde{H} : A \to HF(U)$$

Definition 2.10: Let X be a BCI-algebra. A hesitant fuzzy set, H={(x, h_H(x))/x∈ X} on X is said to be a hesitant fuzzy sub algebra of X if it satisfies

$$(\forall\, x, y \in X)(\, h_H(x*y) \supseteq h_H(x) \cap h_H(y))$$

Definition 2.11: Let X be a BCI-algebra. A hesitant fuzzy set, H={(x, h_H(x))/x∈ X} on X is said to be a hesitant fuzzy ideal of X if it satisfies

$$((\forall\, x, y \in X)(\, h_H(x*y) \cap h_H(y) \subseteq h_H(x) \subseteq h_H(0))$$

Definition 2.12: A hesitant bipolar fuzzy set (BHFSS) μ of a BCH-algebra X is defined as $\mu = \{(x, \mu_H^P(x), \mu_H^N(x)) : x\epsilon X\}$ where $\mu_H^P : X \to [0,1]$ and $\mu_H^N : X \to [-1,0]$ are the mappings. The positive membership function $\mu_H^P(x)$ denote the satisfaction degree of the element x to the property corresponding to the hesitant fuzzy set $\mu = \{(x, \mu_H^P(x), \mu_H^N(x)) : x\epsilon X\}$ and the negative membership degree $\mu_H^N(x)$ denotes the satisfaction degree of an element x to some implicit counter part of $\mu = \mu = \{(x, \mu_H^P(x), \mu_H^N(x)) : x\epsilon X\}$.

Definition 2.13: A pair $(\widetilde{BH}, A)$ is said to be a bipolar hesitant fuzzy soft set over a reference set U, where $\widetilde{BH}$ is a mapping given by

$$B\widetilde{H} : A \to BHF(U).$$

Definition 2.14: Let $\varphi : S \to T$ be a hesitant fuzzy soft homomorphism and B be a bipolar hesitant fuzzy soft set of T. Then the inverse image of B, φ^{-1}(B) is the bipolar hesitant fuzzy soft set of S given by $\varphi\text{-}1(\mu_{BH}^P)(x) = \mu_{BH}^P(\varphi(x))$, $\varphi\text{-}1(\mu_{BH}^N)(x) = \mu_{BH}^N(\varphi(x))$, for all $x \in S$. Conversely, let A be a bipolar fuzzy soft set of S. The image of A, $\varphi(A)$ is bipolar hesitant fuzzy soft set of T defined by

$$\varphi(\mu_{AH}^P)(x) = \begin{cases} \cup\, \mu_{AH}^P(z), \text{if } \varphi - 1(y) \neq \emptyset \\ 0, \quad otherwise. \end{cases}$$

$$\varphi(\mu_{AH}^N)(x) = \begin{cases} \cap\, \mu_{AH}^N(z), \text{if } \varphi - 1(y) \neq \emptyset \\ 0, \quad otherwise. \end{cases} \quad \text{for all } z \in \varphi^{-1}(y)$$

Where $\varphi^{-1}(y)$) =(x∈ $S/\varphi(x) = y\}$.

3. Bipolar Hesitant Fuzzy Soft a-Ideals:

Let X denotes a BCI-algebra unless otherwise specified.

Definition 3.1: A Bipolar hesitant fuzzy soft set $\mu = \{(x, \mu_H^P(x), \mu_H^N(x)) : x\epsilon X\}$ is called a bipolar hesitant fuzzy soft ideal of X if it satisfies

$$\text{(BF1)} \quad \mu_H^P(0) \supseteq \mu_H^P(x) \quad \text{and} \quad \mu_H^N(0) \subseteq \mu_H^N(x) \quad ;$$

$$\text{(BF2)} \quad \mu_H^P(x) \supseteq \{\mu_H^P(x*y) \cap \mu_H^P(y)\} \quad \text{and} \quad \mu_H^N(x) \subseteq \{\mu_H^N(x*y) \cup \mu_H^N(y)\}$$

Definition 3.2: A Bipolar hesitant fuzzy soft set $\mu = \{(x, \mu_H^P(x), \mu_H^N(x)) : x\epsilon X\}$ is called a bipolar hesitant fuzzy soft a-ideal of X if it satisfies

$$\text{(BFS1)} \quad \mu_H^P(y*x) \supseteq \{\mu_H^P((x*z)*(0*y)) \cap \mu_H^P(z)\}$$

$$\text{(BFS2)} \quad \mu_H^N(y*x) \subseteq \{\mu_H^N((x*z)*(0*y)) \cup \mu_H^N(z)\}$$

Example 3.3: Consider a BCI-algebra X= {0, a, b, c} with the following cayley table:

*	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

Define a bipolar hesitant fuzzy soft set $\mu = \left\{\left(x, \mu_H^P(x), \mu_H^N(x)\right) : x\epsilon X\right\}$ by

X	0	a	b	c
μ_H^N	(-0.8,-0.5)	(-0.5, -0.4)	(-0.6, -0.3)	(-0.6,-0.1)
μ_H^P	(0.4, 0.9)	(0.3,0.5)	(0.5,0.8)	(0.6,0.7)

Proposition 3.4 If $\mu = \left\{\left(x, \mu_H^P(x), \mu_H^N(x)\right) : x\epsilon X\right\}$ is a bipolar fuzzy soft a-ideal of X, then $\mu_H^N(x) = \mu_H^N(0 * x)$ and $\mu_H^P(x) = \mu_H^P(0 * x)$ for all $x\epsilon X$.

Proof: Let $\mu = \left\{\left(x, \mu_H^P(x), \mu_H^N(x)\right) : x\epsilon X\right\}$ be a bipolar hesitant fuzzy soft a-ideal of X. Taking y=z=0 in (BFS1 and BFS2) and using the definition of BCI algebra, we get

$$\mu_H^N(0 * x) \subseteq \mu_H^N(x) \text{ and } \mu_H^P(0 * x) \supseteq \mu_H^P(x)$$

Setting x=z=0 in (BFS1 and BFS2) and using the definition of BCI algebra, we have

$$\mu_H^N(y) = \mu_H^N(y * 0) \subseteq \mu_H^N(0 * (0 * y)) \subseteq \mu_H^N(0 * y) \text{ and}$$

$$\mu_H^P(y) = \mu_H^P(y * 0) \supseteq \mu_H^P(0 * (0 * y)) \supseteq \mu_H^P(0 * y) \quad \text{for all } y\epsilon X.$$

Hence $\mu_H^N(x) = \mu_H^N(0 * x)$ and $\mu_H^P(x) = \mu_H^P(0 * x)$ for all $x\epsilon X$.

Hence the Proof

Theorem 3.5 Every bipolar hesitant fuzzy soft a-ideal of X is both a bipolar fuzzy subalgebra of X and a bipolar fuzzy ideal of X.

Proof: Let $\mu = \left\{\left(x, \mu_H^P(x), \mu_H^N(x)\right) : x\epsilon X\right\}$ be a bipolar hesitant fuzzy soft a-ideal of X. Using the definition (BFS1 and BFS2) and using the definition of BCI algebra, Proposition 3.4, we have

$$\mu_H^N(x) = \mu_H^N(0 * x) \subseteq \{\mu_H^N((x * z) * (0 * 0)) \cup \mu_H^N(z)\}$$
$$= \{\mu_H^N(x * z) \cup \mu_H^N(z)\} \qquad \text{and}$$
$$\mu_H^P(x) = \mu_H^P(0 * x) \supseteq \{\mu_H^P((x * z) * (0 * 0)) \cap \mu_H^P(z)\}$$
$$= \{\mu_H^P(x * z) \cap \mu_H^P(z)\} \qquad \text{for all } x,y\epsilon X.$$

Hence $\mu = \left\{\left(x, \mu_H^P(x), \mu_H^N(x)\right) : x\epsilon X\right\}$ is a bipolar hesitant fuzzy ideal of X.

Now for any x, y, zϵ X, we obtain

$$\mu_H^N(x * y) \subseteq \{\mu_H^N((x * y) * x) \cup \mu_H^N(x)\} = \{\mu_H^N(0 * y) \cup \mu_H^N(x)\}$$
$$= \{\mu_H^N(x) \cup \mu_H^N(y)\}$$
$$\mu_H^P(x * y) \supseteq .\{\mu_H^P(x * y) * x) \cap \mu_H^P(x)\} = \{\mu_H^P(0 * y) \cap \mu_H^P(x)\}$$
$$= \{\mu_H^P(x) \cap \mu_H^P(y)\}.$$

Hence $\mu = \left\{\left(x, \mu_H^P(x), \mu_H^N(x)\right) : x\epsilon X\right\}$ is a bipolar hesitant fuzzy subalgebra of X.

Hence the Proof

Example 3.6 Consider a BCI –algebra X = {0, 1, a, b, c} with the following cayley table:

*	0	1	a	b	c
0	0	0	a	b	c
1	1	0	a	b	c
a	a	a	0	c	b
b	b	b	c	0	a
c	c	c	b	a	0

Define a bipolar fuzzy set $\mu = \left\{\left(x, \mu_H^P(x), \mu_H^N(x)\right) : x\epsilon X\right\}$ in X by

X	0	1	a	b	c
μ_H^N	[-0.7, -0.5]	[-0.6, -0.4]	[-0.9,-0.7]	(-0.5, -0.4)	{-0.3}
μ_H^P	[0.9. 0.7]	(0.8, 0.7)	(0.6, 0.4)	(0.9, 0.65]	[0.4, 0.75]

Then $\mu = \left\{\left(x, \mu_H^P(x), \mu_H^N(x)\right) : x\epsilon X\right\}$ is not a bipolar hesitant fuzzy a-ideal of X since

$$\mu_H^N(b*a)=\{-0.3\} \nsubseteq [-0.9, -0.7] \cup (-0.5, -0.4)=\{\mu_H^N((a*b)*(0*b)), \mu_H^N(b)\}$$

Proposition 3.7 Let φ=(X; μ_H^N , μ_H^P) be a hesitant bipolar fuzzy ideal of X . If the inequality $x*y \leq z$ holds in X, then

$$\mu_H^N(x) \subseteq \max\{\mu_H^N(y), \mu_H^N(z)\},$$
$$\mu_H^P(x) \supseteq \min\{(\mu_H^P(y), \mu_H^P(z)\}.$$

Now we extend this condition to hesitant bipolar fuzzy soft a-ideal.

Theorem 3.8 Let φ=(X; μ_H^N , μ_H^P) be a hesitant bipolar fuzzy ideal of X .Then the following are equivalent:

- ✓ φ=(X; μ_H^N , μ_H^P) is a hesitant bipolar fuzzy soft a-ideal of X.
- ✓ φ=(X; μ_H^N , μ_H^P) satisfies the following assertions:

$\mu_H^N(y * (x * z)) \subseteq \mu_H^N((x * z) * (0 * y))$,

$\mu_H^P(y * (x * z)) \supseteq \mu_H^P((x * z) * (0 * y))$ for all x, y, z ∈ X.

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✓ φ=(X; μ_H^N , μ_H^P) satisfies the following assertions:

$$\mu_H^N(y*x) \subseteq \mu_H^N(x*(0*y)),$$
$$\mu_H^P(y*x) \supseteq \mu_H^P(x*(0*y)) \text{ for all } x, y \in X.$$

Proof: Assume that φ =(X; μ_H^N , μ_H^P) be a hesitant bipolar fuzzy soft a- ideal of X and let x, y, z ∈ X.

Using definition 3.2 and proposition 3.4, we get

$$\mu_H^N(y*(x*z)) \subseteq \max\{\mu_H^N(((x*z)*0)*(0*y)), \mu_H^N(0)\}$$
$$= \mu_H^N((x*z)*(0*y))$$

And $$\mu_H^P(y*(x*z)) \supseteq \min\{\mu_H^P(((x*z)*0)*(0*y)), \mu_H^P(0)\}$$
$$= \mu_H^P((x*z)*(0*y))$$

Which proves (2).

Taking z=0 in (2) and using proposition 2.1 induces (3). Suppose that (3) is valid. Note that

$$(x*(0*y))*((x*z)*(0*y)) \le x*(x*z) \le z \text{ for all } x, y, z \in X.$$

It follows from (3) and Proposition 3.7 that

$$\mu_H^N(y*x) \subseteq \mu_H^N(x*(0*y)) \subseteq \min\{\mu_H^N((x*z)*(0*y)), \mu_H^N(z)\}$$

And $$\mu_H^P(y*x) \supseteq \mu_H^P(x*(0*y)) \supseteq \max\{\mu_H^P((x*z)*(0*y)), \mu_H^P(z)\}$$

Hence φ =(X; μ_H^N , μ_H^P) be a hesitant bipolar fuzzy soft a- ideal of X.

Theorem 3.9 Let $\varphi : S \rightarrow T$ be a hesitant fuzzy soft homomorphism and B be a bipolar hesitant fuzzy soft set of T. Then the inverse image of B, φ^{-1}(B) is the bipolar hesitant fuzzy soft set of S.

Proof: Suppose that B = (μ_{BH}^P, μ_{BH}^N) is a bipolar hesitant fuzzy soft a-ideal of T and φ is a hesitant fuzzy soft homomorphism from S to T. Then for all x, y ∈S, we have

(BHFSaI$_1$) $$\varphi^{-1}(\mu_{BH}^P)(x+y) = \mu_{BH}^P(\varphi(x+y))$$
$$= \mu_{BH}^P(\varphi(x) + \varphi(y))$$
$$\supseteq \min\{(\mu_{BH}^P(\varphi(x)), \mu_{BH}^P(\varphi(y))\}$$
$$\supseteq \min\{\varphi^{-1}(\mu_{BH}^P)(x), \varphi^{-1}(\mu_{BH}^P)(y)\}.$$

(BHFSaI$_2$) $$\varphi^{-1}(\mu_{BH}^N)(x+y) = \mu_{BH}^N(\varphi(x+y))$$
$$= \mu_{BH}^N(\varphi(x) + \varphi(y))$$
$$\subseteq \max\{(\mu_{BH}^N(\varphi(x)), \mu_{BH}^N(\varphi(y))\}$$
$$\subseteq \max\{\varphi^{-1}(\mu_{BH}^N)(x), \varphi^{-1}(\mu_{BH}^N)(y)\}.$$

Hence the proof

Theorem 3.10 Let A = (μ_{BH}^P, μ_{BH}^N) and B = (μ_{BH}^P, μ_{BH}^N) be two bipolar hesitant fuzzy soft a-ideal of X. If one is contained in another, then their union A∪B is a bipolar hesitant fuzzy soft a-ideal of X.

Proof: Let A=(X; μ_{AH}^N , μ_{AH}^P) and B=(X; μ_{BH}^N , μ_{BH}^P) be two bipolar hesitant fuzzy soft a-ideal of X satisfies the following assertions:

$$\mu_H^P(0) \supseteq \mu_H^P(x) \text{ and } \mu_H^N(0) \subseteq \mu_H^N(x)$$

We have

$$\mu_{AH}^N(y*(x*z)) \subseteq \mu_{AH}^N((x*z)*(0*y)),$$
$$\mu_{AH}^P(y*(x*z)) \supseteq \mu_{AH}^P((x*z)*(0*y)) \text{ and}$$
$$\mu_{BH}^N(y*(x*z)) \subseteq \mu_{BH}^N((x*z)*(0*y)),$$
$$\mu_{BH}^P(y*(x*z)) \supseteq \mu_{BH}^P((x*z)*(0*y)) \text{ for all } x, y, z \in X.$$

If A⊆B

$$\mu_{(A\cup B)H}^N(y*(x*z)) \subseteq \mu_{(A\cup B)H}^N((x*z)*(0*y)),$$
$$\mu_{(A\cup B)H}^P(y*(x*z)) \supseteq \mu_{(A\cup B)H}^P((x*z)*(0*y)) \text{ for all } x, y, z \in X.$$

Their union A∪B is a bipolar hesitant fuzzy soft a-ideal of X.

Theorem 3.11 Let A = (μ_{BH}^P, μ_{BH}^N) and B = (μ_{BH}^P, μ_{BH}^N) be two bipolar hesitant fuzzy soft a-ideal of X. Then their intersection A∩B is a bipolar hesitant fuzzy soft a-ideal of X.

Proof: Let A=(X; μ_{AH}^N , μ_{AH}^P) and B=(X; μ_{BH}^N , μ_{BH}^P) be two bipolar hesitant fuzzy soft a-ideal of X satisfies the following assertions:

$$\mu_H^P(0) \supseteq \mu_H^P(x) \text{ and } \mu_H^N(0) \subseteq \mu_H^N(x)$$

We have

$$\mu_{AH}^N(y*(x*z)) \subseteq \mu_{AH}^N((x*z)*(0*y)),$$
$$\mu_{AH}^P(y*(x*z)) \supseteq \mu_{AH}^P((x*z)*(0*y)) \text{ and}$$
$$\mu_{BH}^N(y*(x*z)) \subseteq \mu_{BH}^N((x*z)*(0*y)),$$
$$\mu_{BH}^P(y*(x*z)) \supseteq \mu_{BH}^P((x*z)*(0*y)) \text{ for all } x, y, z \in X.$$

We get

$$\mu^{N}_{(A\cap B)H}(y*(x*z)) \subseteq \mu^{N}_{(A\cap B)H}((x*z)*(0*y)),$$
$$\mu^{P}_{(A\cap B)H}(y*(x*z)) \supseteq \mu^{P}_{(A\cap B)H}((x*z)*(0*y)) \text{ for all } x, y, z \in X.$$

Their intersection A∩B is a bipolar hesitant fuzzy soft a-ideal of X.

4. An Application of Bipolar Hesitant Fuzzy Soft A-Ideals:

Bipolar fuzzy soft set has several applications to deal with uncertainties from our different kinds of daily life problems. Here, we discuss such an application for solving a socialistic decision making problem. We apply the concept of bipolar hesitant fuzzy soft set for modelling of a socialistic decision making problem and then we give an algorithm for the choice of optimal object based upon the available sets of information.

Suppose that U = {c_1, c_2, c_3, c_4} be the set of four cars under consideration say U is an initial universe and E = {e_1 =costly, e_2 =Beautiful, e_3 =Fuel efficient, e_4 =Modern technology, e_5 =Luxurious} be a set of parameters.

Suppose a man Mr. X is going to buy a car on the basis of his wishing parameter among the listed above. Our aim is to find out the attractive car for Mr. X.

Suppose the wishing parameters of Mr. X be A ⊂ E where A = {e_1, e_4, e_5}.Consider the bipolar hesitant fuzzy soft set as below.

F (e_1) = {(c_1 , (0.8, 0.7), (-0.5, -0.4)), (c_2 , (0.8, 0.1), (-0.4, -0.3, -0.2)),
(c_3, (0.9,0.7), (-0.5, -0.1)), (c_4 , (0.8, 0.8, 0.5),(-0.8, -0.6))}.

F (e_4) = {(c_1 , (0.7, 0.65), (-0.5, -0.46)), (c_2 , (0.4, 0.7), (-0.45, -0.2)),
(c_3, (0.85, 0.6), (-0.5, -0.1)), (c_4 , (0.8, 0.8),(-0.7, -0.6))}.

F (e_5) = {(c_1 , (0.8, 0.7), (-0.5, -0.4)), (c_2 , (0.6, 0.2), (-0.4, -0.3, -0.2)),
(c_3, (0.9, 0.2), (-0.5, -0.1)), (c_4 , (0.9, 0.6),(-0.8, -0.6))}.

Definition 4.1 For a hesitant fuzzy element $h(x)$,
S (h) = $\frac{1}{l(h)}\sum\gamma$ is called the score function of h where $l(h)(x)$ be the number of values in h(x).

Definition 4.2 Let (F, A) denotes hesitant soft set. Then the fuzzy soft set (F_s, A) in which each entries in the fuzzy element $F_s(e)$ is the score function of the respective entries in the hesitant fuzzy set $F(e)$ is called the score matrix.

Definition 4.3 The table obtained by calculating $F_s(e)$ for each x_j is called the decision table. This table determines the optimal outcome for the decision making problem. Now we will propose an algorithm which is shown by considering score matrix, a bipolar hesitant fuzzy soft based decision making problem can be reduced into simpler bipolar fuzzy soft set.

Definition 4.4. (Comparison table). It is a square table in which number of rows and number of columns are equal and both are labeled by the object name of the universe such as c_1,c_2,c_3,c_4 and the entries dij where dij = the number of parameters for which the value of di exceeds or equal to the value of dj .

Algorithm 4.5

Step 1: Input the set A ∈ E of choice of parameters of Mr. X.

Step 2: Consider the bipolar hesitant fuzzy soft set in tabular form.

Step 3: Obtain the score matrix (F_s, A) corresponds to both positive membership function and negative membership function by S (h) = $\frac{1}{l(h)}\sum\gamma$

Step 4: Compute $F_s(e)$

Step 5: Compute the comparison table of positive information function and negative information function.

Step 6: Compute the positive information score and negative information score.

Step 7: Compute the final score by averaging positive information score and negative information score.

.	e_1	e_4	e_5
c_1	0.75	0.675	0.75
c_2	0.45	0.55	0.4
c_3	0.8	0.725	0.55
c_4	0.7	0.8	0.75

.	e_1	e_4	e_5
c_1	-0.45	-0.48	-0.45
c_2	-0.3	-0.32	-0.3
c_3	-0.3	-0.3	-0.3
c_4	-0.7	-0.65	-0.7

Table 1: Representation of Positive Membership and negative membership function

.	c_1	c_2	c_3	c_4
c_1	3	3	1	2
c_2	0	3	0	0
c_3	2	3	3	1
c_4	1	3	2	3

.	c_1	c_2	c_3	c_4
c_1	3	0	0	3
c_2	3	3	3	3
c_3	3	2	3	3
c_4	0	0	0	3

Table 2: Comparison table of Positive Membership and negative membership function

.	Row sum(c)	Column sum(d)	Non-membership score(c-d)
c_1	9	6	3
c_2	3	12	-9
c_3	9	6	3
c_4	9	6	3

.	Row sum(c)	Column sum(d)	Non-membership score(c-d)
c_1	6	9	-3
c_2	12	5	7
c_3	11	6	5
c_4	3	9	-6

Table 3: Final Score table

.	Positive Information Score(P)	Negative information Score(N)	Final Score((P+N)/2)
c_1	3	-3	0
c_2	-9	7	-1
c_3	3	5	4
c_4	3	-6	-1.5

Finally the maximum score is 4 scored by car c_3.

Mr. X will buy c_3. If he does not want to buy c_3 due to certain reason, his second choice will be c_1.

5. Conclusion:

We establish bipolar hesitant fuzzy soft a-ideal and discuss the correspondence between the hesitant fuzzy soft sets and soft a-ideals in BCK/BCI algebras. We define some operations in bipolar hesitant fuzzy soft a-ideal and its applications in multicriteria decision making problem

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