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ON NS-TOPOLOGICAL SPACES

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Abstract:

In the task of solving complicated problems in engineering, physics, computer science and operation research, the classical mathematical theories are not adequate. To overcome these problems many new sets were introduced by our mathematician like soft sets, fuzzy sets and rough sets. The theory of SN-topological spaces is investigated in this paper. This paper deals some results on SN-topology.

Key Words: Soft Set & NS-Topology

1. Introduction:

In 1999, Molodtsov [1] introduced soft set as a mathematical tool for dealing with uncertainty. It is getting popularity among the researchers and a good number of papers is being published very year. Soft set gives an aggrandize development in problem solving of uncertainty and imprecise problems at different levels. In 1982, Pawlak [2] introduced the concept of rough sets. Rough sets were made by lower and upper approximations. Equivalence relations are the bricks of the above approximations. In 2010, Feng et al., [3] proposed the soft rough set theory as a new mathematical tool for dealing with uncertainties which is free from the difficulties affecting existing methods. This set propagates many algorithms for attribute reductions. L. Thivagar [1] introduced the concept of nano topological spaces with respect to a subset X of a universe U. This topology contains five elements only. Because of its size it got its name as nano. In this paper we introduced new topology named as NS-topology that is nano soft topology. The blossom of new topology makes indiscernibility and binary indiscernibility in same line. The paper is organized as follows: section 2 contains the preliminaries; section 3 comprises the definitions of NS-interior, NS-closure and so on.

2. Preliminaries:

This section provides the needed existing definitions and result that give the correct attention to this work.

Definition 2.2 [1]: Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space.

Let U $\subseteq$X.

- ✓ The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and its is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup_{x\in U}\{R(x) : R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by x.
- ✓ The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by U_R (X). That is, $U_R(X) = \bigcup_{x\in U}\{R(x) / R(x) \cap X \neq \phi\}$
- ✓ The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not-X with respect to R and it is denoted by B_R(X). That is, $B_R(X) = U_R(X) - L_R(X)$.

If B_R(X) is non empty then X is said to be rough otherwise it is crisp.

Definition 2.4 [3]: Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{\phi, U, L_R(X), U_R(X), B_R(X)\}$ where X $\subseteq$U . The collection $\tau_R(X)$ is said to be nanotopology on U with respect to X satisfies the following axioms:

- ✓ $\phi, U \in \tau_R(X)$
- ✓ The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$.
- ✓ The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

We call, (U, $\tau_R(X)$) as a nano topological space and the elements of $\tau_R(X)$ are called as nano-open sets.

Definition 2.1 [1]: A pair S = (F, A) is called a soft set over U, where A$\subseteq$ E and F : A $\rightarrow$ P(U) is a set-valued mapping.

Definition 2.3 [11]: Let (U,R) be a Pawlak approximation space and ς =(F,A) be a soft set over U. The lower and upper approximations of ς in (U, R) are denoted by $R_*(\varsigma) = (F_*, A)$ and $R^*(\varsigma) = (F^*, A)$, which are soft sets over U defined by:

$$F_*(x) = \{y \in U ; [y]_R \subseteq F(x)\}$$

$$F^*(x) = \{y \in U ; [y]_R \cap F(x) \neq \phi\}$$

for all $x \in A$. The operators R^* and R_* are called the lower and upper rough approximation operators on soft sets. If $R^*(\varsigma) = R_*(\varsigma)$, the soft set ς is said to be definable; otherwise ς is called a rough soft set.

3. On NS-Topological Spaces:

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This part contains the introduction of new topology and its related definitions.

Definition 3.1:

Let U be a universe set, R be an equivalence relation on U. Let (U,R) be a pawlak approximation space, ς = (F, A) be a soft set over U and $\tilde{\tau}_R(X) = \{\phi, U, F_*(X), F^*(X), FB(X)\}$ where X $\subseteq$U.

$\tilde{\tau}_R(X)$ Satisfies the following:

- ✓ U and ϕ are in $\tilde{\tau}_R(X)$.
- ✓ The union of the elements of any sub collection of $\tilde{\tau}_R(X)$ is in $\tilde{\tau}_R(X)$.
- ✓ The intersection of the elements of any finite sub collection of $\tilde{\tau}_R(X)$ is in $\tilde{\tau}_R(X)$.

That is, $\tilde{\tau}_R(X)$ forms a topology on U called the NS- topology on U with respect to X. We call (U, $\tilde{\tau}_R(X)$) as the NS-topological space. The elements of $\tilde{\tau}_R(X)$ are called NS-open sets.

Example 3.2:

Let U be the set of all houses in one apartment. The buyers want to select the house based their own expectations. Using soft set we can categorize the houses by some conditional attributes. Let

U ={h_1,h_2,h_3,h_4,h_5,h_6}, U\R ={{h_1,h_3},{h_2,h_4},{h_5},{h_6}}, the discernible relation on U. Let E = {e_1, e_2,e_3, e_4, e_5} and A={e_1,e_2,e_3}. Then (F,A) is soft by F(e_1) = {h_1,h_2}, F(e_2) = {h_3,h_4}, F(e_3) = {h_3,h_4,h_5}, F(e_4) = {h_5,h_6} and F(e_5) = {h_1,h_3}. And (F,A) = {(e1,{h1,h2,h5}),(e2,{h3,h4,h6}),(e3,{h2,h5,h6}}. Then

$$\tilde{\tau}_R(X) = \{U, \tilde{\phi}, F^*(X), F_*(X), FB(X)\} = \{U, \tilde{\phi}, \{h_1, h_2, h_3, h_4, h_5\}, \{h_1, h_3, h_5\}, \{h_2, h_4\}\}$$

Theorem 3.3:

Let U be a non empty, finite universe and X $\subseteq$U. Let $\tilde{\tau}_R(X)$ be the NS-topology on U with respect to X. Then [$\tilde{\tau}_R(X)$]^C whose elements are A^C for A $\in \tilde{\tau}_R(X)$, is a NS-topology on U.

Definition 3.4:

Let (U, $\tilde{\tau}_R(X)$) be a NS-topological space with respect to *X*, where $X \subseteq U$ and if $x \in A$ and $F_A(x) \subseteq U$, then The SN-interior of the set $F_A(x)$ is defined as the union of all SN-open subsets contained in *A* , and is denoted by *SNint* ($F_A(x)$). That is, *SNint* ($F_A(x)$) is the largest SN-open subset $F_A(x)$.

Theorem 3.4:

Let (U, $\tilde{\tau}_R(X)$) be a NS-topological space with respect to *X*, where $X \subseteq U$. Let $F_A(x)$, $G_A(x) \subseteq U$. Then

- ✓ SNint ($F_A(x)$) $\subseteq F_A(x)$
- ✓ $F_A(x)$ is SN-open if and only if $F_A(x)$ = SNint ($F_A(x)$).
- ✓ SNint (ϕ) = ϕ and SNint (U) = U.
- ✓ $F_A(x) \subseteq F_B(x)$ $\Rightarrow$ SNint ($F_A(x)$) $\subseteq$ SNint ($F_B(x)$)
- ✓ SNint ($F_A(x) \cup F_B(x)$) $\supseteq$ SNint ($F_A(x)$) $\cup$ SNint ($F_B(x)$)
- ✓ SNint ($F_A(x) \cap F_B(x)$) = SNint ($F_A(x)$) $\cap$ SNint ($F_B(x)$)
- ✓ SNint(SNint ($F_A(x)$) = SNint($F_A(x)$)

Proof:

Obvious

Definition 3.4:

Let (U, $\tilde{\tau}_R(X)$) be a NS-topological space with respect to *X*, where $X \subseteq U$ and if $x \in A$ and $F_A(x) \subseteq U$, then The SN-closure of the set $F_A(x)$ is defined as the union of all SN-closed subsets contained in *A* , and is denoted by *SNcl* (*A*).

Theorem 3.5:

Let (U, $\tilde{\tau}_R(X)$) be a NS-topological space with respect to *X*, where $X \subseteq U$ and if $x \in A$ and $F_A(x) \subseteq U$. Then

- ✓ U - SNint($F_A(x)$) = *SNcl* (U-$F_A(x)$).
- ✓ U - SNcl($F_A(x)$) = *SNint*(U-$F_A(x)$).

Theorem 3.4:

Let (U, $\tilde{\tau}_R(X)$) be a NS-topological space with respect to *X*, where $X \subseteq U$. Let $F_A(x)$, $G_A(x) \subseteq U$. Then

- ✓ *SNcl* ($F_A(x)$) $\subseteq F_A(x)$
- ✓ $F_A(x)$ is SN-open if and only if $F_A(x)$ = *SNcl* ($F_A(x)$).
- ✓ *SNcl* (ϕ) = ϕ and *SNcl* (U) = U.
- ✓ $F_A(x) \subseteq F_B(x)$ $\Rightarrow$ *SNcl* ($F_A(x)$) $\subseteq$ *SNcl* ($F_B(x)$)
- ✓ *SNcl* ($F_A(x) \cup F_B(x)$) $\supseteq$ *SNcl* ($F_A(x)$) $\cup$ *SNcl* ($F_B(x)$)
- ✓ *SNcl* ($F_A(x) \cap F_B(x)$) = *SNcl* ($F_A(x)$) $\cap$ *SNcl* ($F_B(x)$)
- ✓ *SNcl(SNcl* ($F_A(x)$) = SNcl($F_A(x)$)

Proof:

From the definitions of *SNint*, *SNcl* and previous theorem

Definition 3.5:

Let $(U, \tilde{\tau}_R(X))$ be a NS-topological space with respect to *X*, where $X \subseteq U$. Let $F_A(x) \subseteq U$ Then the NS-boundary of $F_A(x)$ is denoted by $bF_A(x)$ and is defined as $SNbF_A(x) = SNcl(F_A(x)) \cap (SNcl(F_A(x)))^C$.

Definition 3.6:

Let $(U, \tilde{\tau}_R(X))$ be a NS-topological space with respect to *X*, where $X \subseteq U$. Let $F_B(x) \subseteq U$. If every NS nbd of $\alpha \in F_B(x)$ *intersects* F_B in some points other than α itself, then α is called a NS- limit point of F_B. The set of all NS- limit points of F_B is denoted by $(NSF_B)'$.

Definition 3.7:

Let $(U, \tilde{\tau}_R(X))$ be a NS-topological space with respect to *X*, where $X \subseteq U$. Let $F_B(x) \subseteq U$. If there is a NS-open $F_B(x)$ *such that* $\alpha \in F_B(x)$ then $F_B(x)$ is called a NS-open neighborhood (NS-nbd) of α. The set of all NS-nbds of α, denoted NS $(\nu(\alpha))$, is called the family of NS-nbds of α.

Proposition 3.8:

Let $(U, \tilde{\tau}_R(X))$ be a NS-topological space with respect to *X*, where $X \subseteq U$. Let $F_A(x), F_B(x) \subseteq U$. Then, the collection of NS-nbd $NSb(\nu(\alpha))$, at α in $(U, \tilde{\tau}_R(X))$ has the following properties:

- ✓ If $F_A(x) \in NS(\nu(\alpha))$, then $\alpha \in F_A$.
- ✓ If $F_A(x), F_C(x) \in NS(\nu(\alpha))$, then $F_A(x) \cap F_B(x) \in NS(\nu(\alpha))$.
- ✓ If $F_A(x) \in NS(\nu(\alpha))$ and $F_A(x) \subseteq FC$, then $F_B(x) \in NS(\nu(\alpha))$.
- ✓ If $F_A(x) \in NS(\nu(\alpha))$, then there is an $F_B(x) \in NS(\nu(\alpha))$ such that $F_A(x) \in NS(\nu(\beta))$, for each $\beta \in F_{B(x)}$

Proof:

Obvious

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