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ENERGY OF A CYCLE AND ITS MIDDLE GRAPH

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Abstract:

Let G be a finite, undirected and simple graph. If $\{v_{1,}v_2 \dots v_n\}$ is the set of vertices of G, then the adjacency matrix A(G)=$[a_{ij}]$ is an n-by-n matrix where a_{ij}=1 if v_i and v_j are adjacent and a_{ij}=0 otherwise. The energy of a graph ,E(G) is defined as the sum of the absolute values of Eigen values of A(G).Several classes of graphs are known that satisfy the condition E(G)>n ,where n is the number of vertices. This paper contains the energy of the middle graph of the cycle. This paper shows that, if G be a cycle with n vertices and n edges then its middle graph has 2n vertices and 3n edges and the energy of middle graph of c_n is approximately equal to 3n.

Key Words: Regular Graphs, Middle Graphs, Adjacency Matrix, Eigen Values, Cycle & Energy of a Graph

1. Introduction:

Graph theory is one of the most developing branches of mathematics with wide applications to computer science. Graph theory is applied in diverse areas such as chemistry, social science, communication engineering and others. In this paper, all graphs are assumed to be finite, undirected and without loops or multiple edges (i.e.) simple graphs. Let its vertices be $v_1, v_2, \dots v_n$ then the adjacency matrix of G is the sequence matrix A(G) of order n, whose (i,j) entry is defined as

$$a_{ij} = \begin{cases} 0, \text{ if } i=j \\ 1, \text{ if the vertices } v_i \text{ and } v_j \text{ are adjacent} \\ 0, \text{ if the vertices } v_i \text{ and } v_j \text{ are not adjacent} \end{cases}$$

The characteristic polynomial set (XI-A) of the adjacency matrix A(G) is called the characteristic polynomial of G and it is denoted by $P_G(x)$ and the Eigen values $\lambda_1, \lambda_2, \dots \lambda_n$ of A (G) form the spectrum of the graph G. If λ is an eigen value of G then a non zero vector $x \in R^n$ satisfying $AX = \lambda X$ is called an eigen value of A for λ. The energy of the graph G is a spectrum-based graph invariant, defined as

$$E=E(G)= \sum_{i=1}^{n} |\lambda_i|$$

This notion was introduced by Ivan Gutman.[3] and the laplacian energy LE(G) of G is defined as the sum of the distance between Laplacian Eigen values of G and the average degree d(G) of G. A graph G is singular if the adjacency matrix,A(G) is a singular matrix;(i.e.) zero is the eigen value of G[5].From [4] we obtained

$$\sum\{\lambda_i : \lambda_i > 0\} = -\sum\{\lambda_i : \lambda_i < 0\} = \frac{E(G)}{2}$$

The concept of graph theory arose in chemistry where certain numerical quantities, such as the heat of formation of hydrocarbon, are related to Π-electron energy that can be calculated as the energy of appropriate "molecule" [2]. The concept of middle graph was introduced by T. Hamada and I. Yoshimura in 1974[6]. Here I derived the energy of middle graph of c_n is approximately equal to its number of edges (i.e.) 3n.

2. Preliminaries:

Definition 1: A linear graph (or) a graph G=(V,E) consists of a set of objects V=$\{v_1, v_2, \dots v_n\}$ called vertices and another set E=$\{e_1, e_2, \dots e_n\}$ whose elements are called edges, such that each e_k, is identified with an unordered pair (v_i, v_j) of vertices.

Definition 2: The vertices v_i, v_j associated with each edge e_k are called end vertices of e_k.

Definition 3: A graph that has neither self loops nor parallel edges is called a simple graph

Definition 4: A graph in which all vertices are of equal degree is called a regular graph.

Definition 5: The energy of a graph G, denoted by E(G), is defined as the sum of the absolute values of its eigen values (i.e.) E(G)=$\sum_{i=1}^{n} |\lambda_i|$ where $\lambda_1, \lambda_2, \dots \lambda_n$ are eigen values of G.

Definition 6: The number of edges incident on a vertex v_i with self-loops counted twice, is called the degree $d(v_i)$of a vertex v_i.

Definition 7: A vertex of degree one is called a pendent vertex or an end vertex.

In this paper we have to find the energy of the middle graphs of c_n. Therefore, for a middle graph an adjacency matrix is formed and from that matrix the eigen value of the middle graph is calculated. From these eigen values, the energy of the middle graph of c_n is calculated.

2.1 Previous Relations:

(i) Let $T_n, S_n \, and \, P_n$ be tree, a star and a path on n vertices, respectively. In [1] it is proved E(S_n) < E(T_n) < E(P_n) for $T_n \neq S_n, P_n$.

(ii) When some edges are added, the energy of new graph increased relative to the original one E($J_{2,m}$) < E (W_n) (7)

(iii) Let G be the graph with n vertices and m ≥ 1 edges [8].Then E (M_G) ≤ $2\sqrt{2mn}$ + 4m – 2

Theorem 2.1.1:

The sum of the degrees of the vertices of a graph is equal to twice the number of its edges.

Proof:

If e =uv is an edge of G, e is counted once while counting the degrees of each of u and v (even when u = v). Hence each edge contributes two to the sum of the sum of the degrees of the vertices. Thus the m edges of G contribute 2m to the degree sum.

3. Energy of a Middle Graph of a Cycle:

Definition 1: A walk is a finite alternating sequence of vertices and edges, beginning and ending with vertices, such that each edge is incident with the vertices preceding and following it. No edge appears more than once in a walk.

Definition 2: A closed walk in which no vertex (except the initial and the final vertex) appears more than once is called a cycle.

Definition 3: The middle graph M(G) of a graph G is the graph whose vertex set is V(G)∪E(G) and in which two vertices are adjacent iff either they are adjacent edges of G or one is a vertex of G and the other is an edge incident with it.

In this paper we take all graphs as simple 2- regular graphs. Middle graph of a graph G, denoted by M(G) is obtained by inserting a new vertex into every edge of G.

Theorem 3.1.1:

If G be a cycle with n vertices and n edges then its middle graph has 2n vertices and 3n edges.

Proof:

Let G be a cycle with n vertices and n edges. By definition of middle graph, whose vertex set is V(G) ∪ E(G) (i.e.) the number of vertices of M(G) is n + n = 2n. In a middle graph, the two vertices are adjacent iff either they are adjacent edges of G or one is a vertex of G and the other is an edge incident with it.

In any simple graph, every edge is incident with two vertices.

In particular, in a cycle every edge is adjacent to two edges. Therefore, in a middle graph of c_n, every edge create a new vertex, and the degree a newly formed vertex is four.

The sum of the degree of the M(G) is equal to the sum of the degree of the vertices of G plus the sum of the newly formed vertices in M(G).(i.e.) twice the number of vertices of G plus four times the number of vertices of G is equal to six times the number of vertices of G.

But we know that,

The summation of the degree of the vertices of G is equal to twice the number of edges of G.

Here, twice the number of edges of M(G) is equal to six times the number of vertices of G.

Hence the number of edges of M(G) is equal to thrice the number of vertices of G.

Hence if a cycle with n vertices and n edges then its middle graph has 2n vertices and 3n edges.

Example 3.1.1:

Graph: c_3

$$A(G) = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

eig(G) = -1, -1, 2.

Energy of G = $\sum_{i=1}^{3}|\lambda_i|$ = 1 + 1 +2 =4

Middle Graph of C_3:

$$A(M(G)) = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

Eigen values of (M(G)) are as follows: -1.6180, -1.2361, 0.6180, 0.6180, 3.2361.

Energy of M(G) =$\sum_{i=1}^{6}|\lambda_i|$ = 8.9442

Example 3.1.2:

Graph: c_4

$$A(G) = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

Eigen values = -2, 0, 0, 2

E(G)=$\sum_{i=1}^{4}|\lambda_i|$ = 4

Middle Graph Of c_4:

$$A(M(G)) = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

Eigen values = -2, -1.4142, -1.4142, -1.2361, 0, 1.4142, 1.414 , 3.2361.

E(M(G)) = $\sum_{i=1}^{8} |\lambda_i|$ =12.1290

Here the energy of M(G) is approximately equal to its number of edges.

A table of such c_n are given below:

S.No	Name of the graph	No. of vertices & edges	No. of edges of the middle graph	Energy of middle graph
1	c_3	,3	9	8.9442
2	c_4	4,4	12	12.1290
3	c_5	5,5	15	15.1338
4	c_6	6,6	18	18.1554
5	c_7	7,7	21	21.1856

4. Conclusion:

We have showed that if G be a cycle with n vertices and n edges then its middle graph has 2n vertices and 3n edges, in the cycle the energy of graph is less than energy of its middle graph and the energy of middle graph of c_n is approximately equal to 3n.In larger values of n the value of 3n is (i.e.) the energy of middle graph of c_n is approximately equal to the number of edges of M(G).

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