ISSN: 2455 – 5428
International Journal of Current Research and Modern Education
Impact Factor 6.725, Special Issue, July - 2017

A MATHEMATICAL MODEL USING DAGUM DISTRIBUTION ON TRANSIENT NEONATAL THYROID STIMULATING HORMONE

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Cite This Article: D. R. Kirubaharan, A. Venkatesh & S. Udhayakumar, "A Mathematical Model Using Dagum Distribution on Transient Neonatal Thyroid Stimulating Hormone", International Journal of Current Research and Modern Education, Special Issue, July, Page Number 100-103, 2017.

Abstract:

In this article, we initiate the fuzzy transmuted Dagum distribution which can be used for the reliability and hazard analysis upon a serum abnormality due to transplacentally acquired antibody to thyroid stimulating hormone. This distribution model provides the broader range of hazard behavior than the Dagum model. To illustrate utility and potentiality of the proposed model, it has been applied to transient neonatal hyper thyrotrophinaemia.

Key Words: Fuzzy Transmuted Dagum Distribution, Reliability and Hazard Values & Transient Neonatal Hyper TSH.

Introduction:

In the 1970s, Camilo Dagum [2], [3], [9] embarked on a quest for a statistical distribution closely fitting empirical income and wealth distributions. Not satisfied with the classical distributions used to summarize such information – the Pareto distribution and the lognormal distribution– he looked for a model accommodating the heavy tails present in empirical income and wealth distributions as well as permitting an interior mode. The earlier aspect is fine captured by the Pareto but not by the lognormal distribution, the latter by the lognormal but not the Pareto distribution. Experimenting with a shifted log-logistic distribution (Dagum), a generalization of a distribution previously considered by Fisk, he rapidly realized that an additional parameter was needed. This led to the Dagum type I distribution, a three-parameter distribution, and two four-parameter generalizations (Dagum).

Materials & Methods:

The Dagum distribution is a continuous probability distribution defined over positive real numbers. It is named after Camilo Dagum [6], [7], [8], who proposed it in a series of papers in the 1970s. The Dagum distribution arose from several variants of a new model on the size distribution of personal income and is mostly associated with the study of income distribution. There is both a three-parameter specification (Type I) and a four-parameter specification (Type II) of the Dagum distribution; a summary of the origin of this distribution can be originated in "A Guide to the Dagum Distributions". A general source on statistical size distributions often cited in work using the Dagum distribution is statistical size distributions in economics and actuarial sciences.

Fuzzy Reliability and Fuzzy Hazard Model:

Dagum distribution is widely used for modeling a wide range of data in several fields. It is very worthwhile for analyzing income distribution, actuarial, metrological data and equally preferable for survival analysis. Moreover, it is considered to be the most suitable choice as compared to other three parameter distributions in several cases.

A random variable follows the transmuted distribution, if it satisfies the following relationship that is proposed by Shaw et al. [24] named as quadratic rank transmutation map

$$F(y) = G(y)[(1+\lambda) - \lambda G(y)]$$

where G(y) is the cdf of the parent distribution and λ is the additional parameter that is called transmuted parameter. Due to the transmuted parameter the distribution becomes more flexible distribution to model even the complex data sets. The pdf of the Dagum (parent) distribution is as

$$g(y;\alpha,\beta,\rho) = \frac{\alpha\rho y^{\alpha\rho-1}}{\beta^{\alpha\rho}\,(1+({}^{y}/_{\beta})^{\alpha})^{\rho+1}}, 0 \le x \le \infty; \alpha,,\rho > 0$$

and its cdf is as

$$G(y;\alpha,\beta,\rho) = (1 + (1 + ({}^{y}/_{\beta})^{-\alpha})^{-\rho}$$

Where α and β are the shape parameters, ρ is the scale parameter and all the three parameters are positive. Now substituting the (2.3) in (2.1), we obtained the cdf of the transmuted Dagum distribution in the following form

$$F(y;\alpha,\beta,\rho,\lambda) = (1 + (1 + ({}^{y}/_{\beta})^{-\alpha})^{-\rho}[1 + \lambda - \lambda(1 + ({}^{y}/_{\beta})^{-\alpha})^{-\rho}]$$

And its respective pdf of transmuted Dagum distribution is given by

$$f(y;\alpha,\beta,\rho,\lambda) = \frac{\alpha\rho y^{\alpha\rho-1}[(1+\lambda)(1+({}^{y}/_{\beta})^{-\alpha})^{\rho} - 2\lambda]}{\beta^{\alpha\rho}\,(1+({}^{y}/_{\beta})^{\alpha})^{2\rho+1}}$$

The parameter λ has the support $-1 \le y \le 1$ and simply taking $\lambda = 0$ in above pdf and cdf, transmuted distribution reduces to the parent distribution. Dagum distribution due to quadratic rank transmutation map becomes more flexible.

The reliability function R(t) gives the probability of surviving of an item at least reach the age of t time. The cdf F(t) and reliability function are reverse of each other as R(t) + F(t) = 1. The reliability function for transmuted Dugam distribution is given by

$$R(t) = P(T > t)$$
$$= \int_t^{\infty} f(t)dt$$
$$= 1\text{-}F(t)$$

ISSN: 2455 – 5428
International Journal of Current Research and Modern Education
Impact Factor 6.725, Special Issue, July - 2017

$$= 1 + (1 + ({}^{t}/_{\beta})^{-\alpha})^{-2\rho}[\lambda - (1+\lambda)(1 + (t)^{-\alpha})^{-\rho}]$$

With various choices of parametric values for the reliability function pattern of transmuted Dagum distribution.

An important property of a random variable is the hazard function, it measure the inclination towards failure rate. The probability approaches to failure increases as the value of the hazard function increase. Mathematically, the hazard function and the hazard function of transmuted Dagum distribution is defined as

$$h(t) = \frac{f(t)}{1-F(t)} = \frac{\alpha\rho y^{\alpha\rho-1}[(1+\lambda)(1+({}^{y}/_{\beta})^{-\alpha})^{\rho}-2\lambda]}{\beta^{\alpha\rho}(1+({}^{y}/_{\beta})^{\alpha})^{2\rho+1}[1+(1+({}^{t}/_{\beta})^{-\alpha})^{-2\rho}[\lambda-(1+\lambda)(1+(\frac{t}{\beta})^{-\alpha})^{-\rho}]}$$

On simplification we get, the reliability function and hazard rate function for the total damage Dagum distribution at time 't' is as

$$R(t) = 1 + (1 + ({}^{t}/_{\beta})^{-\alpha})^{-2\rho}[\lambda - (1+\lambda)(1+(t)^{-\alpha})^{-\rho}$$

$$H(t) = \frac{\alpha\rho y^{\alpha\rho-1}[(1+\lambda)(1+({}^{y}/_{\beta})^{-\alpha})^{\rho}-2\lambda]}{\beta^{\alpha\rho}(1+({}^{y}/_{\beta})^{\alpha})^{2\rho+1}[1+(1+({}^{t}/_{\beta})^{-\alpha})^{-2\rho}[\lambda-(1+\lambda)(1+(\frac{t}{\beta})^{-\alpha})^{-\rho}]}.$$

The alpha cut of the fuzzy Dagum reliability and the hazard function at time 't' is

R(t)={ $\bar{R}_l(t)$, $\bar{R}_u(t)$ } where

$$\bar{R}_l(t) = min\{ 1 + (1 + ({}^{\bar{t}}/_{\bar{\beta}})^{-\bar{\alpha}})^{-2\bar{\rho}}[\bar{\lambda} - (1+\bar{\lambda})(1+(\bar{t})^{-\bar{\alpha}})^{-\bar{\rho}}]\}$$ and

$$\bar{R}_u(t) = max\{ 1 + (1 + ({}^{\bar{t}}/_{\bar{\beta}})^{-\bar{\alpha}})^{-2\bar{\rho}}[\bar{\lambda} - (1+\bar{\lambda})(1+(\bar{t})^{-\bar{\alpha}})^{-\bar{\rho}}]\}$$

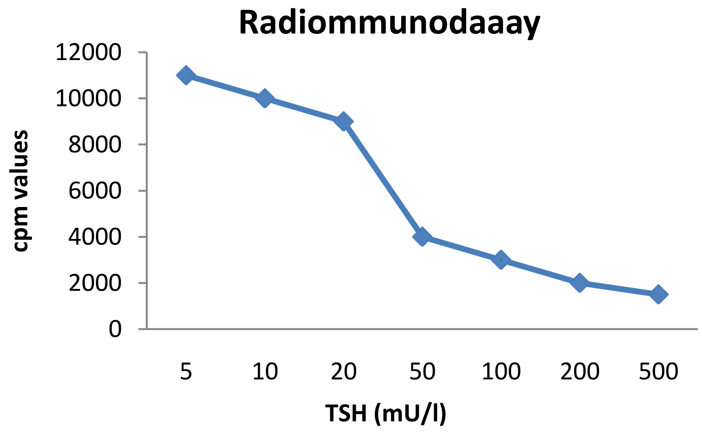
H(t)={ $\bar{H}_l(t)$, $\bar{H}_u(t)$ } where

$$\bar{H}_l(t) = min\{ \frac{\bar{\alpha}\bar{\rho}\bar{t}^{\bar{\alpha}\bar{\rho}-1}[(1+\bar{\lambda})(1+({}^{\bar{t}}/_{\bar{\beta}})^{-\bar{\alpha}})^{\bar{\rho}}-2\bar{\lambda}]}{\bar{\beta}^{\bar{\alpha}\bar{\rho}}(1+({}^{\bar{t}}/_{\bar{\beta}})^{\bar{\alpha}})^{2\bar{\rho}+1}[1+(1+({}^{\bar{t}}/_{\bar{\beta}})^{-\bar{\alpha}})^{-2\bar{\rho}}[\bar{\lambda}-(1+\bar{\lambda})(1+(\frac{t}{\bar{\beta}})^{-\bar{\alpha}})^{-\bar{\rho}}]}\}$$ and

$$\bar{H}_u(t) = max\{ \frac{\bar{\alpha}\bar{\rho}\bar{t}^{\bar{\alpha}\bar{\rho}-1}[(1+\bar{\lambda})(1+({}^{\bar{t}}/_{\bar{\beta}})^{-\bar{\alpha}})^{\bar{\rho}}-2\bar{\lambda}]}{\bar{\beta}^{\bar{\alpha}\bar{\rho}}\left(1+({}^{\bar{t}}/_{\bar{\beta}})^{\bar{\alpha}}\right)^{2\bar{\rho}+1}[1+(1+({}^{\bar{t}}/_{\bar{\beta}})^{-\bar{\alpha}})^{-2\bar{\rho}}[\bar{\lambda}-(1+\bar{\lambda})(1+(\frac{t}{\bar{\beta}})^{-\bar{\alpha}})^{-\bar{\rho}}]}$$

Application:

A full term female infant weighing 3200 g was reviewed because a raised concentration of thyroid stimulating hormone was found on a filter paper whole blood spot taken seven days after birth [1], [4], [5], [10], [12], [14], [15], [16]. The infant had been delivered normally, and there was no history of drug ingestion during pregnancy or of maternal illness. Clinical examination of the child was normal; in particular there was no goitre or features of hypothyroidism. She was being breast fed. The mother, aged 35, was clinically euthyroid with no goitre. There was no history of illness or recent immunization and no family history of thyroid disease or parental consanguinity [11], [13].



Discussion:

This baby had a high concentration of thyroid stimulating hormone as measured by the conventional assay used in screening for neonatal hypothyroidism and confirmed on examination of the serum with two independent radioimmunoassays. The high concentration of the hormone in the maternal serum and the subsequent biochemical investigations on both sera suggested that the concentrations in the baby and mother were falsely raised. In addition the baby had normal serum concentrations of thyroid hormones and the gland was in the normal position on scanning.

Thyroid function was normal in the mother, who appeared to have a normal response of thyroid stimulating hormone to thyroid releasing hormone, though this test is difficult to evaluate because of the high background value.

Any baby found to have a high concentration of thyroid stimulating hormone but in whom the serum thyroxine concentration is within normal limits and who has a demonstrable thyroid gland in the normal position must be evaluated for a

ISSN: 2455 – 5428
International Journal of Current Research and Modern Education
Impact Factor 6.725, Special Issue, July - 2017

false positive result. Maternal concentrations of thyroid stimulating hormone and thyroid function should be checked and, if laboratory facilities permit, appropriate tests done to determine the cause of the raised hormone concentration.

The triangular fuzzy numbers on the scale and location parameters are:

$$\bar{\lambda} = [\,1.3989, \quad 1.9556, \quad 2.6998\,]$$
$$\bar{\rho} = [\,0.18211, \quad 0.23934, \quad 0.30351]$$
$$\bar{\alpha} = [\,0.7851, \quad 0.9098, \quad 1.0584\,]$$
$$\bar{\beta} = [1498.1458, \quad 1500, \quad 1501.6429]$$

The corresponding alpha cut for the scale and location parameters are

$$\bar{\lambda} = [\,1.3989 + (0.5567\alpha), \quad 2.6998 - (0.7442\alpha)]$$
$$\bar{\rho} = [\,0.18211 + (0.05723\alpha), \quad 0.30351 - (0.06417\alpha)]$$
$$\bar{\alpha} = [\,0.7851 + (0.1247\alpha), \quad 1.0584 - (0.1486\alpha)]$$
$$\bar{\beta} = [\,1498.1458 + (1.8542\alpha), \quad 1501.6429 - (1.6429\alpha)]$$

Under the alpha cut zero, the fuzzy Reliability values of the cpm response due to thyroid stimulating hormone for t = 1,2,3,4,5 are calculated from $\bar{R}\{\bar{z}(t)\} = [\bar{R}_l(t),\ \bar{R}_u(t)]$ and the fuzzy hazard rate values of the cpm for t = 1,2,3,4,5 are calculated from $\bar{h}\{\bar{z}(t)\} = [\ \bar{h}_l(t),\ \bar{h}_u(t)]$ is shown in the following table 3.1.

Table 3.1: Fuzzy reliability and fuzzy hazard rate values for lower and upper thyroid.

α	**low(λ)**	**up(λ)**	**low(ρ)**	**up(ρ)**	**low(α)**	**up(α)**	**low(β)**	**up(β)**
0	1.3989	2.6998	0.1821	0.3035	0.7851	1.0584	1498.1458	1501.64
0.1	1.4546	2.6254	0.1878	0.2971	0.7976	1.0435	1498.3312	1501.48
0.2	1.5102	2.5510	0.1936	0.2907	0.8100	1.0287	1498.5166	1501.31
0.3	1.5659	2.4765	0.1993	0.2843	0.8225	1.0138	1498.7021	1501.15
0.4	1.6216	2.4021	0.2050	0.2778	0.8350	0.9990	1498.8875	1500.99
0.5	1.6773	2.3277	0.2107	0.2714	0.8475	0.9841	1499.0729	1500.82
0.6	1.7329	2.2533	0.2164	0.2650	0.8599	0.9692	1499.2583	1500.66
0.7	1.7886	2.1789	0.2222	0.2586	0.8724	0.9544	1499.4437	1500.49
0.8	1.8443	2.1044	0.2279	0.2522	0.8849	0.9395	1499.6292	1500.33
0.9	1.8999	2.0300	0.2336	0.2458	0.8973	0.9247	1499.8146	1500.16
1	1.9556	1.9556	0.2393	0.2393	0.9098	0.9098	1500.0000	1500.0000

Table 3.2: Lower and upper alpha cut for the fuzzy reliability values.

α	$R_l(t=1)$	$R_u(t=1)$	$R_l(t=2)$	$R_u(t=2)$	$R_l(t=3)$	$R_u(t=3)$	$R_l(t=4)$	$R_u(t=4)$	$R_l(t=5)$	$R_u(t=5)$
0	0.3298558	0.67163136	0.2801187	0.5974437	0.2506497	0.54757766	0.2296914	0.5091209	0.2134596	0.4774862
0.1	0.3421237	0.65276031	0.2900670	0.5779393	0.2590915	0.52803885	0.2370036	0.4897508	0.2198648	0.45837511
0.2	0.3550980	0.6335928	0.3007554	0.5583773	0.2682723	0.50860052	0.2450439	0.4705991	0.2269832	0.43957601
0.3	0.3687530	0.61420487	0.3121772	0.5388427	0.2781966	0.48934928	0.2538244	0.4517519	0.2348328	0.4211733
0.4	0.3830553	0.59467883	0.3243173	0.5194239	0.2888607	0.47037379	0.2633494	0.433296	0.2434241	0.40325118
0.5	0.3979644	0.5751026	0.3371532	0.5002124	0.3002532	0.4517638	0.2736155	0.415318	0.2527597	0.3858926
0.6	0.4134338	0.55556889	0.3506552	0.4813016	0.3123553	0.43360904	0.2846121	0.3979031	0.2628355	0.36917815
0.7	0.4294118	0.53617423	0.3647873	0.4627854	0.3251418	0.41599803	0.2963217	0.381134	0.2736403	0.35318492
0.8	0.4458424	0.51701782	0.3795082	0.4447573	0.3385812	0.3990168	0.3087207	0.3650897	0.2851567	0.33798526
0.9	0.4626662	0.49820022	0.3947716	0.4273086	0.3526368	0.3827475	0.3217797	0.3498439	0.2973611	0.32364554
1	0.4798218	0.47982178	0.4105273	0.4105273	0.3672670	0.36726699	0.3354641	0.3354641	0.3102249	0.31022493

Table 3.3: Lower and upper alpha cut for the fuzzy hazard values.

α	**$H_l(t=1)$**	**$H_u(t=1)$**	**$H_l(t=2)$**	**$H_u(t=2)$**	**$H_l(t=3)$**	**$H_u(t=3)$**	**$H_l(t=4)$**	**$H_u(t=4)$**	**$H_l(t=5)$**	**$H_u(t=5)$**
0	0.214935	0.14524	0.129318	0.09783	0.096864	0.07864	0.079264	0.06787	0.068049	0.06087
0.1	0.216294	0.15148	0.131062	0.10150	0.098651	0.08136	0.081045	0.07009	0.069815	0.06279
0.2	0.216907	0.15767	0.132335	0.10511	0.100078	0.08402	0.082529	0.07225	0.071326	0.06463
0.3	0.216793	0.16377	0.133134	0.10862	0.101135	0.08658	0.083701	0.07431	0.072564	0.06638
0.4	0.215982	0.16973	0.133468	0.11199	0.10182	0.08901	0.084554	0.07624	0.073519	0.06801
0.5	0.214515	0.17550	0.133355	0.11520	0.102141	0.09129	0.085092	0.07803	0.074188	0.06949
0.6	0.212439	0.18101	0.132816	0.11819	0.102112	0.09337	0.085321	0.07963	0.074577	0.07080
0.7	0.209806	0.18620	0.131882	0.12092	0.101753	0.09521	0.085256	0.08102	0.074694	0.07189
0.8	0.20667	0.19100	0.130584	0.12333	0.101087	0.09679	0.084915	0.08215	0.074556	0.07274
0.9	0.203085	0.19533	0.128957	0.12539	0.10014	0.09804	0.084319	0.08298	0.074179	0.07332
1	0.199107	0.19911	0.127035	0.12703	0.098939	0.09894	0.083493	0.08349	0.073585	0.07359

Result:

The concentration of thyroid stimulating hormone on the filter paper blood sample obtained from the baby seven days after birth was 104 mU/l. Repeat testing on the same filter paper spot and on a fresh filter paper spot gave similar results. Serum concentrations were: thyroxine 180 nmol/l (14-0 ,ug/100 ml), free thyroxine 22 pmol/l (1-7 ng/100 ml), triiodothyronine 3-1 nmol/l (202 ng/100 ml), and thyroxine binding globulin 25-5 mg/l. These results were all normal. Serum bilirubin concentration was normal. Other serum hormone concentrations were prolactin 1520 mU/l, luteinising hormone 2-8 U/l, follicle stimulating hormone 4-7 U/1, and i-human chorionic gonadotrophin < 2-5 IU/l (normal less than 5 IU/1). No thyroid antibodies were detected in the baby's serum.

Conclusion:

This paper has provided a brief introduction to the Dagum distributions and their applications in economics. The transmuted Dagum distribution proposed in this study of the Dagum distribution. This distribution is quite flexible and its application diversities increased due to the fourth parameter as compared to the standard Dagum distribution. Finally in this paper, if the fuzzy reliability increases in lower alpha cut values, then the fuzzy hazard rate slightly decreases, and in upper alpha cut values if the fuzzy reliability decreases when the hazard increases, then the serum cpm is increased by TSH level and we have dealt with these parameters neonatal.

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