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HESITANT FUZZY SOFT GRAPH

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Abstract:
Fuzzy sets and soft sets are two different soft computing models for representing vagueness and uncertainty. We apply these soft computing models in combination to study vagueness and uncertainty in graphs. We introduce the notions of hesitant fuzzy soft graphs, strong hesitant fuzzy soft graphs, complete hesitant fuzzy soft graphs, order of hesitant fuzzy soft graphs, and size of hesitant fuzzy soft graphs.

Introduction:

Molodtsov introduced the concept of soft set that can be seen as a new mathematical theory for dealing with uncertainties. The traditional soft set is a mapping from parameter to the crisp subset of universe. Maji presented the concept of fuzzy soft sets by embedding ideas of fuzzy set. Later to make decision making evaluation more effective so many generalization are defined such as Intuitionistic fuzzy sets, type 2 fuzzy sets, interval valued fuzzy sets. However, when defining the membership degree of an element to a set, the difficulty of establishing the membership degree is not because we have a margin of error (as in Intuitionistic fuzzy set or interval-valued fuzzy set), or some possibility distribution (as in type-2 fuzzy set) on the possible values, but because we have a set of possible values. To such cases, Torra and Narukawa and Torra introduced another generalization of fuzzy set, hesitant fuzzy set, allowing the membership degree having a set of possible values. In 1736, Euler first introduced the concept of graph theory. The theory of graph is extremely useful tool for solving combinatorial problems in different areas such as geometry, algebra, number theory, topology, operation research, optimization and computer science, etc. The first definition of fuzzy graphs was proposed by Kaffmann in 1973, from Zadeh's fuzzy relations. But Rosenfeld introduced another elaborated definition including fuzzy vertex and fuzzy edges and several fuzzy analogs of graph theoretic concepts. Soft graph was introduced by Thumbakara and George. In 2015, Mohinta and Samanta introduced the concept of fuzzy soft graph. In this paper, My aim is to introduce the notions of hesitant fuzzy soft graph, strong hesitant fuzzy soft graph, complete hesitant fuzzy soft graph, order of hesitant fuzzy soft graphs, and size of hesitant fuzzy soft graphs.

Preliminaries:

Definition 1.1: Let X be a fixed set, a hesitant fuzzy set (HFS) on X is in terms of a function that when applied to X returns a subset of [0, 1]. The HFS is defined by a mathematical symbol:

$$A = \{ < x, h_A(x) > / x \epsilon X \}$$

Where $h_A(x)$ is a set of some values in [0,1], denoting the possible membership degrees of the element $x \epsilon X$ to the set A. h = $h_A(x)$ a hesitant fuzzy element (HFE) and Θ the set of all HFEs.

Definition 1.2: Let U be an initial universe set and E be a set of parameters. Let P (U) denotes the power set of U and A ⊆ E. A pair (F, A) is called a soft set over U, where F is a mapping given by F: A P(U).

Definition 1.3: A pair (F, A) is called fuzzy soft set over U , where F is a mapping given by F : A → I^U ; I^U denotes the collection of all fuzzy subsets of U; A ⊆ P.

Definition 1.4: Let U be a universal set and E be set of parameters. Let HF(U) denotes the set of all hesitant fuzzy sets defined over U. A pair (F, E) is a hesitant fuzzy soft sets if F(e)∈ $HF(U)$ for every e in E.

Definition 1.5: Let V = $\{v_1, v_2, v_3 \ldots v_n\}$ (non-empty set),P(parameter set) and A ⊆ P. Also let

1. ρ: A → I^U (V) (I^U (V) denotes collection of all fuzzy subsets in V)

$$a \rightarrow \rho(a) = \rho_a \text{ (say)}, a \in A \text{ and } \rho_a : V \rightarrow [0,1],$$
(A, ρ) Fuzzy soft vertex.

2. μ: A →I^U (V×V), (I^U (V×V) denotes collection of all fuzzy subsets in V ×V)

$$a \rightarrow \mu(a) = \mu_a \text{ (say)}, a \in A \text{ and } \mu_a : V\times V \rightarrow [0,1]\ (v_i,v_j) \rightarrow \mu_a(v_i,v_j),\ (A, \mu) \text{ Fuzzy soft edge.}$$

Then ((A, ρ),(A,μ)) is called fuzzy soft graph if and only if $\mu_a (v_i,v_j) \leq \rho_a(v_i) \wedge \rho_a(v_j)$ v for each $(v_i,v_j) \in V \times V$, for every a∈ A and i, j =1,2,…,n .

Definition 1.6: A Hesitancy Fuzzy Graph is of the form G = (V,E), where V = $\{v_1, v_2, v_3 \ldots v_n\}$ such that μ_1:V→[0,1], γ_1: V→[0,1] and β_1: V→[0,1] denote the degree of membership, non-membership and hesitancy of the element $v_i \in V$ respectively and $\mu_1(v_i)+\gamma_1(v_i)+\beta_1(v_i)$ = 1for every $v_i \in V$, where $\beta_1(v_i) = 1-[\mu_1 (v_i) + \gamma_1 (v_i)]$ and E ⊆ V×V where μ_2: V×V →[0,1], γ_2: V×V →[0,1] and β_2: V×V →[0,1] are such that,

$$\mu_2(v_i,v_j) \leq \min [\mu_1(v_i), \mu_1(v_j)]$$
$$\gamma_2(v_i,v_j) \leq \max [\gamma_1(v_i), \gamma_1(v_j)]$$
$$\beta_2(v_i,v_j) \leq \min [\beta_1(v_i), \beta_1(v_j)] \text{ and}$$
$$0 \leq \mu_2(v_i,v_j) + \gamma_2(v_i,v_j) + \beta_2(v_i,v_j) \leq 1 \text{ for every } (v_i,v_j) \in E.$$

Hesitant Fuzzy Soft Graph:

Definition 2.1: Let G = (V, E) be a simple graph, where V = $\{v_1, v_2, v_3 \ldots v_n\}$ (non-empty set), E ⊆ VxV, P(parameter set) and A P. Also let

1. μ_1 is a membership function defined on V by

μ_1: A→IF^U(V) (IF^U (V) denotes collection of all hesitant fuzzy subsets in V)

$a \rightarrow \mu_1(a) = \mu_{1a}$ (say), a A and μ_{1a}:V →[0,1], $v_i \rightarrow \mu_{1a}(v_i)$

(A, μ_1) Hesitant Fuzzy soft vertex of membership function and

γ_1 is a non-membership function defined on V by

γ_1: A→IF^U(V) (IF^U (V) denotes collection of all hesitant fuzzy subsets in V)

a → $\gamma_1(a) = \gamma_{1a}$ (say), a A and γ_{1a}:V →[0,1], $v_i \to \gamma_{1a}(v_i)$

(A, γ_1) Hesitant Fuzzy soft vertex of membership function

β_1 is a hesitant function defined on V by

β_1: A→IF^U(V) (IF^U (V) denotes collection of all hesitant fuzzy subsets in V)

a → $\beta_1(a) = \beta_{1a}$ (say), a A and β_{1a}:V →[0,1], $v_i \to \beta_{1a}(v_i)$

(A, β_1) Hesitant Fuzzy soft vertex of membership function

2. μ_2 is a membership function defined on E by

μ_2: A→IF^U(VxV) (IF^U (VxV) denotes collection of all hesitant fuzzy subsets in VxV)

a → $\mu_2(a) = \mu_{2a}$ (say), μ_{2a}: VxV →[0, 1], $(v_i,v_j) \to \mu_{2a}(v_i,v_j)$

γ_2 is a non-membership function defined on E by

γ_2: A→IF^U(VxV) (IF^U (VxV) denotes collection of all hesitant fuzzy subsets in VxV)

a → $\gamma_2(a) = \gamma_{2a}$ (say), a A and γ_{2a}: VxV →[0,1], $(v_i,v_j) \to \gamma_{2a}(v_i,v_j)$

β_2 is a hesitant function defined on E by

β_2: A→IF^U(VxV) (IF^U (VxV) denotes collection of all hesitant fuzzy subsets in VxV)

a → $\beta_2(a) = \beta_{2a}$ (say), a A and β_{2a}: VxV →[0,1], $(v_i,v_j) \to \beta_{2a}(v_i,v_j)$

where (A, μ_2), (A, γ_2) and (A, β_2) Hesitant Fuzzy soft edge of membership function, non-membership function and hesitant function satisfying

$$\mu_{2a}(v_i,v_j) \le \min [\mu_{1a}(v_i), \mu_{1a}(v_j)]$$
$$\gamma_{2a}(v_i,v_j) \le \max [\gamma_{1a}(v_i), \gamma_{1a}(v_j)]$$
$$\beta_{2a}(v_i,v_j) \le \min [\beta_{1a}(v_i), \beta_{1a}(v_j)] \text{ and}$$

$0 \le \mu_{2a}(v_i,v_j) + \gamma_{2a}(v_i,v_j) + \beta_{2a}(v_i,v_j) \le 1$ for every (v_i,v_j) E and a A. Then

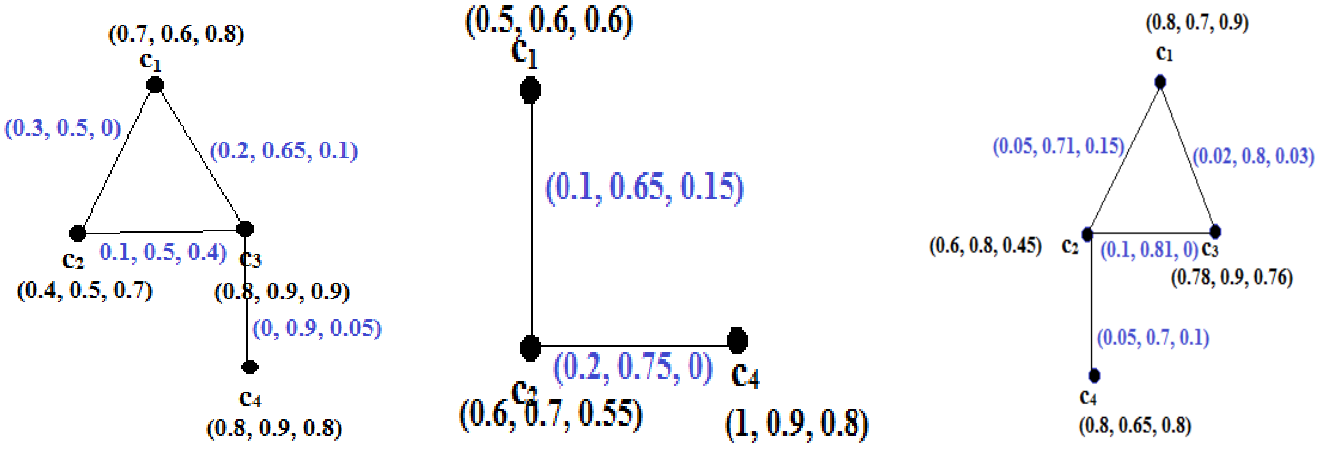
$G^* = (V, E, (A, \mu_1), (A, \gamma_1), (A, \beta_1), (A, \mu_2), (A, \gamma_2), (A, \beta_2))$ is said to be the Hesitant Fuzzy Soft Graph (HFSG) is denoted by $G^*_{A,V,E}$

Example: let U be set of participants performing dance programme. U ={c_1, c_2, c_3, c_4}. Let A = {confident, creative, timing}. Then hesitant fuzzy soft sets (F, A) defined as below gives the evaluation of the performance of candidates by three judges.

F(confident) = {c_1 = {0.7,0.6,0.8}, c_2 = {0.4,0.5,0.7},
c_3 = {0.8,0.9,0.9}, c_4 = {0.8,0.9,0.8}}

F(creative) = {c_1 = {0.5, 0.6, 0.6}, c_2 = {0.6, 0.7, 0.55},
c_3 = {0.8, 0.9, 0.82}, c_4 = {1,0.9,0.8}}

F(timing) = {c_1 = {0.8, 0.7, 0.9}, c_2 = {0.6, 0.8, 0.45},
c_3 = {0.78, 0.9, 0.76}, c_4 = {0.8, 0.65, 0.8}}



(1a) HFSG to the parameter Confident (1b) HFSG to the parameter Creative (1c) HFSG to the parameter Timing

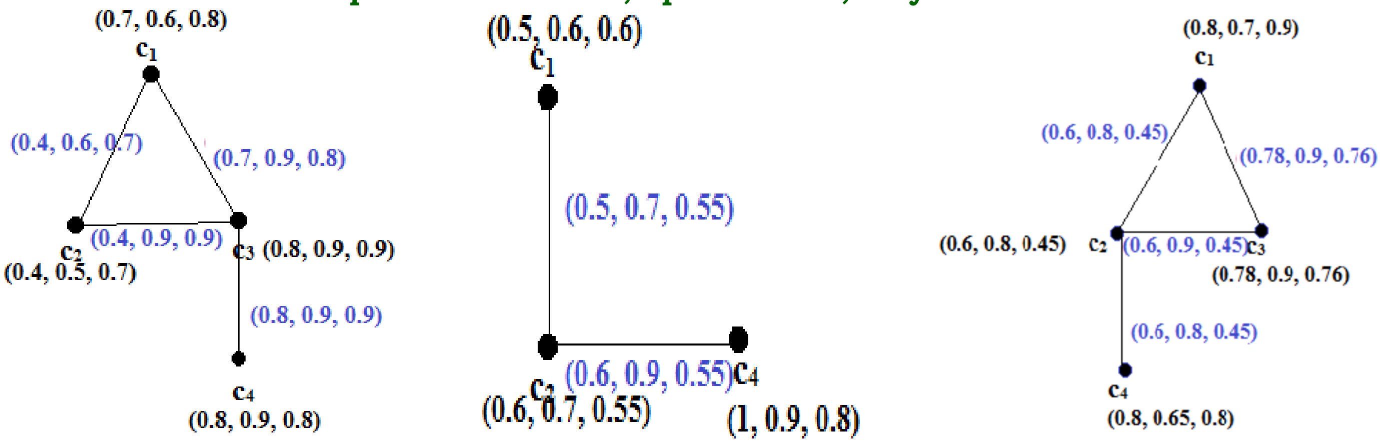
Definition 2.2: A HFSG G = (V, E) is said to be a μ-strong HFSG if $\mu_{2ij} = \min (\beta_{1i}, \beta_{1j})$, for all $(v_i, v_j) \in E$.

Definition 2.3: A HFSG, G = (V, E) is said to be a γ-strong HFSG if $\gamma_{2ij} = \max (\gamma_{1i}, \gamma_{1j})$, for all $(v_i, v_j) \in E$.

Definition 2.4: A HFSG G = (V, E) is said to be a β-strong HFSG if $\beta_{2ij} = \min (\beta_{1i}, \beta_{1j})$, for all $(v_i, v_j) \in E$.

Definition 2.5: A HFSG, G = (V, E) is said to be a strong HFSG if,

$$\mu_{2ij} = \min (\mu_{1i}, \mu_{1j})$$
$$\gamma_{2ij} = \max(\gamma_{1i}, \gamma_{1j})$$
$$\beta_{2ij} = \min(\beta_{1i}, \beta_{1j}), \text{ for all (vi, vj)} \in E.$$



(1a) strong HFSG to the parameter Confident (1b) strong HFSG to the parameter Creative (1c) strong HFSG to the parameter Timing

Definition 2.6: A HFSG, G = (V, E) is said to be a μ-complete HFSG if $\mu_{2ij} = \min(\mu_{1i}, \mu_{1j})$, for all $v_i, v_j \in V$.

Definition 2.7: A HFSG, G = (V,E) is said to be a γ-complete HFSG if $\gamma_{2ij} = \max(\gamma_{1i}, \gamma_{1j})$, for all $v_i, v_j \in V$.

Definition 2.8: A HFSG, G = (V,E) is said to be a β-complete HFSG if $\beta_{2ij} = \min(\beta_{1i}, \beta_{1j})$, for all $v_i, v_j \in V$.

Definition 2.9: A HFSG, G = (V,E) is said to be a complete HFSG if

$\mu_{2ij} = \min(\mu_{1i}, \mu_{1j})$

$\gamma_{2ij} = \max(\gamma_{1i}, \gamma_{1j})$

$\beta_{2ij} = \min(\beta_{1i}, \beta_{1j})$ for all $v_i, v_j \in V$.

Definition 2.10: The order of a hesitant fuzzy soft graph is

$$\text{Ord}(G^*) = \left(\sum_{e_i \in A} \sum_{a \in V} \mu_{E_i}(a), \quad \sum_{e_i \in A} \sum_{a \in V} \gamma_{E_i}(a), \quad \sum_{e_i \in A} \sum_{a \in V} \beta_{E_i}(a) \right)$$

Definition 2.11: The size of a hesitant fuzzy soft graph is

$$\text{Siz}(G^*) = \left(\sum_{e_i \in A} \sum_{ab \in E} \mu_{E_i}(a), \quad \sum_{e_i \in A} \sum_{ab \in E} \gamma_{E_i}(a), \quad \sum_{e_i \in A} \sum_{ab \in E} \beta_{E_i}(a) \right)$$

In Example,

The order of hesitant fuzzy soft graph to the parameter confident is
= (0.7+0.4+ 0.8+ 0.8, 0.6+0.5+0.9+0.9, 0.8+0.7+0.9+0.8)
= (2.7, 2.9, 3.2)

The order of hesitant fuzzy soft graph to the parameter creative is
= (0.5+0.6+ 0.8+1, 0.6+0.7+0.9+0.9, 0.6+0.55+0.82+0.8)
= (2.9, 3.1, 2.77)

The order of hesitant fuzzy soft graph to the parameter timing is
= (0.8+0.6+ 0.78+ 0.8, 0.7+0.8+0.9+0.65, 0.9+0.45+0.76+0.8)
= (2.98, 3.05, 2.91)

The size of fuzzy soft graph to the parameter confident in the figure (1a) is
= (0.3+0.2+0.1, 0.5+0.65+0.5+0.9, 0.1+0.4+0.05)
= (1.1, 2.55, 0.55)

The size of fuzzy soft graph to the parameter creative in the figure (1b)is
= (0.1+0.2, 0.65+0.75, 0.15)
= (0.3, 1.4, 0.15)

The size of fuzzy soft graph to the parameter hesitant in the figure (1c)is
= (0.05+0.02+0.1+0.05, 0.71+0.8+0.81+0.7, 0.15+0.03+0.1)
= (0.22, 3.02, 0.28)

Conclusion:

Soft set is completely a new approach for modeling vagueness and uncertainty. Hesitant fuzzy set is a generalization of fuzzy set allowing the membership degree having a set of possible values. Fuzzy graph has numerous applications in modern sciences and technology, especially in research areas of computer science including database theory, data mining, neural networks, expert systems, cluster analysis, control theory, and image capturing. We have applied these soft computing models in combination to study vagueness and uncertainty in graphs. In this paper we have defined a new fuzzy soft graph called Hesitancy Fuzzy Soft Graph (HFSG) and illustrated with some examples. Although this study is a preliminary proposal concerning the hesitant fuzzy soft graph, we hope it will give rise to a potentially interesting research direction. These particular graph aggregate the hesitation degree raised from the human intuition in decision making process.

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