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R -FUZZY GENERALIZED E-CLOSED SETS AND MAPS IN FUZZY TOPOLOGICAL SPACES

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Abstract:

In this paper, we introduce and study the concept of r -fuzzy generalized e-open (closed) sets in fuzzy topological spaces in the sense of ˇ S ostak's. By using r -fuzzy generalized e-open (closed) sets, we define a new fuzzy closure operator namely r -fuzzy generalized e -interior (closure) operator. Also, we introduce fuzzy generalized e -continuous and fuzzy generalized e-irresolute mappings. Moreover, we investigate the relationship among fuzzy generalized e-continuous and fuzzy generalized e-irresolute mappings. Finally, the concept of fuzzy generalized e -homeomorphisms are introduced and studied.

Key Words: r-fuzzy generalized e-open (closed) sets, r-fuzzy generalized e-interior (closure) operator, fuzzy generalized e-continuous (irresolute) maps & fuzzy generalized e-homeomorphism.

1. Introduction:

In 1965, Zadeh [13] introduced the concept of fuzzy sets in his classical paper. Subsequently, many researchers have been worked in this area and related areas which have applications in different field based on this concept. Chang in [1] introduced the concept of fuzzy topological space. ˇ S ostak [10] introduced the fundamental concept of a fuzzy topological structure, as an extension of both crisp topology and Chang's fuzzy topology [1]. Chattopadhyay et al. [3] have redefined the same concept under the name gradation of openness. Ramadan [7] introduced a similar definition, namely, smooth topological space for lattice L = [0, 1] . It has been developed in many directions [2, 3, 4, 8]. The concept of generalized closed sets plays a significant role in topology. There are many research papers which deals with different types of generalized closed sets. Levine [6] introduced the concept of generalized closed sets (briefly g-closed) in topological spaces. In 2004, Kim and Ko [5] introduced the concept of r-generalized fuzzy closed sets in fuzzy topological spaces. In 2016, Vadivel and Elavarasan [11] introduced the concept of r-generalized regular fuzzy closed sets in fuzzy topological spaces. The concept of r-fuzzy e-open sets and fuzzy e-continuity and their properties were defined by Vadivel et al, [12]. In this paper, the notion of r-fuzzy generalized e-closed set is introduced and its properties are studied. Also fuzzy generalized e-continuous, fuzzy generalized e-irresolute, fuzzy generalized e-homeomorphism and their properties are investigated with the help of r-fuzzy generalized e-open sets.

2. Preliminaries:

Throughout this paper, let X be a non-empty set, $I = [0,1]$, $I_0 = (0,1]$. A fuzzy set $\boldsymbol{\lambda}$ of X is a mapping $\lambda: X \to I$, and I^X be the family of all fuzzy sets on X. The complement of a fuzzy set $\boldsymbol{\lambda}$ is denoted by $1-\boldsymbol{\lambda}$. For $\lambda \in I^X$, $\bar{\lambda}(x) = \boldsymbol{\lambda}$ for all $x \in X$. For each $x \in X$ and $t \in I_0$, a fuzzy point x_t is defined by $x_t(y) = \begin{cases} t, & \text{if } y = x \\ 0, & \text{if } y \neq x. \end{cases}$ Let Pt(X) be the family of all fuzzy points in X. All other notations and definitions are standard in the fuzzy set theory.

Definition 2.1: [10] A function $\tau : I^X \to I$ is called a fuzzy topology on X if it satisfies the following conditions:

(1) $\tau(\bar{0}) = \tau(\bar{1}) = 1$,
(2) $\tau\left(\vee_{i\in J}\mu_i\right) \geq \wedge_{i\in J}\tau(\mu_i)$, for any $\{\mu_i : i \in J\} \subseteq I^X$.
(3) $\tau(\mu_1 \wedge \mu_2) \geq \tau(\mu_1)\wedge\tau(\mu_2)$, for all $\mu_1, \mu_2 \in I^X$.

The pair (X,τ) is called a fuzzy topological space (for short, fts). A fuzzy set $\boldsymbol{\lambda}$ is called r-fuzzy open (for short r-fo) if $\tau(\lambda) \geq r$. A fuzzy set $\boldsymbol{\lambda}$ is called r-fuzzy closed (for short r-fc) if $\tau(\bar{1} - \lambda) \geq r$.

Definition 2.2: [2] Let (X,τ) be a fuzzy topological space. Then for each $\lambda \in I^X$ and $r \in I_0$, we define an operator $C_\tau, I_\tau : I^X \times I_0 \to I^X$ as follows:

$C_\tau(\lambda, r) = \wedge\{\mu \in I^X : \lambda \leq \mu,\ \tau(\bar{1} - \mu) \geq r\}$.
$I_\tau(\lambda, r) = \vee\{\mu \in I^X : \lambda \geq \mu,\ \tau(\mu) \geq r\}$.

Definition 2.3: [9] Let (X,τ) be a fts. For $\lambda \in I^X$ and $r \in I_0$. Then a fuzzy set $\boldsymbol{\lambda}$ is called r-fuzzy regular open (for short, r-fro) (resp. r-fuzzy regular closed (for short, r-frc)) if $\boldsymbol{\lambda} = I_\tau(C_\tau(\lambda, r), r)$ (resp. $\boldsymbol{\lambda} = C_\tau(I_\tau(\lambda, r), r)$).

Definition 2.4: [12] Let (X,τ) be a fts. For $\lambda \in I^X$ and $r \in I_0$. The r-fuzzy δ-closure (resp. δ-interior) of $\boldsymbol{\lambda}$ denoted by $\delta\text{–}C_\tau(\lambda, r)$ (resp. $\delta\text{–}I_\tau(\lambda, r)$), and is defined by

$\delta\text{–}C_\tau(\lambda, r) = \wedge\{\mu \in I^X : \lambda \leq \mu,\ \mu \text{ is } r - frc\}$.
$\delta\text{–}I_\tau(\lambda, r) = \vee\{\mu \in I^X : \lambda \geq \mu,\ \mu \text{ is } r - fro\}$.

Definition 2.5: [12] Let (X,τ) be a fts. For $\lambda \in I^X$ and $r \in I_0$. Then a fuzzy set $\boldsymbol{\lambda}$ is called r-fuzzy e-open (for short, r-feo) (resp. r-fuzzy e-closed (for short, r-fec)) if $\lambda \leq I_\tau(\delta\text{–}C_\tau(\lambda, r), r) \vee C_\tau(\delta\text{–}I_\tau(\lambda, r), r)$ (resp. $\lambda \geq I_\tau(\delta\text{–}C_\tau(\lambda, r), r) \wedge C_\tau(\delta\text{–}I_\tau(\lambda, r), r)$).

Definition 2.4: [12] Let (X,τ) be a fts. For $\lambda \in I^X$ and $r \in I_0$. The r-fuzzy e-closure (resp. e-interior) of $\boldsymbol{\lambda}$ denoted by $e\text{–}C_\tau(\lambda, r)$ (resp. $e\text{–}I_\tau(\lambda, r)$), and is defined by

$e\text{–}C_\tau(\lambda, r) = \wedge\{\mu \in I^X : \lambda \leq \mu,\ \mu \text{ is } r - fec\}$.
$e\text{–}I_\tau(\lambda, r) = \vee\{\mu \in I^X : \lambda \geq \mu,\ \mu \text{ is } r - feo\}$.

Definition 2.5: [12] Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be a mapping from a fts (X,τ) to another fts (Y,σ). Then f is called fuzzy e-continuous (resp. fuzzy e-closed, fuzzy e-irresolute iff $f^{-1}(\lambda)$ is a r-feo set in I^X for any $\lambda\in I^Y$ and $r\in I_0$ with $\sigma(\mu)\geq r$ (resp. $f(\lambda)$ is a r-fec set in I^Y for any $\lambda\in I^X$ and $r\in I_0$ with $\tau(\bar{1}-\mu)\geq r$, $f^{-1}(\lambda)$ is a r-fec set in I^X for any r-fec set $\lambda\in I^Y$ and $r\in I_0$).

3. r-Fuzzy Generalized e-Closed Sets:

In this section, we define r-fuzzy generalized e-closed (open) sets and study some of their basic properties.

Definition 3.1: Let (X,τ) be a fts. A fuzzy set $\lambda\in I^X$, $r\in I_0$ is called:

(1) r-fuzzy generalized e-closed (in short, r-fgec) set if $C_\tau(\lambda,r)\leq\mu$, whenever $\lambda\leq\mu$ and μ is r-feo set.
(2) r-fuzzy generalized e-open (in short, r-fgeo) set if $\mu\leq I_\tau(\lambda,r)$ whenever $\mu\leq\lambda$ and μ is r -fec set.

The complement of a r-fuzzy generalized e-closed set is called a r-fuzzy generalized e -open set.

Example 3.1: Let $X=\{a,b,c\}$, $\lambda,\mu,\alpha,\beta\in I^X$ are defined as $\lambda(a)=0.3,\lambda(b)=0.4,\lambda(c)=0.5$; $\mu(a)=0.6,\mu(b)=0.5,\mu(c)=0.5$; $\alpha(a)=0.4,\alpha(b)=0.5,\alpha(c)=0.5$; $\beta(a)=0.4$, $\beta(b)=0.5,\beta(c)=0.6$. We define smooth topology $\tau: I^X\rightarrow I$ as follows:

$$\tau(\lambda)=\begin{cases}1 & if \quad \lambda\in\bar{0}\ or\ \bar{1}\\ \frac{1}{2} & if \quad \lambda=\lambda\\ \frac{1}{2} & if \quad \lambda=\mu\\ 0 & otherwise\end{cases}$$

Then, for $r=1/2$, the fuzzy set α is r-fgec set, since $C_\tau(\alpha,r)\leq\alpha$, $\alpha\leq\alpha$ and α is r-feo set in X.

Remark 3.1: Every r-fuzzy closed set is r-fgec set but the converse may not be true in general.

Example 3.2: In Example 3.1, the fuzzy set β is r-fgec set but not r-fc set.

Theorem 3.1: If $\boldsymbol{\lambda}$ is r-fgec set and r-feo set in (X,τ), then $\boldsymbol{\lambda}$ is r-fuzzy closed in (X,τ).

Proof: Let $\boldsymbol{\lambda}$ be r-fgec set and r-feo set in X. For $\boldsymbol{\lambda}\leq\boldsymbol{\lambda}$, by Definition 3.1, $C_\tau(\lambda,r)\leq\boldsymbol{\lambda}$. But $\lambda\leq C_\tau(\lambda,r)$ which implies $\lambda=C_\tau(\lambda,r)$. Hence $\boldsymbol{\lambda}$ is r-fuzzy closed set in X.

Definition 3.2: Let (X,τ) be a fts. For $\lambda\in I^X$ and $r\in I_0$.

(1) The r-fuzzy generalized e-closure of $\boldsymbol{\lambda}$ denoted by $FGeC_\tau(\lambda,r)$ and is defined by $FGeC_\tau(\lambda,r)=\wedge\{\mu\in I^X:\lambda\leq\mu,\ \mu\ is\ r\!-\!fgec.$
(2) The r-fuzzy generalized e-interior of $\boldsymbol{\lambda}$ denoted by $FGeI_\tau(\lambda,r)$, and is defined by $FGeI_\tau(\lambda,r)=\vee\{\mu\in I^X:\lambda\geq\mu,\ \mu\ is\ r\!-\!fgeo.$

Theorem 3.2: Let $\lambda\in I^X$, $r\in I_0$. Then (1) λ is r-fgec if $\lambda=FGeC_\tau(\lambda,r)$. (2) $FGeC_\tau(\lambda,r)$ is r-fgec in X.

Proof: Obvious from Definition 3.2.

Theorem 3.3: Union of two r-fgec sets is an r-fgec set.

Proof: Let $\lambda,\mu\in I^X$, $r\in I_0$ be any two r-fgec sets and let $\lambda\vee\mu\leq\gamma$, where γ is r-feo. Then $\lambda\leq\gamma$, $\mu\leq\gamma$ and since λ, μ are r-fgec, we have $C_\tau(\lambda,r)\leq\gamma$, $C_\tau(\mu,r)\leq\gamma$. So $C_\tau(\lambda\vee\mu,r)\leq C_\tau(\lambda,r)\vee C_\tau(\mu,r)\leq\gamma\Rightarrow\lambda\vee\mu$ is r-fgec set.

Remark 3.2: Intersection of two r-fgec sets may not be an r-fgec set as shown by the following example.

Example 3.3: Let $X=\{a,b,c\}$, $\lambda,\mu,\alpha,\beta\in I^X$ are defined as $\lambda(a)=0.3,\lambda(b)=0.4,\lambda(c)=0.5$; $\mu(a)=0.6,\mu(b)=0.5,\mu(c)=0.5$; $\alpha(a)=0.3,\alpha(b)=0.5,\alpha(c)=0.2$; $\beta(a)=0.4$, $\beta(b)=0.4,\beta(c)=0.4$. We define smooth topology $\tau: I^X\rightarrow I$ as follows:

$$\tau(\lambda)=\begin{cases}1 & if \quad \lambda\in\bar{0}\ or\ \bar{1}\\ \frac{1}{2} & if \quad \lambda=\lambda\\ \frac{1}{2} & if \quad \lambda=\mu\\ 0 & otherwise\end{cases}$$

Then, for $r=1/2$, the fuzzy set α is r-fgec set, $C_\tau(\alpha,r)(=\mu)\leq\mu$, $\alpha\leq\mu$ where μ is r-feo.
Also, β is r-fgec, for $C_\tau(\beta,r)(=\mu)\leq\mu$, $\beta\leq\mu$ where μ is r-feo. But, $\alpha\wedge\beta$ is not r-fgec set, since $C_\tau(\alpha\wedge\beta,r)(=\mu)\nleq\lambda$, $\alpha\wedge\beta\leq\boldsymbol{\lambda}$ where λ is r-feo.

Theorem 3.4: If λ is r-fgec set in (X,τ) and $\lambda\leq\mu\leq C_\tau(\lambda,r)$, then μ is r-fgec set in (X,τ).

Proof: Let γ be r-feo set of (X,τ) such that $\mu\leq\gamma$. Then we get $\lambda\leq\gamma$. Since, $\boldsymbol{\lambda}$ is r-fgec set, it follows that $C_\tau(\lambda,r)\leq\gamma$. Now, $\mu\leq C_\tau(\lambda,r)$ implies $C_\tau(\mu,r)\leq C_\tau(C_\tau(\lambda,r),r)=C_\tau(\lambda,r)$. Thus $C_\tau(\mu,r)\leq\gamma$. This proves that μ is also a r-fgec set of (X,τ).

Theorem 3.5: If λ is r-fgeo set in (X,τ) and $I_\tau(\lambda,r)\leq\mu\leq\lambda$, then μ is r-fgeo set in (X,τ).

Proof: Similarly by above Theorem 3.4.

Theorem 3.6: Let (X,τ) be the fts and $\lambda\in I^X$, $r\in I_0$. Then $\boldsymbol{\lambda}$ is r-fgec set if and only if $\lambda\bar{q}\mu$ implies $C_\tau(\lambda,r)\bar{q}\mu$, for every r-fec set μ of X.

Proof: Suppose λ be a r-fgec set of X. Let μ be a r-fec set in X such that $\lambda\bar{q}\mu$. Then, by Definition 3.1. that implies $\lambda\leq\bar{1}-\mu$ and $\bar{1}-\mu$ is r-feo set of X. Therefore, $C_\tau(\lambda,r)\leq C_\tau(\bar{1}-\mu,r)\leq\bar{1}-\mu$, as λ is r-fgec set. Hence, $C_\tau(\lambda,r)\bar{q}\mu$.

Conversely, let γ be a r-feo set in X such that $\lambda\leq\gamma$. Then, by Definition 3.1,
$\lambda\bar{q}\ \bar{1}-\gamma$ and $\bar{1}-\gamma$ is r-fec set in X. By hypothesis, $C_\tau(\lambda,r)\bar{q}(\bar{1}-\gamma)$ which implies $C_\tau(\lambda,r)\leq\gamma$. Hence, $\boldsymbol{\lambda}$ is r-fgec set.

Theorem 3.7: Let $\boldsymbol{\lambda}$ be r-fgec set in (X,τ) and x_p be a fuzzy point of (X,τ), such that $x_p\ q\ C_\tau(\lambda,r)$ then $C_\tau(x_p)\ q\ \boldsymbol{\lambda}$.

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Proof: Let λ be r-fgec set and x_p be a fuzzy point of X. Suppose, $C_\tau(x_p)\bar{q}\,\boldsymbol{\lambda}$, then by Definition3.1, $C_\tau(x_p) \leq \bar{1} - \lambda$ which implies $\lambda \leq \bar{1} - C_\tau(x_p)$. So, $C_\tau(\lambda, r) \leq \bar{1} - C_\tau(x_p) \leq \bar{1} - x_p$, because, $\bar{1} - C_\tau(x_p)$ is r-fuzzy open in X and $\boldsymbol{\lambda}$ is r-fgec set in X. Hence, $x_p\ \bar{q}\ C_\tau(\lambda, r)$, which is a contradiction.

4. Fuzzy Generalized e-Continuous and Fuzzy Generalized e-Irresolute Mappings:

Definition 4.1: Let (X,τ) and (Y,σ) be two fts's. A mapping $f : (X,\tau) \to (Y,\sigma)$ is said to be:

(1) fuzzy generalized e-continuous (for short, fge-continuous) mapping iff $f^{-1}(\lambda)$ is r-fgeo set in I^X for every $\lambda \in I^Y$, $r \in I_0$ with $\tau(\mu) \geq r$.

(2) fuzzy generalized e-irresolute (for short, fge-irresolute) mapping iff $f^{-1}(\lambda)$ is r -fgeo in I^X for every r -fgeo set $\lambda \in I^Y$, $r \in I_0$.

Example 4.1: Let $X = Y = \{a,b,c\}$, $\lambda, \mu \in I^X$ and $\gamma \in I^Y$ are defined as $\lambda(a) = 0.3, \lambda(b) = 0.4, \lambda(c) = 0.5$; $\mu(a) = 0.6, \mu(b) = 0.5, \mu(c) = 0.5$; $\gamma(a) = 0.4, \gamma(b) = 0.5, \gamma(c) = 0.5$. We define smooth topology $\tau, \sigma: I^X \to I$ as follows:

$$\tau(\lambda) = \begin{cases} 1 & if \quad \lambda \in \bar{0}\ or\ \bar{1} \\ \frac{1}{2} & if \quad \lambda = \lambda \\ \frac{1}{2} & if \quad \lambda = \mu \\ 0 & otherwise \end{cases} \qquad \sigma(\lambda) = \begin{cases} 1 & if \quad \lambda \in \bar{0}\ or\ \bar{1} \\ \frac{1}{2} & if \quad \lambda = \gamma \\ 0 & otherwise \end{cases}$$

Consider the identity mapping $f : (X,\tau) \to (Y,\sigma)$ defined by $f(x) = x$, $\forall x \in X$. Then for every r-fuzzy open set γ in Y, $f^{-1}(\gamma)$ is r-fuzzy generalized e-open set in X. Thus f is a fuzzy generalized e-continuous.

Theorem 4.1: Every fge-irresolute map is fge-continuous.

Proof: Let $f : (X,\tau) \to (Y,\sigma)$ be fge-irresolute and let $\boldsymbol{\lambda}$ be r-fuzzy closed set in Y. Since every r-fuzzy closed set is also r-fgec, $\boldsymbol{\lambda}$ is r-fgec in Y. Since, $f : (X,\tau) \to (Y,\sigma)$ is fge-irresolute, we have $f^{-1}(\lambda)$ is r-fgec. Thus, the inverse image of every r-fuzzy closed set in Y is r-fgec in X. Therefore, the function $f : (X,\tau) \to (Y,\sigma)$ is fge-continuous.

The converse of the above theorem need not be true as shown in the following example.

Example 4.2: In Example 4.1, Let us consider the fuzzy set $\delta \in I^Y$ as follows:

$\delta(a) = 0.3, \delta(b) = 0.5, \delta(c) = 0.2$. The identity mapping $f : (X,\tau) \to (Y,\sigma)$ is fuzzy generalized e-continuous but not fuzzy generalized e-irresolute mapping, since the r-fo set in Y, $f^{-1}(\delta)$ is r-fgeo set in X and the fuzzy set δ is r-fgeo set in Y but not r-fgeo set in X.

Theorem 4.2: Every fuzzy continuous map is fge-continuous.

Proof: Obvious.

However, the converse need not be true as shown in the following example.

Example 4.3: In Example 4.1, the function f is fge-continuous but not fuzzy continuous, since $\sigma(\alpha) \nleq \tau(f^{-1}(\alpha))(= \lambda)$.

Theorem 4.3: If $f : (X,\tau) \to (Y,\sigma)$ is fge-continuous if and only if the inverse image of r-fuzzy open set of Y is r-fgeo set of X.

Proof: Let $\boldsymbol{\lambda}$ be a r-fgeo set of Y. Then, $\bar{1} - \boldsymbol{\lambda}$ is r-fgec set in Y. Since, $f : (X,\tau) \to (Y,\sigma)$ is fge-continuous, $f^{-1}(\bar{1} - \boldsymbol{\lambda}) = \bar{1} - f^{-1}(\boldsymbol{\lambda})$ is r-fgec set of X. i.e, $f^{-1}(\boldsymbol{\lambda})$ is r-fgeo set of X.

The converse is obvious.

Theorem 4.4: If $f : (X,\tau) \to (Y,\sigma)$ is fge-continuous, then

(1) for each fuzzy point x_α of X and each $\lambda \in Y$ such that $f(x_\alpha)\ q\ \lambda$, there exists a r-fgeo set μ of X such that $x_\alpha \in \mu$ and $f(\mu) \leq \lambda$.

(2) for each fuzzy point x_α of X and each $\lambda \in Y$ such that $f(x_\alpha)\ q\ \lambda$, there exists a r-fgeo set μ of X such that $x_\alpha q\ \mu$ and $f(\mu) \leq \lambda$.

Proo:. (1) Let x_α be a fuzzy point of X. Then, $f(x_\alpha)$ is a fuzzy point in Y. Now, let $\lambda \in Y$ be r-fgeo set such that $f(x_\alpha)\ q\ \lambda$. For, $\mu = f^{-1}(\boldsymbol{\lambda})$ as f is fge-continuous, we have μ is r-fgeo set of X and $x_\alpha \in \mu$. Therefore, $f(\mu) = f(f^{-1})(\lambda) \leq \boldsymbol{\lambda}$.

(2) Let x_α be a fuzzy point of X and $\lambda \in Y$ such that $f(x_\alpha)\ q\ \lambda$. Taking $\mu = f^{-1}(\boldsymbol{\lambda})$, we get μ is r-fgeo set of X such that $x_\alpha q\ \mu$ and $f(\mu) = f(f^{-1})(\lambda) \leq \boldsymbol{\lambda}$.

Definition 4.2: A fuzzy topological space X is r-fuzzy e-$T_{1/2}$-space (briefly, r-fe$T_{1/2}$-space) if every r-fgec set in X is r-fuzzy closed set in X

Remark 4.1: In the Remark 3.1, we have established by an example that in an fts, r-fgec set may not be a r-fuzzy closed set. Hence, we can remark here that there are spaces which are not r- fe$T_{1/2}$-space.

Theorem 4.5: A fuzzy topological space X is r- fe$T_{1/2}$-space iff every fuzzy set in X is both r-fo and r-fgeo.

Proof: Let X be r- fe$T_{1/2}$-space and let μ be r-fgeo in X. Then $\bar{1} - \mu$ is r-fgec. By hypothesis, every r-fgec set is r-fuzzy closed, $\bar{1} - \mu$ is r-fuzzy closed and hence μ is r-fuzzy open in X.

Conversely, let μ be r-fgec. Then $\bar{1} - \mu$ is r-fgeo and $\bar{1} - \mu$ is r-fuzzy open. Hence, μ is r-fuzzy closed. Every r-fgec set in X is r-fuzzy closed. Therefore, X is r-fe$T_{1/2}$-space.

Definition 4.3: Let (X,τ) and (Y,σ) be two fts's. A mapping $f : (X,\tau) \to (Y,\sigma)$ is said to be:

(1) fuzzy generalized e-open (for short, fge-open) map if $f(\lambda)$ is r-fgeo set in Y for each $\lambda \in I^Y$, $r \in I_0$ with $\tau(\mu) \geq r$.
(2) fuzzy generalized e-closed (for short, fge-closed) map if $f(\lambda)$ is r-fgec set in Y for each $\lambda \in I^X$, $r \in I_0$ with $\tau(\bar{1} - \mu) \geq r$.
(3) fuzzy generalized e-irresolute open (for short, fgei-open) map if $f(\lambda)$ is r-fgeo set Y for each r-fgeo set $\lambda \in I^X$, $r \in I_0$.
(4) fuzzy generalized e-irresolute closed (for short, fgei-closed) map if $f(\lambda)$ is r-fgec set Y for each r-fgec set $\lambda \in I^X$, $r \in I_0$.

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Theorem 4.6: If $\boldsymbol{\lambda}$ is r-fgec in X and $f : (X,\tau) \to (Y,\sigma)$ is bijective, fuzzy e-irresolute and fge-closed map, then $f(\lambda)$ is r-fgec in Y.

Proof: Let $f(\lambda) \leq \mu$ where μ is r-feo in Y. Since f is fuzzy e-irresolute, $f^{-1}(\mu)$ is r-feo containing $\boldsymbol{\lambda}$. Hence $C_\tau(\lambda, r) \leq f^{-1}(\mu)$ as $\boldsymbol{\lambda}$ is r-fgec. Since f is fge-closed map, $f(C_\tau(\lambda, r))$ is r-fgec set contained in the r-feo set μ which implies that $C_\tau(f(C_\tau(\lambda, r), r) \leq \mu$ and hence $C\tau(f(\lambda), r) \leq \mu$. So $f(\lambda)$ is r-fgec in Y.

Theorem 4.7: Let $f : (X,\tau) \to (Y,\sigma)$ be onto, fge-irresolute and fuzzy closed. If X is r- feT$_{1/2}$-space, then (Y,σ) is r-feT$_{1/2}$-space.

Proof: Let μ be a r-fgec set in Y. Since $f : X \to Y$ is fge-irresolute, $f^{-1}(\mu)$ is r-fgec set in X. As X is r-feT$_{1/2}$-space, $f^{-1}(\mu)$ is r-fuzzy closed set in X. Also $f : X \to Y$ is fuzzy closed mapping, $f(f^{-1}(\mu))$ is r-fuzzy closed in Y. Since, $f : X \to Y$ is onto, $f(f^{-1}(\mu)) \leq \mu$. Thus μ is r-fuzzy closed in Y. Hence, (Y,σ) is r- feT$_{1/2}$-space.

Theorem 4.8: Let $f : (X,\tau) \to (Y,\sigma)$. Then the following statements are equivalent.

(1) f is fge-irresolute.

(2) for every r-fgec set $\lambda \in I^Y$, $f^{-1}(\lambda)$ is r-fgec in X with , r ∈ I_0.

(3) for every fuzzy point x_p of X and every r-fgeo set $\lambda \in I^Y$, r ∈ I_0 such that $f(x_p) \in \lambda$ there exists a r-fgeo set such that $x_p \in \mu$ and $f(\mu) \leq \lambda$.

Proof: (1) ⇒ (2) : Obvious.

(2) ⇒ (3) : Let $\boldsymbol{\lambda}$ be a r-fgeo set in Y which implies $\bar{1} - \boldsymbol{\lambda}$ is r-fgec in Y. By (2), $f^{-1}(\lambda)$ is r-fgeo in X, $f^{-1}(\bar{1} - \boldsymbol{\lambda})$ is r-fgec in X. Let x_p be a fuzzy point of X such that $f(x_p) \in \lambda$ implies that $x_p \in f^{-1}(\lambda)$ is r-fgeo in X. Let $\mu = f^{-1}(\lambda)$ which implies that $f(\mu) = f(f^{-1}(\lambda)) \leq \lambda$.

(3) ⇒ (1) : Let $\boldsymbol{\lambda}$ be a r-fgeo set in Y and $x_p \in f^{-1}(\lambda)$ which implies $f(x_p) \in \lambda$. Then there exists a r-fgeo set μ in X such that $x_p \in \mu$ and $f(\mu) \leq \boldsymbol{\lambda}$. Hence $x_p \in \mu \leq f^{-1}(\lambda)$ and $f^{-1}(\lambda)$ is r-fgeo in X. Hence f is fge-irresolute.

Theorem 4.9: Let $f : (X,\tau) \to (Y,\sigma)$, $g : (Y,\sigma) \to (Z,\rho)$ be two maps such that $g \circ f : (X,\tau) \to (Z, \rho)$ is fge-closed map.

(1) If f is fuzzy continuous and surjective, then g is fge-closed map.

(2) If g is fuzzy fge-irresolute and injective, then f is fge-closed map.

Proof: (1) Let $\boldsymbol{\lambda}$ be r-fuzzy closed in Y. Then $f^{-1}(\lambda)$ is r-fuzzy closed in X, as f is fuzzy continuous. Since $g \circ f$ is fge-closed map and f is surjective, $(g \circ f)(f^{-1}(\lambda)) = g(\lambda)$ is r-fgec in Z. Hence $g : Y \to Z$ is fge-closed map.

(2) Let $\boldsymbol{\lambda}$ be a r-fuzzy closed in X. Then $(g \circ f)(\lambda)$ is r-fgec in Z. Since, g is fge-irresolute and injective$f^{-1}(g \circ f)(\lambda) = f(\lambda)$ is r-fgec in Y. Hence f is a fge-closed map.

Theorem 4.10: Let $f : (X,\tau) \to (Y,\sigma)$, $g : (Y,\sigma) \to (Z,\rho)$ be two maps such that $g \circ f : (X,\tau) \to (Z, \rho)$ is fgei-closed map.

(1) If f is fuzzy continuous and surjective, then g is fge-closed map.

(2) If g is fuzzy fge-irresolute and injective, then f is fgei-closed map.

Proof: Obvious.

Theorem 4.11: For the functions $f : X \to Y$ and $g : Y \to Z$ the following relations hold:

(1) If $f : X \to Y$ is fge-continuous and $g : Y \to Z$ is fuzzy continuous then $g \circ f : X \to Z$ is fge-continuous.

(2) If $f : X \to Y$ and $g : Y \to Z$ are fge-irresolute then $g \circ f : X \to Z$ is fge-irresolute.

(3) If $f : X \to Y$ is fge-irresolute and $g : Y \to Z$ is fge-continuous then $g \circ f : X \to Z$ is fge-continuous.

Proof: Omitted.

Theorem 4.12: If $f : X \to Y$ is fge-continuous and $g : Y \to Z$ is fge-continuous then $g \circ f : X \to Z$ is fge-continuous if Y is r-feT$_{1/2}$-space.

Proof: Suppose $\boldsymbol{\lambda}$ is r-fuzzy closed set of Z. Since $g : Y \to Z$ is fge-continuous $g^{-1}(\lambda)$ is r-fgec set of Y. Now since Y is r-feT$_{1/2}$-space, $g^{-1}(\lambda)$ is r-fuzzy closed set of Y. Also since $f : X \to Y$ is fge-continuous, $f^{-1}(g^{-1}(\lambda)) = (g \circ f)^{-1}(\lambda)$ is r-fgec. Thus $g \circ f : X \to Z$ is fge-continuous.

5. Fuzzy Generalized e and e*-Homeomorphisms:

Definition 5.1: A bijective mapping $f : (X, \tau) \to (Y, \sigma)$ is called:

(1) fuzzy generalized e-homeomorphism (briefly, fge-homeomorphism) if f and f^{-1} are fge-continuous.

(2) fuzzy generalized e*-homeomorphism (briefly, fge*-homeomorphism) if f and f^{-1} are fge-irresolute.

Theorem 5.1: Every fuzzy homeomorphism is fge-homeomorphism.

Proof: Follows from Definition 5.1.

The converse of the above theorem need not be true as seen from the following example.

Example 5.1: Let $X = Y = \{a,b,c\}$, $\lambda \in I^X$ and $\mu \in I^Y$ are defined as $\lambda(a) = 0.3, \lambda(b) = 0.4, \lambda(c) = 0.5$; $\mu(a) = 0.7, \mu(b) = 0.6, \mu(c) = 0.5$. We define smooth topology $\tau, \sigma: I^X \to I$ as follows:

$$\tau(\lambda) = \begin{cases} 1 & if \quad \lambda \in \bar{0} \; or \; \bar{1} \\ \frac{1}{2} & if \quad \lambda = \lambda \\ 0 & otherwise \end{cases} \qquad \sigma(\lambda) = \begin{cases} 1 & if \quad \lambda \in \bar{0} \; or \; \bar{1} \\ \frac{1}{2} & if \quad \lambda = \mu \\ 0 & otherwise \end{cases}$$

The identity mapping $f : (X,\tau) \to (Y,\sigma)$ defined by $f(x) = x, \forall x \in X$. Then f is fge-continuous as for any r-fuzzy open set μ in Y, $f^{-1}(\mu)$ is r-fgeo set in X. Similarly, f^{-1} is also fge-continuous as for any r-fuzzy open set $\boldsymbol{\lambda}$ in X, $f(\lambda)$ is r-fgeo set in Y.

This implies f is fge-homeomorphism but not fuzzy homeomorphism as the fuzzy set μ is r-fuzzy open set in Y, but $f^{-1}(\mu) = \mu$ is not r-fuzzy open set in Y. Hence $f : Y \rightarrow X$ is not fuzzy continuous.

Theorem 5.2: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a bijective mapping. Then the following are equivalent:

(1) f is fge-homeomorphism.

(2) f is fge-continuous and fge-open map.

(3) f is fge-continuous and fge-closed map.

Proof: (1) ⇒(2): Let f be fge-homeomorphism. Then f and f^{-1} are fge-continuous. To prove that f is fge-open map, let $\boldsymbol{\lambda}$ be a r-fuzzy open set in X. Since $f^{-1} : Y \rightarrow X$ is fge-continuous, $(f^{-1})^{-1}(\lambda) = f(\lambda)$ is r-fgeo in Y. Therefore $f(\lambda)$ is r-fgeo in Y. Hence f is fge-open map.

(2) ⇒ (3): Let f be fge-continuous and fge-open map. To prove that f is fge-closed map. Let μ be a r-fuzzy closed set in X. Then $\bar{1} - \mu$ is r-fuzzy open set in X. Since f is fge-open map, $f(\bar{1} - \mu)$ is r-fgeo set in Y. Now, $f(\bar{1} - \mu) = \bar{1} - f(\mu)$. Therefore $f(\mu)$ is r-fgec in Y. Hence, f is a fge-closed map.

(3) ⇒(1): Let f be fge-continuous and fge-closed map. To prove that f^{-1} is fge-continuous. Let $\boldsymbol{\lambda}$ be a r-fuzzy open set in X. Then $\bar{1} - \boldsymbol{\lambda}$ is a r-fuzzy closed set in X. Since f is fge-closed map, $f(\bar{1} - \boldsymbol{\lambda})$ is r -fgec set in Y. Now, $(f^{-1})^{-1}(\bar{1} - \boldsymbol{\lambda}) = f(\bar{1} - \boldsymbol{\lambda}) = \bar{1} - f(\boldsymbol{\lambda})$**.** Therefore $f(\boldsymbol{\lambda})$ is r-fgeo set in Y. Hence $f^{-1} : Y \rightarrow X$ is fge-continuous. Thus f is fge-homeomorphism.

Theorem 5.3: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a bijective mapping. Then the following are equivalent:

(1) f is fge*-homeomorphism.

(2) f is fge-irresolute and fgei-open map.

(3) f is fge-irresolute and fgei-closed map.

Proof: Proof follows from the above theorem 5.2.

Conclusion:

It is interesting to work on r -fge-closed sets, fge-continuous and fge-irresolute mappings and various properties of these things. Compositions of mappings can be tried with other forms of fge-irresolute mappings.

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