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E -LOCAL FUNCTIONS AND ψ_e-OPERATOR IN IDEAL TOPOLOGICAL SPACES

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Abstract:

The main goal of this paper is to introduce another local function to give possibility of obtaining a Kuratowski closure operator. On the other hand, e-local functions defined for ideal topological spaces have not been found in the current literature. e-local functions for the ideal topological spaces has been described within this work. Moreover, with the help of e-local functions Kuratowski closure operators cl_I^{*e} and τ^{*e} topology are obtained. Many theorems in the literature have been revised according to the definition of e-local functions.

Key Words: e-Open Set, Ideal Topological Space, Local Function, e-Local Function, eO-Codense Ideal & e-Compatibility Topology with an Ideal.

1. Introduction:

The studies about ideal topological space has been enriched by so many mathematicians. Hamlett and Jankovic [6] were able to define a closure operator with the help of local functions, and hence defined a new topology.

Lately, local functions on a spaces in which topology is replaced by its generalized open sets worked by many mathematicians [1, 8, 14].

Jain introduced totally continuous functions in classical topology as a generalization of continuous function and as consequence of this e-open sets, fuzzy e-continuous and e-compactness was introduced and studied by Ekici [3, 4, 5]. This concept was found to be useful and many results in general topology were included. Many researchers have worked on this and related problems in general topology.

This paper deals with a space in which topology is replaced by the family of e-open sets.

2. Preliminaries:

Let (X,τ) be a topological space with no separation properties assumed. For a subset A of a topological space (X,τ), $cl(A)$ and $\text{int}(A)$ denote the closure and interior of A in (X,τ), respectively.

An ideal I on a topological space (X,τ) is a non-empty collection of subsets of X which satisfies the following properties [9]:

(1) $A\in I$ and $B\subseteq A$ implies $B\in I$ (heredity),

(2) $A\in I$ and $B\in A$ implies $A\cup B\in I$ (finite additivity).

An ideal topological space is a topological space (X,τ) with an ideal I on X andis denoted by (X,τ,I). For a subset $A\in X$, $A^*(I,\tau)=\{x\in X : A\cap U\notin I, \text{forevery}\, U\in\tau(X,x)\}$ is called the local function of A with respect to I and τ, where $\tau(X,x)=\{U\in\tau : x\in U\}$ [7]. We simply write A^* instead of $A^*(I)$ in case there is no chance for confusion. For every ideal topological space (X,τ,I), there exists a topology $\tau^*(I)$, finer than τ, generated by the base $\beta(I,\tau)=\{U-J : U\in\tau \text{ and } J\in I\}$. It is known in [7] that $\beta(I.\tau)$ is not always a topology. When there is no ambiguity, $\tau^*(I)$ is denoted by τ^*. For a subset $A\subseteq X$, $cl^*(A)$ and $int^*(A)$ will, respectively, denote the closure and interior of A in (X,τ^*).

Definition12.1: [16] Let (X,τ) be a topological space. A subset A of X is said to be regular open if $A=int(cl(A))$. The complement of a regular open set is said to be regular closed. The collection of all regular open (resp. regular closed) sets in X is denoted by $RO(X)$ (resp. $RC(X)$.) The regular closure of A in (X,τ) is denoted by the intersection of all regular closed sets containing A and is denoted by $rcl(A)$.

Definition 2.2: 2[17] Let (X,τ) be a topological space. The δ-interior of a subset A of X is the union of all regular open set of X contained in A and is denoted by $Int_\delta(A)$. The subset A is called δ-open if $A=Int_\delta(A)$. i.e., a set is δ-open if itis the union of regular open sets. The complement of a δ-open is called δ-closed. Alternatively a set $A\subseteq(X,\tau)$ is called δ-closed if $A=Cl_\delta(A)$ where $Cl_\delta(A)=\{x\in X : Int(Cl(U))\cap A\neq\phi, U\in\tau \text{and } x\in U\}$.

Definition 2.3: [10,12]**3**Let (X,τ) be a topological space. A subset A of X is said to be semi open if there exists an open set U in X such that $U \subseteq X \subseteq cl(U)$. The other definition of semi open set is that: A subset A of X is said to be semi open if $A \subseteq cl(int(A))$. The complement of a semi open set is said to be semi closed. The collection of all semi open(resp. semi closed) sets in X is denoted by $SO(X)$ (resp. $SC(X)$).

Definition 2.4: [3]**4**Let (X,τ) be a topological space. A subset A of a space X is said to be e-open if $A \subseteq cl\,\mathrm{int}_\delta(A) \cup \mathrm{int}\,cl_\delta(A)$.

Definition 2.5: [2]5Let (X,τ,I) be an Ideal topological spaces and $A \subseteq X$. If $\tau \cap I = \{\phi\}$ then we say the I is codense ideal.

Definition 2.6: [13]**6**Let (X,τ,I) be an ideal topological spaces. We say the τ is compatible with the ideal I, denoted τ ~ I if the following holds for every $A \subseteq X$, if for every $x \in A$ there exists $U \in \tau(x)$ such that $U \cap A \in I$, then $A \in I$.

Definition 2.7: [8]**7** Let (X,τ,I) be a Ideal space and A a subset of X. Then $A_*(I,\tau) = \{x \in X : A \cap U \notin I \text{ for every } U \in SO(X,x)\}$ is called the semi local function of A with respect to I and τ where $SO(X,x) = \{U \in SO(X) : x \in U\}$. .

When there is no ambiguity, we will write simply A_* for $A_*(I,\tau)$.

3. e **-Local Functions:**

In this section we shall introduce e-ideal space and $()^{*e}$ operator and discuss various properties of this operator. Let (X,τ) be a topological space and I be an ideal on X, then $(X, eO(X,\tau), I)$ is called e-ideal space. Now we shall define the operator $()^{*e}$.

Definition 3.1: **8**Let $(X, eO(X,\tau), I)$ be a e-ideal space and A a subset of X. Then $A^{*e}(I, eO(X,\tau)) = \{x \in X : A \cap U \notin I \text{ for every } U \in eO(X,x)\}$ is called the e-local function of A with respect to I and τ where $eO(X,x) = \{U \in eO(X) : x \in U\}$. When there is no ambiguity, we will write simple A^{*e} for $A^{*e}(I,\tau)$.

Theorem 3.1: **9**Let $(X, eO(X,\tau), I)$ be a e-ideal space and A a subset of X.

(i) $A^{*e} \subseteq A^{*} \subseteq A^{*r}$ for every $A \subseteq X$.

(ii) $A_* = A^{*e}$ if $O(X,\tau) = eO(X,\tau)$.

(iii) If $A \in I$, then $A^{*e} = \phi$.

(iv) $(\phi)^{*e} = \phi$.

Proof: (i). Let $x \in A^{*}(I,\tau)$. Then, $A \cap U \notin I$ for every $U \in \tau$. Since every open set is e-open, therefore $x \in A^{*e}(I,\tau)$. Converse is not true in general, it is shown in Example 3.1

(ii). It is obvious from definition of local and e-local functions.

(iii). Let $A \in I$ and $x \in A^{*e}$. Then for every e-open set U containing x, $U \cap A \notin I$. On the other hand X is also e-open set. So $X \cap A = A \notin I$. It is contradiction.

(iv). Because of Theorem 3.1: 9(iii). it is obvious.

Example 3.1: **10**Let $X = \{a,b,c\}$ and $\tau = \{\phi, \{c\}, \{a,b\}, X\}$ with $I = \{\phi, \{c\}\}$. Let $A = \{a\}$ then $A^{*} = \{a,b\} = Cl(A^{*})$ and $A^{*e} = \{a\} = eCl(A^{*e})$. So $A^{*} \not\subseteq A^{*e}$.

Remark 3.1: **11**Let $(X, eO(X,\tau), I)$ be a e-ideal space and A a subset of X. Neither $A \subseteq A^{*e}$ nor $A^{*e} \subseteq A$ in general.

The following is an example that supports this remark.

Example 3.2: **12**Let $X = \{a,b,c,d\}$ and $\tau = \{\phi, \{b\}, \{c,d\}, \{b,c,d\}, X\}$ with $I = \{\phi, \{a\}\}$. For $A = \{a\}, A^{*e} = \phi$ and so $A^{*e} \subset A$. For $A = \{b,c\}, A^{*e} = \{a,b,c\}$ and so $A \subset A^{*e}$. For $A = \{a,b,c\}, A^{*e} = \{a,b,c\}$ and so $A^{*e} = A$.

Theorem 3.2: **13**Let (X,τ,I) be an ideal topological space and A, B subsets of X. Then, for e-local functions, the following properties hold:

(i)If $A \subseteq B$, then $A^{*e} \subseteq B^{*e}$,

(ii) If I, J ideal on X and $I \subseteq J$, then $A^{*e}(J) \subseteq A^{*e}(I)$.

Proof: (i). Let $x \in A^{*e}$. Then for every e-open set U_x containing x, $U_x \cap A \notin I$. Since $U_x \cap A \subseteq U_x \cap B$, then $U_x \cap B \notin I$.

(ii). Let $x \in A^{*e}(J)$. Then $A \cap U \notin J$, for every $U \in eO(X, x)$. Since $J \supseteq I$, $A \cap U \notin I$ and hence $x \in A^{*e}(I)$.

Theorem 3.3: **14**Let $(X, eO(X,\tau), I)$ be an e-ideal space and A, B subsets of X. Then, for e-local functions, the following properties hold:

(i) $A^{*e} = cl(A^{*e}) \subseteq ecl(A)$ and A^{*e} is closed in (X, τ).

(ii) $(A^{*e})^{*e} \subseteq A^{*e}$.

(iii) $A^{*e} \cup B^{*e} = (A \cup B)^{*e}$.

(iv) $(A \cap B)^{*e} \subseteq A^{*e} \cap B^{*e}$.

(v) $A^{*e} \setminus B^{*e} = (A \setminus B)^{*e} \setminus B^{*e} \subseteq (A \setminus B)^{*e}$,

Proof: (i) We have $A^{*e} \subseteq cl(A^{*e})$ in general. Let $x \in cl(A^{*e})$. Also given the set $U \in eO(X, x)$. Then $A^{*e} \cap T \neq \phi, T \in \tau(x)$. Since U is open, $A^{*e} \cap U \neq \phi$. Therefore, there exists some $y \in (A^{*e} \cap U)$ and $U \in eO(X, y)$. Since $y \in A^{*e}, A \cap U \notin I$ and hence $x \in A^{*e}$. Hence we have $cl(A^{*e}) \subseteq A^{*e}$ and hence $A^{*e} = cl(A^{*e})$. Again, let $x \in A^{*e} = cl(A^{*e})$, then $A \cap U \notin I$, for every $U \in eO(X.x)$. This implies $A \cap U \neq \phi$ for every $U \in eO(X, x)$. Therefore, $x \in ecl(A)$. This shows that $A^{*e} = cl(A^{*e}) \subseteq ecl(A^{*e})$. Since $A^{*e} = cl(A^{*e}), A^{*e}$ is closed.

(ii) Let $x \in (A^{*e})^{*e}$. Then for every $U \in eO(X, x), U \cap A^{*e} \notin I$ and hence $U \cap A^{*e} \neq \phi$. Let $y \in U \cap A^{*e}$. Then $U \in eO(X, y)$ and $y \in A^{*e}$. Hence we have $U \cap A \neq I$ and $x \in A^{*e}$. This shows that $(A^{*e})^{*e} \subseteq A^{*e}$.

(iii)By Theorem: 13 3.2(i)., we have $A^{*e} \cup B^{*e} \subseteq (A \cup B)^{*e}$. To prove the reverse inclusion, let $x \notin A^{*e} \cup B^{*e}$. Then x belongs neither to A^{*e} nor to B^{*e}. Therefore there exists $U_x, V_x \in eO(X, x)$ such that $A \cap U_x \in I$ and $B \cap V_x \in I$. Since I is additive, $(A \cap U_x) \cup (B \cap V_x) \in I$.

$$(A \cap U_x) \bigcup (B \cap V_x) = [(A \cap U_x) \bigcup V_x] \bigcap [(A \cap U_x) \bigcup B]$$

$$= (U_x \cup Vx) \bigcap (A \cup V_x) \bigcap (U_x \cup B) \bigcap (A \cup B)$$

On the other hand since $U_x \cap V_x \subseteq U_x \cup V_x, V_x \subseteq A \cup V_x$ and $U_x \subseteq B \cup U_x$, we have

$$(U_x \cup V_x) \bigcap (A \cup V_x) \bigcap (U_x \cup B) \bigcap (A \cup B) \supseteq (U_x \cap V_x) \bigcap (A \cup B)$$

Since I is heredity, $(U_x \bigcap V_x) \bigcap (A \cup B) \in I$. Since e-open sets closed under the finite intersections, $U_x \cap V_x \in eO(X, x)$ and so $x \notin (A \cup B)^{*e}$. Hence $(X \setminus A^{*e}) \bigcap (X \setminus B^{*e}) \subseteq X \setminus (A \cup B)^{*e}$ or $(A \cup B)^{*e} \subseteq A^{*e} \cup B^{*e}$.

(iv)By Theorem: 13 3.2(i)., $(A \cap B)^{*e} \subseteq A^{*e}$ and $(A \cap B)^{*e} \subseteq B^{*e}$ so $(A \cap B)^{*e} \subseteq A^{*e} \cup B^{*e}$.

(v)We have by Theorem: 14 3.3(iii)., $A^{*e} = [(A \setminus B) \bigcup (A \cap B)]^{*e} = (A \setminus B)^{*e} \bigcup (A \cap B)^{*e} \subseteq (A \setminus B)^{*e} \bigcup B^{*e}$. Thus $A^{*e} \setminus B^{*e} \subseteq (A \setminus B)^{*e} \setminus B^{*e}$.

On the other hand, by Theorem: 13 3.2(i)., $(A \setminus B)^{*e} \subseteq A^{*e}$ and hence $(A \setminus B)^{*e} \setminus B^{*e} \subseteq A^{*e} \setminus B^{*e}$. Hence $A^{*e} \setminus B^{*e} = (A \setminus B)^{*e} \setminus B^{*e} \subseteq (A \setminus B)^{*e}$.

Theorem 3.4: **15**Let $(X, eO(X,\tau), I)$ be an e-ideal space and A, B subsets of X. Then, for e-local functions, the following properties hold:

(i) If $I_0 \in I$, then $(A \setminus I_0)^{*e} = A^{*e} = (A \cup I_0)^{*e}$.

(ii) If $U \subseteq X$, then $U \cap (U \cap A)^{*e} \subseteq U \cap A^{*e}$.

(iii) If $A \subseteq X$ and $U \in eO(X, \tau)$, then $U \cap A \in I \Rightarrow U \cap A^{*e} = \phi$.

(iv) If $A \subseteq X$, then $(A \cap A^{*e})^{*e} \subseteq A^{*e}$.

Proof: (i). Since $I_0 \in I$, by Theorem: 9 3.1(iii)., $I_0^{*e} = \phi$. By Theorem 3.3(v)., $A^{*e} = (A \setminus I_0)^{*e}$ and by Theorem 3.3: 14 (iii)., $(A \cup I_0)^{*e} = A^{*e} \cup I_0^{*e} = \phi \cup A^{*e} = A^{*e}$.

(ii). Since $U \cap A \subseteq A$, by Theorem: 13 3.2(i)., $(U \cap A)^{*e} \subseteq A^{*e}$ and hence $U \bigcap (U \cap A)^{*e} \subseteq U \cap A^{*e}$.

(iii). Let $U \cap A \in I$, then for every $x \in U, x \notin A^{*e}$ because of $U \in eO(X, \tau)$. So $U \cap A^{*e} = \phi$.

(iv). By Theorem: 13 3.2(i) $(A \cap A^{*e})^{*e} \subseteq (A^{*e})^{*e}$. On the other hand, from Theorem 3.3(ii)., we have $(A \cap A^{*e})^{*e} \subseteq (A^{*e})^{*e} \subseteq A^{*e}$.

In literature [7] for ideal topological spaces we will obtain $cl^* = A \cup A^*$ Kuratowski Closure operator. But in [8,14] and [15] we are not able to define a Kuratowski Closure operator with the help of () local function. Because that functions do not provide a Theorem: 14 3.3(iii), given above for $()^{*e}$ -Operator.

We are able to define a closure operator with the help of e-local function. Because the $()^{*e}$ operator satisfy the conditions of Theorem: 9 3.1(iv)., Theorem: 14 3.3(ii). and Theorem: 14 3.3(iii). And thus

$Cl_I^{*e} : \wp(X) \to \wp(X), Cl_I^{*e} = A \cup A^{*e}, \forall A \in \wp(X)$ is a Kuratowski closure operator. Hence it generates a τ^{*e} topology:

$$\tau^{*e}(I) = \{A \in \wp(X) : cl^{*e}(X \setminus A) = X \setminus A\}$$

4. ψ_e -Operator:

In topological space $cl(A) = X \setminus int(X \setminus A)$ [9] is remarkable result. Many useful result have been proved with the help of this result. This relation is the motivation of defining the operator ψ_e.

Definition 4.1: 16 Let $(X, eO(X,\tau), I)$ be a e -ideal space. An operator $\psi_e : \wp(X) \to \tau$ is defined as: $\psi_e(A) = \{x \in X \mid \exists U_x \in eO(X,x) : U_x \setminus A \in I\}$, for every $A \in \wp(X)$.

We observe that $\psi_e(A) = X \setminus (X \setminus A)^{*e}$.

Theorem 4.1: 17 Let $(X, eO(X,\tau), I)$ be a e -ideal space and $A, B \in \wp(X)$.

(i) $\psi_e(A) \supseteq eint(A)$.

(ii) $\psi_e(A)$ is open.

(iii) If $A \subseteq B$, then $\psi_e(A) \subseteq \psi_e(B)$.

(iv) $\psi_e(A) \cup \psi_e(B) \subseteq \psi_e(A \cup B)$.

(v) $\psi_e(A \cap B) = \psi_e(A) \cap \psi_e(B)$.

(vi) $\psi_e(A) \subseteq \psi(A)$.

Proof: (i). $\psi_e(A) = X \setminus (X \setminus A)^{*e} \supseteq X \setminus ecl(X \setminus A)$ by Theorem: 14 3.3(i). So $\psi_e(A) \supseteq eint(A)$.

(ii). Since A^{*e} is closed, then $(X \setminus A)^{*e}$ is closed. So $X \setminus (X \setminus A)^{*e} = \psi_e(A)$ is a open set.

(iii). $A \subseteq B \Rightarrow X \setminus A \supseteq X \setminus B \Rightarrow (X \setminus A)^{*e} \supseteq (X \setminus B)^{*e}$

$\Rightarrow X \setminus (X \setminus A)^{*e} \subseteq X \setminus (X \setminus B)^{*e}$

$\Rightarrow \psi_e(A) \subseteq \psi_e(B)$

(iv). Proof is obvious from Theorem: 17 4.1(iii).

(v). $\psi_e(A \cap B) = X \setminus [X \setminus (A \cap B)]^{*e}$

$= X \setminus [(X \setminus A) \cup (X \setminus B)]^{*e}$

$= X \setminus [(X \setminus A)^{*e} \cup (X \setminus B)^{*e}]$

$= [X \setminus (X \setminus A)^{*e}] \cap [X \setminus (X \setminus B)^{*e}]$

$= \psi_e(A) \cap \psi_e(B)$

(vi). From Theorem: 9 3.1(i)., we have that

$(X\setminus A)^{*}\subseteq(X\setminus A)^{*e}\Rightarrow X\setminus(X\setminus A)^{*e}\subseteq X\setminus(X\setminus A)^{*}$

$\Rightarrow\psi_e(A)\subseteq\psi(A)$

Theorem 4.218: Let $(X,eO(X,\tau),I)$ be a e-ideal space and $A,B\in\wp(X)$.

(i) $\psi_e(A)=\psi_e(\psi_e(A))$ if and only if $(X\setminus A)^{*e}=[(X\setminus A)^{*e}]^{*e}$.

(ii) If $I_0\in I$, then $\psi_e(A\setminus I_0)=\psi_e(A)$.

(iii) If $I_0\in I$, then $\psi_e(A\cup I_0)=\psi_e(A)$.

(iv) If $(A\setminus B)\cup(B\setminus A)\in I$, then $\psi_e(A)=\psi_e(B)$.

(v) If $A\in eO(X,\tau)$, then $A\subseteq\psi_e(A)$.

Proof: (i). Proof is obvious from definition of $\psi_e(A)$ and the fact:

$\psi_e(\psi_e(A)) = X\setminus[X\setminus(X\setminus(X\setminus A)^{*e})]^{*e} = X\setminus[(X\setminus A)^{*e}]^{*e}$.

(ii). By Theorem: 15 3.4(ii)., we have

$\psi_e(A\setminus I_0) = X\setminus[X\setminus(A\setminus I_0)]^{*e}$

$= X\setminus[(X\setminus A)\cup I_0]^{*e}$

$= X\setminus(X\setminus A)^{*e}$

$=\psi_e(A)$

(iii). By Theorem: 15 3.4(ii)., we have

$\psi_e(A\cup I_0) = X\setminus[X\setminus(A\cup I_0)]^{*e}$

$= X\setminus[(X\setminus A)\setminus I_0]^{*e}$

$= X\setminus(X\setminus A)^{*e}$

$=\psi_e(A)$

(iv). Assume $(A\setminus B)\cup(B\setminus A)\in I$. Let $A\setminus B=I_1$ and $B\setminus A=I_2$. Observe that by heredity $I_1,I_2\in I$. Also observe that $B=(A\setminus I_1)\cup I_2$. Thus $\psi_e(A)=\psi_e(A\setminus I_1)=\psi[(A\setminus I_1)\cup I_2]=\psi_e(B)$ by Theorem18 4.2(ii) and Theorem18 4.2(iii).

(v). Since $A\in eO(X,\tau),(X\setminus A)\in eC(X,\tau)$. So $(X\setminus A)=ecl(X\setminus A)$. From Theorem 3.3: 14(i), we have

$(X\setminus A)^{*e}\subseteq ecl(X\setminus A)=X\setminus A\Rightarrow(X\setminus A)^{*e}\subseteq X\setminus A$

$\Rightarrow A\subseteq X\setminus(X\setminus A)^{*e}$

$\Rightarrow A\subseteq\psi_e(A)$.

Example 4.1: 19 Let $X=\{a,b,c\}$ and $\tau=\{\phi,\{a\},\{b\},\{a,b\},X\}$ with $I=\{\phi,\{b\},\{c\},\{b,c\}\}$. Then for $A=\{c\}$, we have $\psi_e(A)=\{a,b,c\}\supseteq A$ but $A=\{c\}$ is not a e-open set.

Theorem 4.320: Let $(X,eO(X,\tau),I)$ be a e-ideal space and $A\subseteq X$.

(i) $\psi_e(A)=\bigcup\{U\in eO(X,\tau):U\setminus A\in I\}$.

(ii) $\psi_e(A)\supseteq\bigcup\{U\in eO(X,\tau):(U\setminus A)\cup(A\setminus U)\in I\}$

Proof: (i). Proof is obvious from definition of $\psi_e(A)$.

(ii). Since I is heredity, we have

$$\bigcup\{U\in eO(X,\tau):(U\setminus A)\cup(A\setminus U)\in I\}\subseteq\bigcup\{U\in eO(X,\tau):U\setminus A\in I\}=\psi_e(A).$$

5. eO -Codense Ideal:

Definition 5.1: 21 Let $(X,eo(X,\tau),I)$ be a e-ideal spaces and $A\subseteq X$. If $eO(X,\tau)\cap I=\{\phi\}$ then we say that I is eO-codense ideal.

Theorem 5.1:22 Let $(X,eO(X,\tau),I)$ be a e-ideal spaces. If I is eO-codense ideal with $eO(X,\tau)$, then $X=X^{*e}$.

Proof: It is obvious that $X^{*e}\subseteq X$. Let $x\notin X^{*e}$, for $x\in X$. Then there is atleast one $U_x\in eO(X.\tau)$ that provide

$U_x \cap X \in I$. Hence $U_x \cap X = U_x \in I$. But $eO(X.\tau) \cap I = \{\phi\}$. It is a contradiction. So $X = X^{*e}$.

Theorem 5.2: **23**The following are equivalent for $(X, eO(X,\tau), I)$ e-ideal space.

(i) $eO(X,\tau) \cap I = \{\phi\}$,

(ii) $\psi_e(\phi) = \phi$,

(iii) If $I_0 \in I, \psi_e(I_0) = \phi$.

Proof: (i) $\Rightarrow$ (ii): Let $eO(X,\tau) \cap I = \{\phi\}$. From definition of ψ_e operator and Theorem :225.1., we have $\psi_e(\phi) = X \setminus (X \setminus \phi)^{*e} = X \setminus X^{*e} = \phi.$

(ii)$\Rightarrow$ (iii): Let $I_0 \in I$ and $\psi_e(\phi) = \phi.$ Also because of Theorem: 15 3.4(i), we have obtained $(X \setminus I_0)^{*e} = X^{*e}$. So we have $\psi_e(I_0) = X \setminus (X \setminus I_0)^{*e} = X \setminus X^{*e} = \psi_e(\phi) = \phi.$

(iii)$\Rightarrow$ (i): Let $A \in eO(X.\tau) \cap I.$ Then because of $A \in I$ and Theorem: 23 5.2(iii)., we have $\psi_e(A) = \phi.$ Also $A \subseteq \psi_e(A) = \phi$ since $A \in eO(X,\tau)$ and $A \subseteq \psi_e(A)$. And so $A = \phi.$ Hence we have $eO(X,\tau) = \{\phi\}.$

Theorem 5.3: **24**Let $(X, eO(X,\tau), I)$ be a e-ideal spaces. If I is eO-codense ideal with $eO(X,\tau),$ then $\psi_e(A) \subseteq A^{*e}$ for every $A \subseteq X.$

Proof: Let $x \in \psi_e(A)$ and $x \notin A^{*e}$ for a least one $x \in X$. Then we obtain $x \notin A^{*e} \Rightarrow \exists T_x \in eO(X,x); T_x \cap A \in I.$

Since $x \in \psi_e(A),$ we have $x \in \bigcup\{U \in eO(X,\tau) : U \setminus A \in I\}$ from Theorem20 4.3(i). Hence there is $V \in eO(X,\tau)$ which satisfy $x \in V$ and $V \setminus A \in I.$ Since $x \in T_x \cap V$ is a e-open set, we obtain $(T_x \cap V) \cap A \in I$ and $(T_x \cap V) \setminus A \in I$ from heredity of I . Also since I is finite additivity, we obtain $T_x \cap V = [(T_x \cap V) \cap A] \bigcup [(T_x \cap V) \setminus A] \in I$

Since $T_x \cap V \neq \phi$ is a e-open set, $I \cap eO(X,\tau) \neq \phi.$ But it contradict with the fact I is eO-codense. So $x \in A^{*e}.$

Remark 5.1: **25**Let $(X, eO(X,\tau), I)$ be a e-ideal spaces and $A \subseteq X.$ If I is eO-codense ideal, then $\psi_e(A) \subseteq ecl(A).$

6. e-Compatibility Topology with an Ideal:

Definition 6.1: **26**Let $(X, eO(X,\tau), I)$ be an e-ideal spaces. We say the τ is e-compatible with the ideal $I,$ denoted $\tau \sim_e I$ if the following holds for every $A \subseteq X,$ if for every $x \in A$ there exists $U \in eO(X,x)$ such that $U \cap A \in I,$ then $A \in I$.

Theorem 6.1: **27**Let $(X, eO(X,\tau), I)$ be a e-ideal space. $\tau \sim_e I$ if and only if $\psi_e(A) \setminus A \in I,$ for every $A \subseteq X.$

Proof: Let $\tau \sim_e I$ and $\psi_e(A) \setminus A \in I$, for every $A \subseteq X$. Then we have

$$x \in \psi_e(A), x \notin A \Rightarrow x \in \setminus (X \setminus A)^{*e}, x \notin A$$
$$\Rightarrow x \notin (X \setminus A)^{*e}, x \notin A$$
$$\Rightarrow \exists U \in eO(X,x); U \cap (X \setminus A) \in I, x \notin A$$
$$\Rightarrow X \setminus A \in I, x \in X \setminus A$$

So $\psi_e(A) \setminus A \subseteq X \setminus A \in I.$

Conversely, let $\psi_e(A) \setminus A \in I$ for every $A \subseteq X.$ Also there is $U \in eO(X,x)$ which $U \cap A \in I$ for every $x \in A$. Then

$$x \notin A^{*e} \Rightarrow x \in X \setminus A^{*e} \Rightarrow A \subseteq X \setminus A^{*e}.$$

Hence because of the following equation and the fact $A \subseteq X \setminus A^{*e}$ we have $\psi_e(X \setminus A) \setminus (X \setminus A) = A.$

$$\psi_e(X \setminus A) \setminus (X \setminus A) = [X \setminus (X \setminus (X \setminus A))^{*e}] \setminus (X \setminus A) = (X \setminus A^{*e}) \cap A$$

Since $\psi_e(A) \setminus A \in I$ for every $A \subseteq X$, $\psi_e(X \setminus A) \setminus (X \setminus A) = A \in I.$

From the above theorem we will give the following remark.

Remark 6.1: 28 Let $(X, eO(X,\tau), I)$ be a e ideal space and $\tau :\ _e I$. Then $\psi_e(\psi_e(A)) = \psi_e(A),$ for every $A \subseteq X.$

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