ISSN: 2455 – 5428
International Journal of Current Research and Modern Education
Impact Factor 6.725, Special Issue, July - 2017

$(\in, \in \vee q)$ - INTERVAL VALUED FUZZY SUBNEAR-RINGS AND $(\in, \in \vee q)$ -INTERVAL VALUED FUZZY IDEALS OF NEAR RINGS

V. Vetrivel* & P. Murugadas**
* Department of Mathematics, Annamalai University, Annamalainagar, Tamilnadu
** Department of Mathematics, Govt. Arts and Science College, Karur, Tamilnadu

Cite This Article: V. Vetrivel & P. Murugadas, "$(\in, \in \vee q)$- Interval Valued Fuzzy Subnear-Rings and $(\in, \in \vee q)$-Interval Valued Fuzzy Ideals of Near Rings", International Journal of Current Research and Modern Education, Special Issue, July, Page Number 127-135, 2017.

Abstract:

In this paper, the notions of $(\in, \in \vee q)$ - interval valued fuzzy subnear-rings, $(\in, \in \vee q)$ -interval valued fuzzy ideal and $(\in, \in \vee q)$ - interval valued fuzzy quasi-ideal of near-rings are studied. The characterization of such $(\in, \in \vee q)$ - interval valued fuzzy ideals are also obtained.

Key Words: Interval-Valued Fuzzy Subnear Ring, Fuzzy Ideal & Quasi-Ideal

1. Introduction:

The theory of fuzzy set which was introduced by Zadeh[20] is applied to many Mathematical brances. The notion of fuzzy subgroup was introduced by Rosenfeld[17] in 1971. Fuzzy ideals and rings were introduced by W.Liu[13] and it has been studied by several authors[9,11,12]. The notions of fuzzy subnear-rings, fuzzy ideals of near-rings were introduced by Salah Abou-Zaid[18], A.L. Narayanan introduced the notion of fuzzy quasi-ideal of near-rings. A new type of fuzzy subgroup (viz, $(\in, \in \vee q)$ - fuzzy subgroup) was introduced in an earlier paper of Bhakat and Das[1] by using the combined notions of "belonginess" and"quasicoincidence" of fuzzy points and fuzzy sets. The concept has been studied further in[1,2,3,4,5]. As a generalization of fuzzy set Zadeh[20] in 1975 introduced a new notion of fuzzy subsets viz., interval valued (i-v) fuzzy subset, where the values of the membership function are closed intervals of numbers instead of a number. Thillaigovindan et.al.,[19] introduced the notion of i-v fuzzy subnear-ring and i-v fuzzy left (right) ideal of near-ring and investigated some of their properties. Further Chinnadurai and Kadalarasi[6] studied interval valued fuzzy quasi-ideal of near-rings. In this paper, we extend the i-v fuzzy set notion to $(\in, \in \vee q)$ - fuzzy subnear-rings and $(\in, \in \vee q)$- fuzzy ideals of near-rings.

2. Preliminaries:

We first recall some basic concepts for the sake of completeness. By a near-ring [10] we mean a non-empty set N with two binary operations `$+'$ and `$.'$ satisfying the following axioms:

(i) $(N,+)$ is a group,

(ii) $(N,.)$ is a semigroup,

(iii) $(x+y).z = x.z + y.z \ \forall\, x, y, z \in N.$

Precisely speaking, it is a right near-ring because it satisfies the right distributive law. We will use the word "near-ring" to mean "right near-ring". We denote xy instead of $x.y$. Note that $0x = 0$ but in general $x0 \neq 0$ for some $x \in N$. If P and Q are two non-empty subsets of N we define $PQ = \{ab / a \in P, b \in Q\}$ and $P*Q = \{a(b+i) - ab / a, b \in P, i \in Q\}$.

A subgroup M of a near-ring N is called a subnear-ring of N if $MM \subseteq M$.

A near-ring N is called zero-symmetric if $x0 = 0 \ \forall x \in N$. A subset I of a near-ring N is called an ideal of N if

(i) $(I,+)$ is a normal subgroup of $(N,+)$,

(ii) $IN \subseteq I$,

(iii) $a(b+i) - ab \in I \ \forall\, a, b \in N$ and $i \in I$, that is, $N*I \subseteq I$.

A normal subgroup R of $(N,+)$ with (ii) is called a right ideal of N while a normal subgroup L of $(N,+)$ with (iii) is called a left ideal of N. A subgroup Q of $(N,+)$ is called a quasi-ideal of near-ring N if $QN \cap NQ \cap N*Q \subseteq Q$.

We now review some fuzzy logic concepts.

Definition 2.1 [19] 1 An interval number $\bar{a}$ on [0,1] is a closed subinterval of [0,1], that is, $\bar{a} = [a^-, a^+]$ such that $0 \leq a^- \leq a^+ \leq 1$ where a^- and a^+ are the lower and upper end limits of $\bar{a}$ respectively. The set of all closed subintervals of $[0,1]$ is denoted by $D[0,1]$. We also identify the interval $[a,a]$ by the number $a \in [0,1]$. For any interval numbers $\bar{a}_i = [a_i^-, a_i^+], \bar{b}_i = [b_i^-, b_i^+] \in D[0,1], i \in I$, we define

$$max^i\{\bar{a}_i, \bar{b}_i\} = [max^i\{a_i^-, b_i^-\}, max^i\{a_i^+, b_i^+\}],$$

$$min^i\{\bar{a}_i, \bar{b}_i\} = [min^i\{a_i^-, b_i^-\}, min^i\{a_i^+, b_i^+\}],$$

$inf^i \bar{a}_i = [\bigcap_{i\in I} a_i^-, \bigcap_{i\in I} a_i^+], sup^i \bar{a}_i = [\bigcup_{i\in I} a_i^-, \bigcup_{i\in I} a_i^+]$

In this notation $\bar{0} = [0,0]$ and $\bar{1} = [1,1]$. For any interval numbers $\bar{a} = [a^-, a^+]$ and $\bar{b} = [b^-, b^+]$ on $[0,1]$, define

(1) $\bar{a} \le \bar{b}$ if and only if $a^- \le b^-$ and $a^+ \le b^+$.

(2) $\bar{a} = \bar{b}$ if and only if $a^- = b^-$ and $a^+ = b^+$.

(3) $\bar{a} < \bar{b}$ if and only if $\bar{a} \le \bar{b}$ and $\bar{a} \ne \bar{b}$

(4) $k\bar{a} = [ka^-, ka^+]$, whenever $0 \le k \le 1$.

Definition 2.2 [19]2 Let X be any set. A mapping $\bar{A}: X \to D[0,1]$ is called an interval-valued fuzzy subset (briefly, i-v fuzzy subset) of X where $D[0,1]$ denotes the family of all closed subintervals of $[0,1]$ and $\bar{A}(x) = [A^-(x), A^+(x)]$ for all $x \in X$, where A^- and A^+ are fuzzy subsets of X such that $A^-(x) \le A^+(x)$ for all $x \in X$. Note that $\bar{A}(x)$ is an interval (a closed subset of [0,1]) and not a number from the interval [0,1] as in the case of fuzzy subset.

Definition 2.3. [19] 3A mapping $min^i : D[0,1] \times D[0,1] \to D[0,1]$ defined by

$min^i(\bar{a}, \bar{b}) = [min\{a^-, b^-\}, min\{a^+, b^+\}]$ for all $\bar{a}, \bar{b} \in D[0,1]$ is called an interval min-norm.

Definition 42.4. [19] A mapping $max^i : D[0,1] \times D[0,1] \to D[0,1]$ defined by

$max^i(\bar{a}, \bar{b}) = [max\{a^-, b^-\}, max\{a^+, b^+\}]$ for all $\bar{a}, \bar{b} \in D[0,1]$ is called an interval max-norm.

Let min^i and max^i be the interval min-norm and max-norm on $D[0,1]$ respectively. Then the following are true.

1. $min^i\{\bar{a}, \bar{a}\} = \bar{a}$ and $max^i\{\bar{a}, \bar{a}\} = \bar{a}$ for all $\bar{a} \in D[0,1]$.
2. $min^i\{\bar{a}, \bar{b}\} = min^i\{\bar{b}, \bar{a}\}$ and $max^i\{\bar{a}, \bar{b}\} = max^i\{\bar{b}, \bar{a}\}$ for all $\bar{a}, \bar{b} \in D[0,1]$.
3. If $\bar{a} \ge \bar{b} \in D[0,1]$, then $min^i\{\bar{a}, \bar{c}\} \ge min^i\{\bar{b}, \bar{c}\}$ and $max^i\{\bar{a}, \bar{c}\} \ge max^i\{\bar{b}, \bar{c}\}$ for all $\bar{c} \in D[0,1]$. Let $\bar{A}$ and $\bar{B}$ be two i-v fuzzy subsets of semigroup X. We define the relation $\subseteq$ between $\bar{A}$ and $\bar{B}$, the intersection and product of $\bar{A}$ and $\bar{B}$, respectively as follows:

(i) $\bar{A} \subseteq \bar{B}$ if $\bar{A}(x) \le \bar{B}(x)\ \forall x \in X$,

(ii) $(\bar{A} \cap \bar{B})(x) = min^i\{\bar{A}(x), \bar{B}(x)\}\ \forall\ x \in X$,

(iii) $(\bar{A} \circ \bar{B})(x) = \begin{cases} sup^i_{x=yz}[min^i\{\bar{A}(y), \bar{B}(z)\}] & \text{if } x = yz, \text{for } y, z \in X, \\ \bar{0} & \text{Otherwise} \end{cases}$

It is easily verified that the "product" of i-v fuzzy subsets is associative. Throughout this paper, N will denote a near-ring unless otherwise specified.

Definition 52.5. [19] For an i-v fuzzy subset $\bar{A}$ of a set X and $\bar{t} \in D[0,1]$, the subset $\bar{A}_{\bar{t}} = \{x \in X / \bar{A}(x) \ge \bar{t}\}$ is called a level subset of X determined by $\bar{A}$ and $\bar{t}$.

Definition 6 2.6. [19] An i-v fuzzy subset $\bar{A}$ of a set X of the form

$$\bar{A}(y) = \begin{cases} \bar{t}(\ne \bar{0}) & \text{if } y = x, \\ \bar{0} & \text{if } y \ne x \end{cases}$$

is said to be an i-v fuzzy point with support x and value $\bar{t}$ and is denoted by $x_{\bar{t}}$.

Definition 7 2.7. [19] An i-v fuzzy subset $\bar{A}$ of a group G is said to be an i-v fuzzy subgroup of G if $\forall\ x, y \in G$, (i) $\bar{A}(xy) \ge min^i\{\bar{A}(x), \bar{A}(y)\}$, (ii) $\bar{A}(x^{-1}) \ge \bar{A}(x)$.

Definition 8 2.8. [19] An i-v fuzzy subset $\bar{A}$ of N is called an i-v fuzzy subnear-ring of N if $\forall\ x, y \in N$, (i) $\bar{A}(x-y) \ge min^i\{\bar{A}(x), \bar{A}(y)\}$, (ii) $\bar{A}(xy) \ge min^i\{\bar{A}(x), \bar{A}(y)\}$.

Definition 9 2.9. [19] An i-v fuzzy subset $\bar{A}$ of N is said to be an i-v fuzzy ideal of N if

ISSN: 2455 – 5428
International Journal of Current Research and Modern Education
Impact Factor 6.725, Special Issue, July - 2017

(i) $\bar{A}$ is an i-v fuzzy subnear-ring of N. (ii) $\bar{A}(y+x-y)\geq\bar{A}(x)\ \forall\ x,y\in N$,

(iii) $\bar{A}(xy)\geq\bar{A}(x)\ \forall\ x,y\in N$, (iv) $\bar{A}(a(b+i)-ab)\geq\bar{A}(i)\ \forall\ a,b,i\in N$.

An i-v fuzzy subset with (i), (ii) and (iii) is called an i-v fuzzy right ideal of N whereas an i-v fuzzy subset with (i), (ii) and (iv) is called an i-v fuzzy left ideal of N.

Definition 10 2.10. [19] Let $\bar{A}$ be an i-v fuzzy subset of N. We define

$$(N*\bar{A})(x)=\begin{cases} sup^i_{x=a(b+i)-ab}\bar{A}(i) & \text{if } x=a(b+i)-ab,\ a,b,i\in N,\\ \bar{0} & \text{Otherwise}\end{cases}$$

Definition 11 2.11. [19] An i-v fuzzy subgroup $\bar{A}$ of N is called an i-v fuzzy quasi-ideal of N if $(\bar{A}\circ N)\cap(N\circ\bar{A})\cap(N*\bar{A})\subseteq\bar{A}$.

Example 2.12.12 Let $N=\{0,a,b,c\}$ be the near-ring with $(N,+)$ as the Klein's four group and $(N,.)$ as defined below

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

·	0	a	b	c
0	0	0	0	0
a	a	a	a	a
b	0	0	0	b
c	a	a	a	c

Let $Q=\{0,c\}$. Then $QN=\{0,a,c\}, NQ=\{0,a,b,c\}$ and $N*Q=\{0,b\}$.

Therefore $QN\cap NQ\cap N$ å $Q=\{0\}\subseteq Q$. Hence Q is a quasi-ideal of N.

Define an i-v fuzzy subset $\bar{A}:N\rightarrow D[0,1]$ by $\bar{A}(0)=\bar{A}(c)=\bar{1}$ and $\bar{A}(a)=\bar{A}(b)=\bar{0}$. Clearly, $\bar{A}$ is an i-v fuzzy quasi-ideal of N.

3. An $(\in,\in\vee q)$ - i-v Fuzzy Subnear-Rings and $(\in,\in\vee q)$ - i-v Fuzzy Ideals:

In this section, we introduce the notions of an $(\in,\in\vee q)$ i-v fuzzy subnear-ring and an $(\in,\in\vee q)$ i-v fuzzy ideal of a near-ring.

Definition 3.1. 13 An i-v fuzzy point $x_{\bar{t}}$ is said to belong to (resp. be fuzzy quasi-coincident with) an i-v fuzzy subset $\bar{A}$, written as $x_{\bar{t}}\in\bar{A}$ (resp. $x_{\bar{t}}\,q\,\bar{A}$) if $\bar{A}(x)\geq\bar{t}\ (resp.\ \bar{A}(x)+\bar{t}>\bar{1})$. $x_{\bar{t}}\in\bar{A}$ or $x_{\bar{t}}\,q\,\bar{A}$" will be denoted by $x_{\bar{t}}\in\vee q\,\bar{A}$. $x_{\bar{t}}\,\overline{\in}\bar{A},\ x_{\bar{t}}\,\overline{\in\vee q}\,\bar{A}$" will respectively mean $x_{\bar{t}}\in\bar{A}$ and $x_{\bar{t}}\in\vee q\,\bar{A}$ do not hold.

Definition 3.2. 14 An i-v fuzzy subset $\bar{A}$ of a group G is said to be an $(\in,\in\vee q)$- i-v fuzzy subgroup of G if $\forall\ x,y\in G$ and $\bar{t},\bar{r}\in D(0,1]$, (i) $x_{\bar{t}},y_{\bar{r}}\in\bar{A}\Rightarrow(xy)_{min(\bar{t},\bar{r})}\in\vee q\,\bar{A}$, (ii) $x_{\bar{t}}\in\bar{A}\Rightarrow x^{-1}_{\bar{t}}\in\vee q\,\bar{A}$.

Remark 3.3. 15(I) The conditions (i) and (ii) of Definition 3.2 are respectively equivalent to

(i) $\bar{A}(xy)\geq min^i\{\bar{A}(x),\bar{A}(y),\overline{0.5}\}\ \forall\ x,y\in G$ and (ii) $\bar{A}(x^{-1})\geq min^i\{\bar{A}(x),\overline{0.5}\}\ \forall\ x\in G$.

(ii) For any $(\in,\in\vee q)$ - i-v fuzzy subgroup $\bar{A}$ of G such that $\bar{A}(x)\geq\overline{0.5}$ for some $x\in G,\ \bar{A}(e)\geq\overline{0.5}$.

(iii) Note that if $\bar{A}$ is an i-v fuzzy subgroup of a group G, then $\bar{A}$ is an $(\in,\in\vee q)$ - i-v fuzzy subgroup of G. However, the converse is not necessarily true.

Remark 3.4.16 An i-v fuzzy subset $\bar{A}$ of a group G is an $(\in,\in\vee q)$ - i-v fuzzy subgroup of G if and only if the level subset $\bar{A}_{\bar{t}}=\{x\in G/\bar{A}(x)\geq\bar{t}\}$ is a subgroup of G $\forall\ \bar{t}\in D(0,0.5]$. But the level subset $\bar{A}_{\bar{t}},\bar{t}\in D(0.5,1]$ may not be a subgroup of G. Here we define the notion of an $(\in,\in\vee q)$ - i-v fuzzy subnear-ring of N.

Definition 3.5. 17 An i-v fuzzy subset $\bar{A}$ is said to be an $(\in,\in\vee q)$ - i-v fuzzy subnear-ring of N if $\forall\ x,y\in N$ and $\bar{t},\bar{r}\in D(0,1]$, (i) $x_{\bar{t}},y_{\bar{r}}\in\bar{A}\Rightarrow(x+y)_{min(\bar{t},\bar{r})}\in\vee q\,\bar{A}$, (ii) $x_{\bar{t}}\in\bar{A}\Rightarrow(-x)_{\bar{t}}\in\vee q\,\bar{A}$.

(iii) $x_{\bar{t}},y_{\bar{r}}\in\bar{A}\Rightarrow(xy)_{min(\bar{t},\bar{r})}\in\vee q\,\bar{A}$,

Lemma 3.6. 18 Let $\bar{A}$ be an i-v fuzzy subset of N and $\bar{t},\bar{r}\in D(0,1]$. Then

(i) (ia) $x_{\bar t}, y_{\bar r} \in \bar{A} \Rightarrow (x+y)_{min(\bar t, \bar r)} \in \vee q\, \bar{A}$ if and only if

(1a) $\bar{A}(x+y) \geq min^i\{\bar{A}(x), \bar{A}(y), \overline{0.5}\}, \forall\, x, y \in N.$

(ii) (ib) $x_{\bar t} \in \bar{A} \Rightarrow (-x)_{\bar t} \in \vee q\, \bar{A}$ if and only if

(1b) $\bar{A}(-x) \geq min^i\{\bar{A}(x), \overline{0.5}\}, \forall\, x \in N.$

(iii) (ic) $x_{\bar t}, y_{\bar r} \in \bar{A} \Rightarrow (xy)_{min(\bar t, \bar r)} \in \vee q\, \bar{A}$ if and only if

(1c) $\bar{A}(xy) \geq min^i\{\bar{A}(x), \bar{A}(y), \overline{0.5}\}, \forall\, x, y \in N.$

Proof. $(ia \Rightarrow 1a)$: Suppose that $x, y \in N.$ We consider the following two cases:

(a) $\bar{A}(x) \wedge \bar{A}(y) < \overline{0.5}$, (b) $\bar{A}(x) \wedge \bar{A}(y) \geq \overline{0.5}$.

Case a: Assume that $\bar{A}(x+y) < \bar{A}(x) \wedge \bar{A}(y) \wedge \overline{0.5}$, which implies $\bar{A}(x+y) < \bar{t} < \bar{A}(x) \wedge \bar{A}(y)$. Then $x_{\bar t}, y_{\bar t} \in \bar{A}$, but $(x+y)_{\bar r} \,\overline{\in \vee q}\, \bar{A}$ which contradicts (ia).

Case b: Assume that $\bar{A}(x+y) < \overline{0.5}$, then $x_{\overline{0.5}}, y_{\overline{0.5}} \in \bar{A}$, but $(x+y)_{\overline{0.5}} \,\overline{\in \vee q}\, \bar{A}$, a contradiction. Hence (1a) holds.

$(ib \Rightarrow 1b)$: Suppose that $x \in N.$ We consider the following cases: (a) $\bar{A}(x) < \overline{0.5}$, (b) $\bar{A}(x) \geq \overline{0.5}$.

Case a: Assume that $\bar{A}(x) = \bar{t} < \overline{0.5}$ and $\bar{A}(-x) = \bar{r} < \bar{A}(x)$. Choose $\bar{s}$ such that $\bar{r} < \bar{s} < \bar{t}$ and $\bar{r} + \bar{s} < \bar{1}$. Then $x_{\bar s} \in \bar{A}$, but $(-x)_{\bar s} \,\overline{\in \vee q}\, \bar{A}$ which contradicts (ib). So $\bar{A}(-x) \geq \bar{A}(x) = \bar{A}(x) \wedge \overline{0.5}$.

Case b: Let $\bar{A} \geq \overline{0.5}$. If $\bar{A}(-x) < \bar{A} \wedge \overline{0.5}$, then $x_{\overline{0.5}} \in \bar{A}$, but $(-x)_{\overline{0.5}} \,\overline{\in \vee q}\, \bar{A}$, which contradicts (ib). So $\bar{A}(-x) \geq \bar{A}(x) \wedge \overline{0.5}$. (ic)$\Rightarrow$(1c): The proof is similar to (ia $\Rightarrow 1a.$)

Theorem 3.7. 19 An i-v fuzzy subset $\bar{A}$ of N is an $(\in, \in \vee q)$ - i-v fuzzy subnear-ring of N if and only if (i) $\bar{A}(x-y) \geq min^i\{\bar{A}(x), \bar{A}(y), \overline{0.5}\}, \forall\, x, y \in N.$ (ii) $\bar{A}(xy) \geq min^i\{\bar{A}(x), \bar{A}(y), \overline{0.5}\}, \forall\, x, y \in N.$

Proof. (i) Suppose that $x, y \in N.$ We consider the following cases:

(a) $\bar{A}(x) \wedge \bar{A}(y) < \overline{0.5}$, (b) $\bar{A}(x) \wedge \bar{A}(y) \geq \overline{0.5}$.

Case a: Assume that $\bar{A}(x-y) < \bar{A}(x) \wedge \bar{A}(y) \wedge \overline{0.5}$, which implies $\bar{A}(x-y) < \bar{t} < \bar{A}(x) \wedge \bar{A}(y)$. Then $x_{\bar t}, y_{\bar t} \in \bar{A}$, but $(x-y)_{\bar r} \,\overline{\in \vee q}\, \bar{A}$ which contradicts $x_{\bar t}, y_{\bar r} \in \bar{A} \Rightarrow (x+y)_{min(\bar t, \bar r)} \in \vee q\, \bar{A}$.

Case b: Assume that $\bar{A}(x-y) < \overline{0.5}$. then $x_{\overline{0.5}}, y_{\overline{0.5}} \in \bar{A}$, but $(x-y)_{\overline{0.5}} \,\overline{\in \vee q}\, \bar{A}$, a contradiction. Hence $\bar{A}(x-y) \geq min^i\{\bar{A}(x), \bar{A}(y), \overline{0.5}\}$ holds.

(ii) Suppose that $x, y \in N.$ We consider the following cases:

(a) $\bar{A}(y) < \overline{0.5}$, (b) $\bar{A}(y) \geq \overline{0.5}$.

Case a: Assume that $\bar{A}(y) = \bar{t} < \overline{0.5}$ and $\bar{A}(xy) = \bar{r} < \bar{A}(x)$. Choose $\bar{s}$ such that $\bar{r} < \bar{s} < \bar{t}$ and $\bar{r} + \bar{s} < \bar{1}$. Then $y_{\bar s} \in \bar{A}$, but $(xy)_{\bar s} \,\overline{\in \vee q}\, \bar{A}$ which contradicts $y_{\bar r} \in \bar{A}$ and $x \in N$ implies $(xy)_{\bar r} \in \vee q\, \bar{A}$. So $\bar{A}(xy) \geq \bar{A}(y) = \bar{A}(y) \wedge \overline{0.5}$.

Case b: Let $\bar{A}(y) \geq \overline{0.5}$. If $\bar{A}(xy) < \bar{A} \wedge \overline{0.5}$, then $y_{\overline{0.5}} \in \bar{A}$, but $(xy)_{\overline{0.5}} \,\overline{\in \vee q}\, \bar{A}$, which contradicts $y_{\bar r} \in \bar{A}$ and $x \in N$ implies $(xy)_{\bar r} \in \vee q\, \bar{A}$. So $\bar{A}(xy) \geq \bar{A}(y) = \bar{A}(y) \wedge \overline{0.5}$.

Remark 3.8. 20 Every i-v fuzzy subnear-ring of N (according to Definition 2.8) is an $(\in, \in \vee q)$ - i-v fuzzy subnear-ring of N. But the converse is not necessarily true as shown by the following example.

Example 3.9. 21 Consider the near-ring $(N, +, .)$ as defined in Example 2.12. Define an i-v fuzzy subset $\bar{A}: N \rightarrow [0,1]$ by $\bar{A}(0) = \overline{0.7}, \bar{A}(a) = \overline{0.4}, \bar{A}(b) = \overline{0.8}, \bar{A}(c) = \overline{0.4}$. Then $\bar{A}$ is an $(\in, \in \vee q)$ i-v fuzzy subnear-ring of N. But since $\bar{A}(0) = \bar{A}(b-b) \not\geq min^i\{\bar{A}(b), \bar{A}(b)\}, \bar{A}$ is not an i-v fuzzy subnear-ring of N.

Definition 3.10. 22An i-v fuzzy subset $\overline{A}$ of N is said to be an $(\in, \in \vee q)$ - i-v fuzzy ideal of N if

(i) $\overline{A}$ is an $(\in, \in \vee q)$ - i-v fuzzy subnear-ring of N.

(ii) $x_{\bar{t}} \in \overline{A} \Rightarrow (y + x - y)_{\bar{t}} \in \vee q \overline{A} \; \forall \, x, y \in N,$

(iii) $x_{\bar{t}} \in \overline{A} \Rightarrow (xy)_{\bar{t}} \in \vee q \; \overline{A} \; \forall \, x, y \in N,$

(iv) $i_{\bar{t}} \in \overline{A} \Rightarrow (x(y+i) - xy)_{\bar{t}} \in \vee q \; \overline{A}$ for any $x, y, i \in N.$ and $t \in (0,1]$

An i-v fuzzy subset $\overline{A}$ with conditions (i), (ii) and (iii) is called an $(\in, \in \vee q)$ - i-v fuzzy right ideal of N. If $\overline{A}$ satisfies (i), (ii) and (iv), then it is called an $(\in, \in \vee q)$ i-v fuzzy left ideal of N.

Lemma 3.11. 23*Let* $\overline{A}$ be an i-v fuzzy subset of N. Then

(I) (a) $\overline{A}$ is an $(\in, \in \vee q)$ - i-v fuzzy subnear-ring of N and

(b) $\overline{A}(x-y), \overline{A}(xy) \geq min^i\{\overline{A}(x), \overline{A}(y), \overline{0.5}\} \; \forall \, x, y \in N$ are equivalent.

(II) (c) $x_{\bar{t}} \in \overline{A} \Rightarrow (y + x - y)_{\bar{t}} \in \vee q \; \overline{A}$ and

(d) $\overline{A}(y + x - y) \geq min^i\{\overline{A}(x), \overline{0.5}\} \; \forall \, x, y \in N$ are equivalent.

(III) (e) $x_{\bar{t}} \in \overline{A} \Rightarrow (xy)_{\bar{t}} \in \vee q \; \overline{A}$ and

(f) $\overline{A}(xy) \geq min^i\{\overline{A}(x), \overline{0.5}\} \; \forall \, x, y \in N$ are equivalent.

(IV) (g) $i_{\bar{t}} \in \overline{A} \Rightarrow (y(x+i) - yx)_{\bar{t}} \in \vee q \; \overline{A}$ and

(h) $\overline{A}(y(x+i) - yx) \geq min^i\{\overline{A}(i), \overline{0.5}\}$ for any $x, y, i \in N$ are equivalent.

Proof. $(a) \Leftrightarrow (b)$ It follows from Theorem 3.7.

$(c) \Rightarrow (d)$ Let $x, y \in N$ and $\overline{A}(x) < \overline{0.5}.$ Assume that $\overline{A}(y + x - y) < \overline{A}(x).$

Choose $\bar{t}$ such that $\overline{A}(y + x - y) < \bar{t} \leq \overline{A}(x).$

Then $x_{\bar{t}} \in \overline{A}$ and $(y + x - y)_{\bar{t}} \overline{\in \vee q} \; \overline{A}$ which contradicts $(c).$ Thus $\overline{A}(y + x - y) \geq \overline{A}(x).$

Next, let $\overline{A}(x) \geq \overline{0.5}.(1)$ Assume that $\overline{A}(y + x - y) < \overline{0.5}.$ (2)

From (1) we have $x_{\overline{0.5}} \in \overline{A}.$ But from (2) we have $(y + x - y)_{\overline{0.5}} \overline{\in \vee q} \; \overline{A},$ a contradiction. Hence (d) holds. (d)$\Rightarrow$(c) Let $x_{\bar{t}} \in \overline{A}$ and $y \in N.$ Then $\overline{A}(x) \geq \bar{t}.$ By (d),

$\overline{A}(y + x - y) \geq min^i\{\overline{A}(x), \overline{0.5}\} \geq min^i\{\bar{t}, \overline{0.5}\}.$

Then $\overline{A}(y + x - y) \geq \bar{t}$ if $\bar{t} \leq \overline{0.5}$ and $\overline{A}(y + x - y) \geq \overline{0.5}$ if $\bar{t} > \overline{0.5}.$ Hence $(y + x - y)_{\bar{t}} \in \vee q \; \overline{A}.$ Thus (c) holds.

(e)$\Rightarrow$(f) Assume that (e) is valid and let $x, y \in N.$ Let $\overline{A}(x) < \overline{0.5}.$ Assume that $\overline{A}(xy) < \overline{A}(x).$ Choose $\bar{t}$ such that $\overline{A}(xy) < \bar{t} \leq \overline{A}(x),$

then $x_{\bar{t}} \in \overline{A}$ and $(xy)_{\bar{t}} \overline{\in \vee q} \; \overline{A}$ which contradicts (e). So $\overline{A}(xy) \geq \overline{A}(x).$ Next, let $\overline{A}(x) \geq \overline{0.5}.(3)$

Assume that $\overline{A}(xy) < \overline{0.5}.$ (4)

From (3), we have $x_{\overline{0.5}} \in \overline{A}.$ From (4), we have $(xy)_{\overline{0.5}} \overline{\in \vee q} \; \overline{A},$ which contradicts (e). Hence (f) holds.

(f)$\Rightarrow$(e) Let $x_{\bar{t}} \in \overline{A}$ and $y \in N.$ By (f),

$\overline{A}(xy) \geq min^i\{\overline{A}(x), \overline{0.5}\} \geq min^i\{\bar{t}, \overline{0.5}\}.$

Then $\overline{A}(xy) \geq \bar{t}$ if $\bar{t} \leq \overline{0.5}$ and $\overline{A}(xy) \geq \overline{0.5}$ if $\bar{t} > \overline{0.5}.$ Hence $(xy)_{\bar{t}} \in \vee q \; \overline{A}.$

Thus (e) holds.

Similarly it can be shown that (g) and (h) are equivalent.

Theorem 3.12. **24**An i-v fuzzy subset $\overline{A}$ of N is an $(\in, \in \vee q)$ - i-v fuzzy ideal of N if and only if $\forall \, x, y, i \in N,$

(i) $\overline{A}(x - y) \geq min^i\{\overline{A}(x), \overline{A}(y), \overline{0.5}\},$

(ii) $\overline{A}(y+x-y) \geq min^i\{\overline{A}(x), \overline{0.5}\}$,

(iii) $\overline{A}(xy) \geq min^i\{\overline{A}(x), \overline{0.5}\}$,

(iv) $\overline{A}(y(x+i)-yx) \geq min^i\{\overline{A}(i), \overline{0.5}\}$.

Proof. The proof is straightforward from Lemma 3.11.

Remark 3.13 25 An i-v fuzzy ideal of N according to the Definition 2.9 is an $(\in, \in\vee q)$ - i-v fuzzy ideal of N. But the converse, in general, is not true as shown by the following example.

Example 3.14 26 Consider the $(\in, \in\vee q)$ - i-v fuzzy subnear-ring $\overline{A}$ of N as defined in example 3.9. Then $\overline{A}$ is an $(\in, \in\vee q)$ - i-v fuzzy ideal of N. But since

$\overline{A}(0) = \overline{A}(b0) \not\geq \overline{A}(b), \overline{A}$ is not an i-v fuzzy ideal of N.

Theorem 3.15 27 A non-empty subset I of N is a subnear-ring (ideal) of N if and only if $\overline{K}_I$ is a characteristic function an $(\in, \in\vee q)$ - i-v fuzzy subnear-ring (ideal) of N.

Proof. We prove the result for ideals. Let I be an ideal of N. It is clear that $\overline{K}_I$ (characteristic function) is an i-v fuzzy ideal of N. By Remark 3.8, $\overline{K}_I$ is an $(\in, \in\vee q)$ - i-v fuzzy ideal of N.

Conversely, let $\overline{K}_I$ be an $(\in, \in\vee q)$ - i-v fuzzy ideal of N. For any $x, y \in I$, we have

$\overline{K}_I(x-y) \geq min^i\{\overline{K}_I(x), \overline{K}_I(y), \overline{0.5}\} = min\{\bar{1}, \bar{1}, \overline{0.5}\} = \overline{0.5}$,

and so $\overline{K}_I(x-y) = \bar{1}$. Thus $x-y \in I$. Let $a \in N$ and $x \in I$. Then

$\overline{K}_I(a+x-a) \geq min^i\{\overline{K}_I(x), \overline{0.5}\} = \overline{0.5}$,

and thus $\overline{K}_I(a+x-a) = \bar{1}$. This shows that $a+x-a \in I$, and therefore $(I,+)$ is a normal subgroup of $(N,+)$. Now let $a \in N$ and $x \in I$. Then

$\overline{K}_I(xa) \geq min^i\{\overline{K}_I(x), \overline{0.5}\} = \overline{0.5}$,

and so $xa \in I$. Finally let $a, b \in N$ and $i \in I$. Then

$\overline{K}_I(a(b+i)-ab) \geq min^i\{\overline{K}_I(i), \overline{0.5}\} = \overline{0.5}$,

which implies that $a(b+i)-ab \in I$. Consequently, I is an ideal of N.

Theorem 3.16 28 29 An i-v fuzzy subset $\overline{A}$ of N is an $(\in, \in\vee q)$ - i-v fuzzy subnear-ring (ideal) of N if and only if the level subset $\overline{A}_{\bar{t}}$ is a subnear-ring (ideal) of $N \,\forall\, \bar{0} < \bar{t} < \overline{0.5}$.

Proof. $\overline{A}(xy) \geq min^i\{\overline{A}(x), \overline{A}(y), \overline{0.5}\} \geq min^i\{\bar{t}, \bar{t}, \overline{0.5}\} = \bar{t}$, oof. We prove the result in the case of $(\in, \in\vee q)$ - i-v fuzzy ideals. Let $\overline{A}$ be an $(\in, \in\vee q)$ - i-v fuzzy ideal of N. Let $\bar{t} \leq \overline{0.5}$ and $x, y, i \in \overline{A}_{\bar{t}}$. Then

(i) $\overline{A}(x-y) \geq min^i\{\overline{A}(x), \overline{A}(y), \overline{0.5}\} = min^i\{\bar{t}, \bar{t}, \overline{0.5}\} = \bar{t}$, and so $x-y \in \overline{A}_{\bar{t}}$.

(ii) and so $xy \in \overline{A}_{\bar{t}}$.

(iii) $\overline{A}(y+x-y) \geq min^i\{\overline{A}(x), \overline{0.5}\} \geq min^i\{\bar{t}, \overline{0.5}\} = \bar{t}$, and so $y+x-y \in \overline{A}_{\bar{t}}$.

(iv) For every $z \in N$, we have $A(xz) \geq min^i\{\overline{A}(x), \overline{0.5}\} \geq min^i\{\bar{t}, \overline{0.5}\} = \bar{t}$, and so $xz \in \overline{A}_{\bar{t}}$.

(v) For every $a, b \in N$, we have $\overline{A}(a(b+i)-ab) \geq min^i\{\overline{A}(i), \overline{0.5}\} = min^i\{\bar{t}, \overline{0.5}\} = \bar{t}$, and thus $a(b+i)-ab \in \overline{A}_{\bar{t}}$. So $\overline{A}_{\bar{t}}$ is an ideal of N.

Conversely, let $\overline{A}_{\bar{t}}$ be an ideal of $N \,\forall\, t \leq \overline{0.5}$. If possible, let there exist $x, y \in N$ such that

$\overline{A}(x-y) < min^i\{\overline{A}(x), \overline{A}(y), \overline{0.5}\}$.

Choose $\bar{t}$ such that

$\overline{A}(x-y) < \bar{t} < min^i\{\overline{A}(x), \overline{A}(y), \overline{0.5}\}$.

Then $x, y \in \overline{A}_{\bar{t}}$. Since $\overline{A}_{\bar{t}}$ is an ideal of N, we have $x-y \in \overline{A}_{\bar{t}}$. Thus $\overline{A}(x-y) \geq \bar{t}$, a contradiction to our assumption. So $\overline{A}(x-y) \geq min^i\{\overline{A}(x), \overline{A}(y), \overline{0.5}\} \,\forall\, x, y \in N$.

ISSN: 2455 – 5428
International Journal of Current Research and Modern Education
Impact Factor 6.725, Special Issue, July - 2017

Similarly it can be shown that

$\overline{A}(xy) \geq min^i\{\overline{A}(x), \overline{A}(y), \overline{0.5}\}$,

$\overline{A}(y+x-y) \geq min^i\{\overline{A}(x), \overline{0.5}\}$,

$\overline{A}(x(y+i)-xy) \geq min^i\{\overline{A}(i), \overline{0.5}\}$,

$\forall x, y, i \in N$. Therefore, $\overline{A}$ is an $(\in, \in \vee q)$ i-v fuzzy ideal of N.

Remark 3.17 3031 Let $\overline{A}$ be an $(\in, \in \vee q)$ - i-v fuzzy subnear-ring (ideal) of N, then the level subset $\overline{A}_{\bar{t}}$ may not be a subnear-ring(ideal) of $N \ \forall \bar{t} \in D(0.5, 1]$. Since by Remark 3.4, $\overline{A}_{\bar{t}}, \bar{t} \in D(0.5, 1]$, may not be a subgroup of N.

4. An $(\in, \in \vee q)$ - i-v Fuzzy Quasi-Ideals:

In this section, we introduce the notion of an $(\in, \in \vee q)$ - i-v fuzzy quasi-ideal of a near-ring which is a generalization of $(\in, \in \vee q)$ - i-v fuzzy quasi-ideal of a near-ring.

Definition 4.1 3233 An $(\in, \in \vee q)$ i-v fuzzy subgroup $\overline{A}$ of N is called an $(\in, \in \vee q)$ - i-v fuzzy quasi-ideal of N if $\forall x \in N$,

$\overline{A}(x) \geq min^i\{((\overline{A} \circ N) \cap (N \circ \overline{A}) \cap (N * \overline{A}))(x), \overline{0.5}\}$, that is, $\overline{A}(x) \geq min^i\{(\overline{A} \circ N)(x), (N \circ \overline{A})(x), (N * \overline{A})(x), \overline{0.5}\}$.

Remark 4.2 34 Every i-v fuzzy quasi-ideal (according to the Definition 3.11 of N is an $(\in, \in \vee q)$ - i-v fuzzy quasi-ideal of N. But the converse is not necessarily true as shown by the following example.

Example 4.3 35 Consider the set integer modulo 4, $Z_4 = \{0, 1, 2, 3\}$ with the following operations.

+	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

·	0	1	2	3
0	0	0	0	0
1	0	2	0	1
2	0	2	0	3
3	0	0	0	2

Clearly $(Z_4, +, .)$ is a near-ring. Let an i-v fuzzy subset $\overline{A}: Z_4 \rightarrow D[0,1]$ be defined by $\overline{A}(0) = \overline{0.6}, \overline{A}(1) = \overline{0.4}\ \overline{A}(2) = \overline{0.8}, \overline{A}(3) = \overline{0.4}$. Then $\overline{A}$ is an $(\in, \in \vee q)$ - i-v fuzzy quasi-ideal of Z_4. But $\overline{A}$ is not an i-v fuzzy quasi-ideal of Z_4.

Since $\overline{A}(0) \not\geq min^i\{(\overline{A} \circ N)(0), (N \circ \overline{A})(0), (N * \overline{A})(0)\}$.

Remark 4.4 36 If $\overline{Q}$ is a quasi-ideal of N, then $\overline{K}_Q$ is an i-v fuzzy quasi-ideal and also an $(\in, \in \vee q)$ - i-v fuzzy quasi-ideal of N.

Theorem 4.5 37 A non-empty i-v fuzzy subset $\overline{Q}$ of N is a quasi-ideal of N if and only if $\overline{K}_Q$ is an $(\in, \in \vee q)$ - i-v fuzzy quasi-ideal of N.

Proof. Let $\overline{Q}$ be a quasi-ideal of $N. \overline{K}_Q$ is an $(\in, \in \vee q)$ - i-v fuzzy quasi-ideal of N.

conversely, let $\overline{K}_Q$ be an $(\in, \in \vee q)$ - i-v fuzzy quasi-ideal of N. Let a be any element of $\overline{Q}N \cap N\overline{Q} \cap N * \overline{Q}$. Then there exist element c, x, y of N and elements b, i of $\overline{Q}$ such that $a = bc = x(y+i) - xy$. Now we have

$(\overline{K}_Q \circ N)(a) = sup_{a=pq}[min^i\{\overline{K}_Q(p), N(q)\}] \geq min^i\{\overline{K}_Q(b), N(c)\} = min\{\bar{1}, \bar{1}\} = \bar{1}$.

So $(\overline{K}_Q \circ N)(a) = \bar{1}$. Similarly, $(N \circ \overline{K}_Q)(a) = \bar{1}$. Moreover, $(N * \overline{K}_Q)(a) = (N * \overline{K}_Q)(x(y+i) - xy) \geq \overline{K}_Q(i) = \bar{1}$.

Hence, $\overline{K}_Q(a) \geq min^i\{(\overline{K}_Q \circ N)(a), (N \circ \overline{K}_Q)(a), (N * \overline{K}_Q)(a), \overline{0.5}\} = \overline{0.5}$, and so $\overline{K}_Q(a) = \bar{1}$ which means $a \in Q$.

Therefore $\overline{Q}N \cap N\overline{Q} \cap N * \overline{Q} \subseteq \overline{Q}$. Hence $\overline{Q}$ is a quasi-ideal of N.

Remark 4.6 38 Note that in general an $(\in, \in \vee q)$ - i-v fuzzy quasi-ideal in a near-ring N is not an $(\in, \in \vee q)$ - i-v fuzzy subnear-ring of N. In fact, we obtain an example of an $(\in, \in \vee q)$ - i-v fuzzy quasi-ideal which is not an $(\in, \in \vee q)$ - i-v fuzzy subnear-ring and obtain conditions for an $(\in, \in \vee q)$ - i-v fuzzy quasi-ideal in a near-ring to be an $(\in, \in \vee q)$ - i-v fuzzy subnear-ring.

ISSN: 2455 – 5428
International Journal of Current Research and Modern Education
Impact Factor 6.725, Special Issue, July - 2017

Example 4.7 39 Consider the non-zero-symmetric near-ring $(N,+,.)$ as defined in Example . Define an i-v fuzzy subset $\overline{A}: N \to D[0,1]$ by $\overline{A}(0)=\overline{0.7}, \overline{A}(a)=\overline{0.3}=\overline{A}(b), \overline{A}(c)=\overline{0.6}$. Then $\overline{A}$ is an $(\in,\in\vee q)$ - i-v fuzzy quasi-ideal of N. But since $\overline{A}(a)=\overline{A}(co) \not\geq min^i\{\overline{A}(c),\overline{A}(o),\overline{0.5}\}$, $\overline{A}$ is not an $(\in,\in\vee q)$ - i-v fuzzy subnear-ring of N.

Theorem 4.8 40 Every $(\in,\in\vee q)$ - i-v fuzzy quasi-ideal in a zero-symmetric near-ring is an $(\in,\in\vee q)$ - i-v fuzzy subnear-ring.

Proof. Let $\overline{A}$ be an $(\in,\in\vee q)$ - i-v fuzzy quasi-ideal in a zero-symmetric near-ring N. Choose $a,b,c,x,y,i \in N$ such that $a=bc=x(y+i)-xy$. Then

$\overline{A}(a) \geq min^i\{(\overline{A}\circ N)(a),(N\circ\overline{A})(a),(N å \overline{A})(a),\overline{0.5}\}$

$= min^i\{sup^i_{a=bc}[min^i\{\overline{A}(b),N(c)\}], sup^i_{a=bc}[min^i\{N(b),\overline{A}(c)\}], sup^i_{a=x(y+i)-xy}\{A(i),\overline{0.5}\}\}$

$\geq min^i\{sup^i_{a=bc}[min^i\{\overline{A}(b),N(c)\}], sup^i_{a=bc}[min^i\{N(b),\overline{A}(c)\}], sup^i_{a=b(0+c)-b0}\{\overline{A}(c),\overline{0.5}\}\}$

$= min^i\{sup\overline{A}(b), sup\overline{A}(c), sup\overline{A}(c),\overline{0.5}\}$

since N is zero-symmetric

$\geq min^i\{\overline{A}(b),\overline{A}(c),\overline{0.5}\}$.

Therefore $\overline{A}(bc) \geq min\{\overline{A}(b),\overline{A}(c),\overline{0.5}\}$. Hence $\overline{A}$ is an $(\in,\in\vee q)$ - i-v fuzzy subnear-ring of N.

Theorem 4.9 41 Every $(\in,\in\vee q)$ - i-v fuzzy right ideal of N is an $(\in,\in\vee q)$ - i-v fuzzy quasi-ideal of N.

Proof. Let $\overline{A}$ be an $(\in,\in\vee q)$ - i-v fuzzy right ideal of N. Choose $a,b,c,x,y,i \in N$, such that $a=bc=x(y+i)-xy$. Then

$((\overline{A}\circ N)\cap(N\circ\overline{A})\cap(N*\overline{A}))(a)$

$= min^i\{(\overline{A}\circ N)(a),(N\circ\overline{A})(a),(N*\overline{A})(a)\}$

$= min^i\{sup^i_{a=bc}\{min^i\{\overline{A}(b),N(c)\}\}, sup^i_{a=bc}\{min^i\{N(b),\overline{A}(c)\}\},(N*\overline{A})(x(y+i)-xy)\}\}$

(Since $N(z)=1 \forall z\in N$)

$= min^i\{sup^i\overline{A}(b),\overline{A}(c),(N*\overline{A})(x(y+i)-xy)\}.(5)$

Now,

$= min^i\{((\overline{A}\circ N)\cap(N\circ\overline{A})\cap(N*\overline{A}))(a),\overline{0.5}\}$

$= min^i\{min^i\{sup\,\overline{A}(b), sup\,\overline{A}(c),(N*\overline{A})(x(y+i)-xy)\},\overline{0.5}\}$

$= min^i\{min^i\{sup\,\overline{A}(b),\overline{0.5}\}, sup\,\overline{A}(c),(N*\overline{A})(x(y+i)-xy)\}\}$

(since $\overline{A}$ is an $(\in,\in\vee q)$ - i-v fuzzy right ideal, $\overline{A}(bc)\geq min^i\{\overline{A}(b),\overline{0.5}\}$)

$\leq min^i\{\overline{A}(bc),N(c),N(x(y+i)-xy)\}=\overline{A}(bc)$.

Thus $\overline{A}(a) \not\geq min^i\{((\overline{A}\circ N)\cap(N\circ\overline{A})\cap(N*\overline{A}))(a),\overline{0.5}\}$. So, $\overline{A}$ is an $(\in,\in\vee q)$ i-v fuzzy quasi-ideal of N.

Theorem 4.10 42 Every $(\in,\in\vee q)$ - i-v fuzzy left ideal of N is an $(\in,\in\vee q)$ - i-v fuzzy quasi-ideal of N.

Proof. Let $\overline{A}$ be an $(\in,\in\vee q)$ - i-v fuzzy left ideal of N. Choose $a,b,c,x,y,i\in N$, such that $a=bc=x(y+i)-xy$.

Then $((\overline{A}\circ N)\cap(N\circ\overline{A})\cap(N*\overline{A}))(a)$

$= min^i\{sup^i\overline{A}(b),\overline{A}(c),(N*\overline{A})(x(y+i)-xy)\}(by(5))$

$= min^i\{sup^i\overline{A}(b), sup^i\overline{A}(c), sup^i\overline{A}(i)\}(6)$

Now,

$= min^i\{((\overline{A}\circ N)\cap(N\circ\overline{A})\cap(N*\overline{A}))(a),\overline{0.5}\}$

$= min^i\{min^i\{sup^i\,\overline{A}(b), sup^i\,\overline{A}(c), sup^i\overline{A}(i)\},\overline{0.5}\}$

$= min^i\{min^i\{sup^i\,\overline{A}(b), sup^i\,\overline{A}(c), min^i\{sup^i\overline{A}(i),\overline{0.5}\}\}$

(since $\overline{A}$ is an $(\in,\in\vee q)$ - i-v fuzzy left ideal, $\overline{A}(x(y+i)-xy)\geq min^i\{\overline{A}(i),\overline{0.5}\}$)

$\leq min^i\{N(b),N(c),\overline{A}(x(y+i)-xy)\}=\overline{A}(x(y+i)-xy)=\overline{A}(a)$.

Thus, $\overline{A}(a) \not\geq min^i\{((\overline{A}\circ N)\cap(N\circ\overline{A})\cap(N*\overline{A}))(a), \overline{0.5}\}$. Hence $\overline{A}$ is an $(\in, \in\vee q)$ - i-v fuzzy quasi-ideal of N.

Theorem 4.11 43 Every $(\in, \in\vee q)$ - i-v fuzzy ideal of N is an $(\in, \in\vee q)$ - i-v fuzzy quasi-ideal of N. The proof is straight forward from Theorem 4.9 and Theorem 4.10

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