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ON (1, 2)*- g''' - CLOSED SETS IN BITOPOLOGICAL SPACE

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Abstract:

In this paper, we offer a new class of sets called (1, 2)*- g''' -closed sets in bitopological spaces and we study some of its basic properties. It turns out that this class lies between the class of $\tau_{1,2}$-closed sets and the class of (1, 2)*-g-closed sets.

Key Words: bitopological space, (1, 2)*-g-closed set, (1, 2)*- g''' -closed set, (1, 2)*- g''' -open set & (1, 2)*- ω -closed set

1. Introduction:

In 1963 Levine [19] introduced the notion of semi-open sets. According to Cameron [7] this notion was Levine's most important contribution to the field of topology. The motivation behind the introduction of semi-open sets was a problem of Kelley which Levine has considered in [20], i.e., to show that cl(U) = cl(U $\cap$ D) for all open sets U and dense sets D. He proved that U is semi-open if and only if cl(U) = cl(U $\cap$ D) for all dense sets D and D is dense if and only if cl(U) = cl(U $\cap$ D) for all semi-open sets U. Since the advent of the notion of semi-open sets, many mathematicians worked on such sets and also introduced some other notions, among others, preopen sets [22], α -open sets [25] and β-open sets [1] (Andrijevic [3] called them semi-pre open sets). It has been shown in [11] recently that the notion of preopen sets and semi-open sets are important with respect to the digital plane.

Levine [18] also introduced the notion of g-closed sets and investigated its fundamental properties. This notion was shown to be productive and very useful. For example it is shown that g-closed sets can be used to characterize the extremally disconnected spaces and the submaximal spaces (see [8] and [9]). Moreover the study of g-closed sets led to some separation axioms between T_0 and T_1 which proved to be useful in computer science and digital topology (see [17] and [14])).

Recently, Bhattacharya and Lahiri [5], Arya and Nour [4], Sheik John [31] and Rajamani and Viswanathan [28] introduced sg-closed sets, gs-closed sets, ω -closed sets and αgs -closed sets respectively.

In this paper, we introduce a new class of sets namely (1, 2)*- g''' -closed sets in bitopological spaces. This class lies between the class of closed sets and the class of (1, 2)*-g-closed sets. This class also lies between the class of closed sets and the class of (1, 2)*- ω -closed sets.

2. Preliminaries:

Throughout this paper (X, τ) and (Y, σ) (or X and Y) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X, τ), cl(A), int(A) and A^c denote the closure of A, the interior of A and the complement of A respectively. We recall the following definitions which are useful in the sequel.

Definition 2.1:

A subset A of a space (X, τ) is called:

(i) (1,2)*-semi-open set [19] if A $\subseteq$ $\tau_{1,2}$-cl($\tau_{1,2}$-int(A));
(ii) (1,2)*-preopen set [22] if A $\subseteq$ $\tau_{1,2}$-int($\tau_{1,2}$-cl(A));
(iii) (1,2)*- α -open set [25] if A $\subseteq$ $\tau_{1,2}$-int($\tau_{1,2}$-cl($\tau_{1,2}$-int(A)));
(iv) (1,2)*-β-open set [1] (= (1,2)*-semi-preopen [3]) if A $\subseteq$ $\tau_{1,2}$-cl($\tau_{1,2}$-int($\tau_{1,2}$-cl(A)));
(v) regular (1,2)*-open set [32] if A = $\tau_{1,2}$-int($\tau_{1,2}$-cl(A)).

The complements of the above mentioned $\tau_{1,2}$-open sets are called their respective $\tau_{1,2}$-closed sets.

The (1,2)*-preclosure [26] (resp. (1,2)*-semi-closure [10], (1,2)*- α -closure [25], (1,2)*-semi-pre-closure [3]) of a subset A of X, denoted by (1,2)*-pcl(A) (resp. (1,2)*-scl(A), (1,2)*- α cl(A), (1,2)*-spcl(A)) is defined to be the intersection of all (1,2)*-preclosed (resp. (1,2)*-semi-closed, (1,2)*- α -closed, (1,2)*-semi-preclosed) sets of X containing A. It is known that (1,2)*-pcl(A) (resp. (1,2)*-scl(A), (1,2)*- α cl(A), (1,2)*-spcl(A)) is a (1,2)*-preclosed (resp. (1,2)*-semi-closed, (1,2)*- α -closed, (1,2)*-semi-preclosed) set. For any subset A of an arbitrarily chosen bitopological space, the (1,2)*-semi-interior [10] (resp. (1,2)*- α -interior [25], (1,2)*-preinterior [26]) of A, denoted by (1,2)*-sint(A) (resp. (1,2)*- α int(A), (1,2)*-pint(A)), is defined to be the union of all (1,2)*-semi-open (resp. (1,2)*- α -open, (1,2)*-preopen) sets of X contained in A.

Definition 2.2:

A subset A of a bitopological space X is called

(i) (1,2)*-generalized closed (briefly, (1,2)*-g-closed) set [18] if $\tau_{1,2}$-cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is $\tau_{1,2}$-open in X.
The complement of (1,2)*-g-closed set is called (1,2)*-g-open set;
(ii) (1,2)*-semi-generalized closed (briefly, (1,2)*-sg-closed) set [5] if (1,2)*-scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is (1,2)*-semi-open in X.
The complement of sg-closed set is called sg-open set;
(iii) (1,2)*-generalized semi-closed (briefly, (1,2)*-gs-closed) set [4] if (1,2)*-scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is $\tau_{1,2}$-open in X.

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The complement of (1,2)*-gs-closed set is called (1,2)*-gs-open set;

(iv) (1,2)*-α-generalized closed (briefly, (1,2)*-α g-closed) set [21] if (1,2)*-α cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is $\tau_{1,2}$-open in X.
The complement of (1,2)*-α g-closed set is called (1,2)*-α g-open set;

(v) (1,2)*-generalized semi-preclosed (briefly, (1,2)*-gsp-closed) set [26] if (1,2)*-spcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is $\tau_{1,2}$-open in X.
The complement of (1,2)*-gsp-closed set is called (1,2)*-gsp-open set;

(vi) (1,2)*-ĝ-closed set [33] ((1,2)*-ω-closed [31]) if $\tau_{1,2}$-cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is (1,2)*-semi-open in X.
The complement of (1,2)*-ĝ-closed set is called (1,2)*-ĝ-open set;

(vii) (1,2)*-αgs-closed set [28] if (1,2)*-α cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is (1,2)*-semi-open in X.
The complement of (1,2)*-αgs-closed set is called (1,2)*-αgs-open set;

(viii) (1,2)*-g*s-closed set [23] if (1,2)*-scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is (1,2)*-gs-open in X.
The complement of (1,2)*-g*s-closed set is called (1,2)*-g*s-open set;

(ix) (1,2)*-g_α'''-closed set [29] if (1,2)*-α cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is (1,2)*-gs-open in X.
The complement of (1,2)*-g_α'''-closed set is called (1,2)*-g_α'''-open set.

Remark 2.3:

The collection of all (1,2)*-g'''-closed (resp. (1,2)*-g_α'''-closed, (1,2)*-ω-closed, (1,2)*-g-closed, (1,2)*-gs-closed, (1,2)*-gsp-closed, (1,2)*-α g-closed, (1,2)*-αgs-closed, (1,2)*-sg-closed, (1,2)*-g*s-closed, (1,2)*-α-closed, (1,2)*-semi-closed) sets is denoted by (1,2)*-G''' C(X) (resp. (1,2)*-G_α''' C(X), (1,2)*-ω C(X), (1,2)*-G C(X), (1,2)*-GS C(X), (1,2)*-GSP C(X), (1,2)*-αg C(X), (1,2)*-αGS C(X), (1,2)*-SG C(X), (1,2)*-G*SC(X), (1,2)*-α C(X), (1,2)*-S C(X)).

The collection of all (1,2)*-g'''-open (resp. (1,2)*-g_α'''-open, (1,2)*-ω-open, (1,2)*-g-open, (1,2)*-gs-open, (1,2)*-gsp-open, (1,2)*-α g-open, (1,2)*-αgs-open, (1,2)*-sg-open, (1,2)*-g*s-open, (1,2)*-α-open, (1,2)*-semi-open) sets is denoted by (1,2)*-G''' O(X) (resp. (1,2)*-G_α''' O(X), (1,2)*-ω O(X), (1,2)*-G O(X), (1,2)*-GS O(X), (1,2)*-GSP O(X), (1,2)*-αg O(X), (1,2)*-αGS O(X), (1,2)*-SG O(X), (1,2)*-G*SO(X), (1,2)*-α O(X), (1,2)*-S O(X)).

We denote the power set of X by P(X).

Definition 2.4 [16]:

A subset S of X is said to be (1,2)*-locally closed if S = U $\cap$ F, where U is $\tau_{1,2}$-open and F is $\tau_{1,2}$-closed in X.

Result 2.5:

(1) Every $\tau_{1,2}$-open set is (1,2)*-g*s-open [23].
(2) Every (1,2)*-semi-open set is (1,2)*-g*s-open [23].
(3) Every (1,2)*-g*s-open set is (1,2)*-sg-open [23].
(4) Every (1,2)*-semi-closed set is (1,2)*-gs-closed [24].
(5) Every $\tau_{1,2}$-closed set is (1,2)*-gs-closed [12].

Corollary 2.6 [27]:

Let A be both $\tau_{1,2}$-open and (1,2)*-sg-closed set and suppose that F is $\tau_{1,2}$-closed set. Then A $\cap$ F is (1,2)*-gs-closed set.

3. (1,2)*-g'''-Closed Sets:

We introduce the following definition.

Definition 3.1:

A subset A of X is called a (1,2)*-g'''-closed set if $\tau_{1,2}$-cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is (1,2)*-gs-open in X.

Proposition 3.2:

Every $\tau_{1,2}$-closed set is (1,2)*-g'''-closed.

Proof:

If A is any $\tau_{1,2}$-closed set in X and G is any (1,2)*-gs-open set containing A, then G $\supseteq$ A = $\tau_{1,2}$-cl(A). Hence A is (1,2)*-g'''-closed. The converse of Proposition 3.2 need not be true as seen from the following example.

Example 3.3:

Let X = {a, b, c} with τ = {ϕ, {a, b}, X}. Then G''' C(X) = {ϕ, {c}, {a, c}, {b, c}, X}. Here, A = {a, c} is g'''-closed set but not closed.

Proposition 3.4:

Every (1,2)*-g'''-closed set is (1,2)*-g_α'''-closed.

Proof:

If A is a (1,2)*- g''' -closed subset of X and G is any (1,2)*-gs-open set containing A, then G $\supseteq$ $\tau_{1,2}$-cl(A) $\supseteq$ (1,2)*- α cl(A). Hence A is (1,2)*- g'''_{α} -closed in X. The converse of Proposition 3.4 need not be true as seen from the following example.

Example 3.5:

Let X = {a, b, c} with τ = {ϕ, {b}, X}. Then (1,2)*- G''' C(X) = {ϕ, {a, c}, X} and (1,2)*- G'''_{α} C(X) = {ϕ, {a}, {c}, {a, c}, X}. Here, A = {a} is (1,2)*- g'''_{α} -closed but not (1,2)*- g''' -closed set in X.

Proposition 3.6:

Every (1,2)*- g''' -closed set is (1,2)*-g*s-closed.

Proof:

If A is a (1,2)*- g''' -closed subset of X and G is any (1,2)*-gs-open set containing A, then G $\supseteq$ $\tau_{1,2}$-cl(A) $\supseteq$ (1,2)*-scl(A). Hence A is (1,2)*-g*s-closed in X. The converse of Proposition 3.6 need not be true as seen from the following example.

Example 3.7:

In Example 3.5, (1,2)*-G*SC(X) = {ϕ, {a}, {c}, {a, c}, X}. Here, A = {c} is (1,2)*-g*s-closed but not (1,2)*- g''' -closed set in X.

Proposition 3.8:

Every (1,2)*- g''' -closed set is (1,2)*- ω -closed.

Proof:

Suppose that A $\subseteq$ G and G is (1,2)*-semi-open in X. Since every (1,2)*-semi-open set is (1,2)*-gs-open and A is (1,2)*- g''' -closed, therefore $\tau_{1,2}$-cl(A) $\subseteq$ G. Hence A is (1,2)*- ω -closed in X. The converse of Proposition 3.8 need not be true as seen from the following example.

Example 3.9:

Let X = {a, b, c } with τ = {ϕ, {a}, {b, c}, X}. Then (1,2)*- G''' C(X) = {ϕ, {a}, {b, c}, X} and (1,2)*- ω C(X) = P(X). Here, A = {a, c} is (1,2)*- ω -closed but not (1,2)*- g''' -closed set in X.

Proposition 3.10:

Every (1,2)*-g*s-closed set is (1,2)*-sg-closed.

Proof:

Suppose that A $\subseteq$ G and G is (1,2)*-semi-open in X. Since every (1,2)*-semi-open set is (1,2)*-gs-open and A is (1,2)*-g*s-closed, therefore (1,2)*-scl(A) $\subseteq$ G. Hence A is (1,2)*-sg-closed in X. The converse of Proposition 3.10 need not be true as seen from the following example.

Example 3.11:

Let X = {a, b, c} with τ = {ϕ, {a}, {b, c}, X}. Then (1,2)*-G*SC(X) = {ϕ, {a}, {b, c}, X} and (1,2)*-SGC(X) = P(X). Here, A = {a, b} is (1,2)*-sg-closed but not (1,2)*-g*s-closed set in X.

Proposition 3.12:

Every (1,2)*- ω -closed set is (1,2)*- αgs -closed.

Proof:

If A is a (1,2)*- ω -closed subset of X and G is any (1,2)*-semi-open set containing A, then G $\supseteq$ $\tau_{1,2}$-cl(A) $\supseteq$ (1,2)*- α cl(A). Hence A is (1,2)*- αgs -closed in X. The converse of Proposition 3.12 need not be true as seen from the following example.

Example 3.13:

Let X = {a, b, c} with τ = {ϕ, {a}, X}. Then (1,2)*- ω C(X) = {ϕ, {b, c}, X} and (1,2)*- αGS C(X) = {ϕ, {b}, {c}, {b, c}, X}. Here, A = {b} is (1,2)*- αgs -closed but not (1,2)*- ω -closed set in X.

Proposition 3.14:

Every (1,2)*- g''' -closed set is (1,2)*-g-closed.

Proof:

If A is a (1,2)*- g''' -closed subset of X and G is any open set containing A, since every $\tau_{1,2}$-open set is (1,2)*-gs-open, we have G $\supseteq$ $\tau_{1,2}$-cl(A). Hence A is (1,2)*-g-closed in X. The converse of Proposition 3.14 need not be true as seen from the following example.

Example 3.15:

Let X = {a, b, c} with τ = {ϕ, {a}, {b, c}, X}. Then (1,2)*- G''' C(X) = {ϕ, {a}, {b, c}, X} and (1,2)*- G C(X) = P(X). Here, A = {a, b} is (1,2)*-g-closed but not (1,2)*- g''' -closed set in X.

Proposition 3.16:

Every (1,2)*- g''' -closed set is (1,2)*- αgs -closed.

Proof:

If A is a (1,2)*- g''' -closed subset of X and G is any (1,2)*-semi-open set containing A, since every (1,2)*-semi-open set is (1,2)*-gs-open, we have G $\supseteq$ $\tau_{1,2}$-cl(A) $\supseteq$ (1,2)*- α cl(A). Hence A is (1,2)*- αgs -closed in X. The converse of Proposition 3.16 need not be true as seen from the following example.

Example 3.17:

Let X = {a, b, c} with τ = {ϕ, {a}, {b, c}, X}. Then (1,2)*- G''' C(X) = {ϕ, {a}, {b, c}, X} and (1,2)*- αGS C(X) = P(X). Here, A = {a, c} is (1,2)*- αgs -closed but not (1,2)*- g''' -closed set in X.

Proposition 3.18:

Every (1,2)*- g''' -closed set is (1,2)*- α g-closed.

Proof:

If A is a (1,2)*- g''' -closed subset of X and G is any $\tau_{1,2}$-open set containing A, since every $\tau_{1,2}$-open set is (1,2)*-gs-open, we have G $\supseteq$ $\tau_{1,2}$-cl(A) $\supseteq$ (1,2)*- α cl(A). Hence A is (1,2)*- α g-closed in X. The converse of Proposition 3.18 need not be true as seen from the following example.

Example 3.19:

Let X = {a, b, c} with τ = {ϕ, {c}, {a, b}, X}. Then (1,2)*- G''' C(X) = {ϕ, {c}, {a, b}, X} and (1,2)*- αg C(X) = P(X). Here, A = {a, c} is (1,2)*- α g-closed but not (1,2)*- g''' -closed set in X.

Proposition 3.20:

Every (1,2)*- g''' -closed set is (1,2)*-gs-closed.

Proof:

If A is a (1,2)*- g''' -closed subset of X and G is any $\tau_{1,2}$-open set containing A, since every $\tau_{1,2}$-open set is (1,2)*-gs-open, we have G $\supseteq$ $\tau_{1,2}$-cl(A) $\supseteq$ (1,2)*-scl(A). Hence A is (1,2)*-gs-closed in X. The converse of Proposition 3.20 need not be true as seen from the following example.

Example 3.21:

Let X = {a, b, c} with τ = {ϕ, {a}, X}. Then (1,2)*- G''' C(X) = {ϕ, {b, c}, X} and (1,2)*- GS C(X) = {ϕ, {b}, {c}, {a, b}, {a, c}, {b, c}, X}. Here, A = {c} is (1,2)*-gs-closed but not (1,2)*- g''' -closed set in X.

Proposition 3.22:

Every (1,2)*- g''' -closed set is (1,2)*-gsp-closed.

Proof:

If A is a (1,2)*- g''' -closed subset of X and G is any $\tau_{1,2}$-open set containing A, every $\tau_{1,2}$-open set is (1,2)*-gs-open, we have G $\supseteq$ $\tau_{1,2}$-cl(A) $\supseteq$ (1,2)*-spcl(A). Hence A is (1,2)*-gsp-closed in X.

The converse of Proposition 3.22 need not be true as seen from the following example.

Example 3.23:

In Example 3.21, (1,2)*- GSP C(X) = {ϕ, {b}, {c}, {a, b}, {a, c}, {b, c}, X}. Here, A = {c} is (1,2)*-gsp-closed but not (1,2)*- g''' -closed set in X.

Remark 3.24:

The following example shows that (1,2)*- g''' -closed sets are independent of (1,2)*- α -closed sets and (1,2)*-semi-closed sets.

Example 3.25:

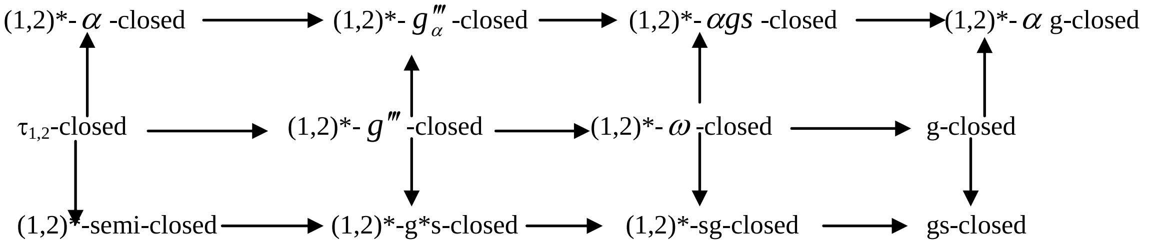
Let X = {a, b, c} with τ = {ϕ, {a, b}, X}. Then (1,2)*- G''' C(X) = {ϕ, {c}, {a, c}, {b, c}, X} and (1,2)*- α C(X) = S C(X) = {ϕ, {c}, X}. Here, A = {a, c} is (1,2)*- g''' -closed but it is neither (1,2)*- α -closed nor (1,2)*-semi-closed in X.

Example 3.26:

Let X = {a, b, c} with τ = {ϕ, {a}, X}. Then (1, 2)*- G''' C(X) = {ϕ, {b, c}, X} and (1, 2)*- α C(X) = S C(X) = {ϕ, {b}, {c}, {b, c}, X}. Here, A = {b} is (1, 2)*- α -closed as well as (1, 2)*-semi-closed in X but it is not (1, 2)*- g''' -closed in X.

Remark 3.27:

From the above discussions and known results in [28, 31, 33], we obtain the following diagram, where A $\rightarrow$ B (resp. A $\longleftrightarrow$ B) represents A implies B but not conversely (resp. A and B are independent of each other).



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None of the above implications is reversible as shown in the remaining examples and in the related papers [28, 31, 33].

4. Properties of (1,2)*- g''' -Closed Sets:

In this section, we have proved that an arbitrary intersection of (1,2)*- g''' -closed sets is (1,2)*- g''' -closed. Moreover, we discuss some basic properties of (1,2)*- g''' -closed sets.

Definition 4.1:

The intersection of all (1,2)*-gs-open subsets of X containing A is called the (1,2)*-gs-kernel of A and denoted by (1,2)*-gs-ker(A).

Lemma 4.2:

A subset A of X is (1,2)*- g''' -closed if and only if $\tau_{1,2}$-cl(A) $\subseteq$ (1,2)*-gs-ker(A).

Proof:

Suppose that A is (1,2)*- g''' -closed. Then $\tau_{1,2}$-cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is (1,2)*-gs-open. Let x $\in$ $\tau_{1,2}$-cl(A). If x $\notin$ (1,2)*-gs-ker(A), then there is a (1,2)*-gs-open set U containing A such that x $\notin$ U. Since U is a (1,2)*-gs-open set containing A, we have x $\notin$ $\tau_{1,2}$-cl(A) and this is a contradiction.

Conversely, let $\tau_{1,2}$-cl(A) $\subseteq$ (1,2)*-gs-ker(A). If U is any (1,2)*-gs-open set containing A, then $\tau_{1,2}$-cl(A) $\subseteq$ (1,2)*-gs-ker(A) $\subseteq$ U. Therefore, A is (1,2)*- g''' -closed.

Proposition 4.3:

For any subset A of X, $X_2 \cap \tau_{1,2}$-cl(A) $\subseteq$ (1,2)*-gs-ker(A), where X_2 = {x $\in$ X : {x} is (1,2)*-preopen}.

Proof:

Let x $\in$ $X_2 \cap \tau_{1,2}$-cl(A) and suppose that x $\notin$ (1,2)*-gs-ker(A). Then there is a (1,2)*-gs-open set U containing A such that x $\notin$ U. If F = X – U, then F is (1,2)*-gs-closed. Since $\tau_{1,2}$-cl({x}) $\subseteq$ $\tau_{1,2}$-cl(A), we have $\tau_{1,2}$-int($\tau_{1,2}$-cl({x})) $\subseteq$ A $\cup$ $\tau_{1,2}$-int($\tau_{1,2}$-cl(A)). Again since x $\in$ X_2, we have x $\notin$ X_1 and so $\tau_{1,2}$-int($\tau_{1,2}$-cl({x})) = ϕ. Therefore, there has to be some y $\in$ A $\cap$ $\tau_{1,2}$-int($\tau_{1,2}$-cl({x})) and hence y $\in$ F $\cap$ A, a contradiction.

Theorem 4.4:

A subset A of X is (1,2)*- g''' -closed if and only if $X_1 \cap \tau_{1,2}$-cl(A) $\subseteq$ A, where X_1 = {x $\in$ X : {x} is (1,2)*-nowhere dense}.

Proof:

Suppose that A is (1,2)*- g''' -closed. Let x $\in$ $X_1 \cap \tau_{1,2}$-cl(A). Then x $\in$ X_1 and x $\in$ $\tau_{1,2}$-cl(A). Since x $\in$ X_1, $\tau_{1,2}$-int($\tau_{1,2}$-cl({x})) = ϕ. Therefore, {x} is (1,2)*-semi-closed, since $\tau_{1,2}$-int($\tau_{1,2}$-cl({x})) $\subseteq$ {x}. Since every (1,2)*-semi-closed set is (1,2)*-gs-closed [Result 2.5 (4)], {x} is (1,2)*-gs-closed. If x $\notin$ A and if U = X \ {x}, then U is a (1,2)*-gs-open set containing A and so $\tau_{1,2}$-cl(A) $\subseteq$ U, a contradiction.

Conversely, suppose that $X_1 \cap \tau_{1,2}$-cl(A) $\subseteq$ A. Then $X_1 \cap \tau_{1,2}$-cl(A) $\subseteq$ (1,2)*-gs-ker(A), since A $\subseteq$ (1,2)*-gs-ker(A). Now $\tau_{1,2}$-cl(A) = X $\cap$ $\tau_{1,2}$-cl(A) = ($X_1 \cup X_2$) $\cap$ $\tau_{1,2}$-cl(A) = ($X_1 \cap \tau_{1,2}$-cl(A)) $\cup$ ($X_2 \cap \tau_{1,2}$-cl(A)) $\subseteq$ (1,2)*-gs-ker(A), since $X_1 \cap \tau_{1,2}$-cl(A) $\subseteq$ (1,2)*-gs-ker(A) and Proposition 4.3. Thus, A is (1,2)*- g''' -closed by Lemma 4.2.

Theorem 4.5:

An arbitrary intersection of (1,2)*- g''' -closed sets is (1,2)*- g''' -closed.

Proof:

Let F = {A_i : i $\in$ $\wedge$} be a family of (1,2)*- g''' -closed sets and let A = $\cap_{i \in \wedge} A_i$. Since A $\subseteq$ A_i for each i, $X_1 \cap \tau_{1,2}$-cl(A) $\subseteq$ $X_1 \cap \tau_{1,2}$-cl(A_i) for each i. Using Theorem 4.4 for each (1,2)*- g''' -closed set A_i, we have $X_1 \cap \tau_{1,2}$-cl(A_i) $\subseteq$ A_i. Thus, $X_1 \cap \tau_{1,2}$-cl(A) $\subseteq$ $X_1 \cap \tau_{1,2}$-cl(A_i) $\subseteq$ A_i for each i $\in$ $\wedge$. That is, $X_1 \cap \tau_{1,2}$-cl(A) $\subseteq$ A and so A is (1,2)*- g''' -closed by Theorem 4.4.

Corollary 4.6:

If A is a (1,2)*- g''' -closed set and F is a $\tau_{1,2}$-closed set, then A $\cap$ F is a (1,2)*- g''' -closed set.

Proof:

Since F is closed, it is (1,2)*- g''' -closed. Therefore by Theorem 4.5, A $\cap$ F is also a (1,2)*- g''' -closed set.

Proposition 4.7:

If A and B are (1,2)*- g''' -closed sets in X, then A $\cup$ B is (1,2)*- g''' -closed in X.

Proof:

If A $\cup$ B $\subseteq$ G and G is (1,2)*-gs-open, then A $\subseteq$ G and B $\subseteq$ G. Since A and B are (1,2)*- g''' -closed, G $\supseteq$ $\tau_{1,2}$-cl(A) and G $\supseteq$ $\tau_{1,2}$-cl(B) and hence G $\supseteq$ $\tau_{1,2}$-cl(A) $\cup$ $\tau_{1,2}$-cl(B) = $\tau_{1,2}$-cl(A $\cup$ B). Thus A $\cup$ B is (1,2)*- g''' -closed set in X.

Proposition 4.8:

If a set A is (1,2)*- g''' -closed in X, then $\tau_{1,2}$-cl(A) – A contains no nonempty $\tau_{1,2}$-closed set in X.

Proof:

Suppose that A is (1,2)*- g''' -closed. Let F be a $\tau_{1,2}$-closed subset of $\tau_{1,2}$-cl(A) – A. Then A $\subseteq F^c$. But A is (1,2)*- g''' -closed, therefore $\tau_{1,2}$-cl(A) $\subseteq F^c$. Consequently, F $\subseteq (\tau_{1,2}\text{-cl}(A))^c$. We already have F $\subseteq \tau_{1,2}$-cl(A). Thus F $\subseteq \tau_{1,2}\text{-cl}(A) \cap (\tau_{1,2}\text{-cl}(A))^c$ and F is empty.

The converse of Proposition 4.8 need not be true as seen from the following example.

Example 4.9:

Let X = {a, b, c} with $\tau = \{\phi, \{a\}, X\}$. Then (1,2)*- G''' C(X) = {ϕ, {b, c}, X}. If A = {b}, then $\tau_{1,2}$-cl(A) – A = {c} does not contain any nonempty $\tau_{1,2}$-closed set. But A is not (1,2)*- g''' -closed in X.

Theorem 4.10:

A set A is (1,2)*- g''' -closed if and only if $\tau_{1,2}$-cl(A) – A contains no nonempty (1,2)*-gs-closed set.

Proof:

Necessity. Suppose that A is (1,2)*- g''' -closed. Let S be a (1,2)*-gs-closed subset of $\tau_{1,2}$-cl(A) – A. Then A $\subseteq S^c$. Since A is (1,2)*- g''' -closed, we have $\tau_{1,2}$-cl(A) $\subseteq S^c$. Consequently, S $\subseteq (\tau_{1,2}\text{-cl}(A))^c$. Hence, S $\subseteq \tau_{1,2}\text{-cl}(A) \cap (\tau_{1,2}\text{-cl}(A))^c = \phi$. Therefore S is empty.

Sufficiency. Suppose that $\tau_{1,2}$-cl(A) – A contains no nonempty (1,2)*-gs-closed set. Let A $\subseteq$ G and G be both $\tau_{1,2}$-closed and (1,2)*-sg-open. If $\tau_{1,2}$-cl(A) $\not\subset$ G, then $\tau_{1,2}\text{-cl}(A) \cap G^c \neq \phi$. Since $\tau_{1,2}$-cl(A) is a $\tau_{1,2}$-closed set and G^c is both $\tau_{1,2}$-open and (1,2)*-sg-closed set, $\tau_{1,2}\text{-cl}(A) \cap G^c$ is a nonempty (1,2)*-gs-closed subset of $\tau_{1,2}$-cl(A) – A (from Corollary 2.6). This is a contradiction. Therefore, $\tau_{1,2}$-cl(A) $\subseteq$ G and hence A is (1,2)*- g''' -closed.

Proposition 4.11:

If A is (1,2)*- g''' -closed in X and A $\subseteq$ B $\subseteq \tau_{1,2}$-cl(A), then B is (1,2)*- g''' -closed in X.

Proof:

Since B $\subseteq \tau_{1,2}$-cl(A), we have $\tau_{1,2}$-cl(B) $\subseteq \tau_{1,2}$-cl(A). Then, $\tau_{1,2}$-cl(B) – B $\subseteq \tau_{1,2}$-cl(A) – A. Since $\tau_{1,2}$-cl(A) – A has no nonempty (1,2)*-gs-closed subsets, neither does $\tau_{1,2}$-cl(B) – B. By Theorem 4.10, B is (1,2)*- g''' -closed.

Proposition 4.12:

Let A $\subseteq$ Y $\subseteq$ X and suppose that A is (1,2)*- g''' -closed in X. Then A is (1,2)*- g''' -closed relative to Y.

Proof:

Let A $\subseteq$ Y $\cap$ G, where G is (1,2)*-gs-open in X. Then A $\subseteq$ G and hence $\tau_{1,2}$-cl(A) $\subseteq$ G. This implies that Y $\cap$ $\tau_{1,2}$-cl(A) $\subseteq$Y $\cap$ G. Thus A is (1,2)*- g''' -closed relative to Y.

Proposition 4.13:

If A is a (1,2)*-gs-open and (1,2)*- g''' -closed in X, then A is closed in X.

Proof:

Since A is (1,2)*-gs-open and (1,2)*- g''' -closed, $\tau_{1,2}$-cl(A) $\subseteq$ A and hence A is $\tau_{1,2}$-closed in X.

Recall that a bitopological space X is called (1,2)*-extremally disconnected if $\tau_{1,2}$-cl(U) is $\tau_{1,2}$-open for each U $\in \tau_{1,2}$.

Theorem 4.14:

Let X be (1,2)*-extremally disconnected and A a (1,2)*-semi-open subset of X. Then A is (1,2)*- g''' -closed if and only if it is (1,2)*-gs-closed.

Proof:

It follows from the fact that if X is (1,2)*-extremally disconnected and A is a (1,2)*-semi-open subset of X, then (1,2)*-scl(A) = $\tau_{1,2}$-cl(A) (Lemma 0.3 [15]).

Theorem 4.15:

Let A be a (1,2)*-locally closed set of X. Then A is $\tau_{1,2}$-closed if and only if A is (1,2)*- g''' -closed.

Proof:

(i) ⇒ (ii). It is fact that every $\tau_{1,2}$-closed set is (1,2)*- g''' -closed.

(ii) ⇒ (i). By Proposition 5.1.3.3 of Bourbaki [6], A ∪ (X – $\tau_{1,2}$-cl(A)) is $\tau_{1,2}$-open in X, since A is (1,2)*-locally closed. Now A ∪ (X – $\tau_{1,2}$-cl(A)) is (1,2)*-gs-open set of X such that A $\subseteq$ A ∪ (X – $\tau_{1,2}$-cl(A)). Since A is (1,2)*- g''' -closed, then $\tau_{1,2}$-cl(A) $\subseteq$ A ∪ (X – $\tau_{1,2}$-cl(A)). Thus, we have $\tau_{1,2}$-cl(A) $\subseteq$ A and hence A is a $\tau_{1,2}$-closed.

Proposition 4.16:

For each x ∈ X, either {x} is (1,2)*-gs-closed or $\{x\}^c$ is (1,2)*- g''' -closed in X.

Proof:

Suppose that {x} is not (1,2)*-gs-closed in X. Then $\{x\}^c$ is not (1,2)*-gs-open and the only (1,2)*-gs-open set containing $\{x\}^c$ is the space X itself. Therefore $\tau_{1,2}$-cl($\{x\}^c$) $\subseteq$ X and so $\{x\}^c$ is (1,2)*- g''' -closed in X.

Theorem 4.17:

Let A be a (1,2)*- g''' -closed set of a bitopological space X. Then,

(i) (1,2)*-sint(A) is (1,2)*- g''' -closed.

(ii) If A is regular (1,2)*-open, then (1,2)*-pint(A) and (1,2)*-scl(A) are also (1,2)*- g''' -closed sets.

(iii) If A is regular (1,2)*-closed, then (1,2)*-pcl(A) is also (1,2)*- g''' -closed.

Proof:

(i) Since $\tau_{1,2}$-cl($\tau_{1,2}$-int(A)) is a closed set in X, by Corollary 4.6, (1,2)*-sint(A) = A ∩ $\tau_{1,2}$-cl($\tau_{1,2}$-int(A)) is (1,2)*- g''' -closed in X.

(ii) Since A is regular (1,2)*-open in X, A = $\tau_{1,2}$-int($\tau_{1,2}$-cl(A)). Then (1,2)*-scl(A) = A ∪ $\tau_{1,2}$-int($\tau_{1,2}$-cl(A)) = A. Thus, (1,2)*-scl(A) is (1,2)*- g''' -closed in X. Since (1,2)*-pint(A) = A ∩ $\tau_{1,2}$-int($\tau_{1,2}$-cl(A)) = A, (1,2)*-pint(A) is (1,2)*- g''' -closed.

(iii) Since A is regular (1,2)*-closed in X, A = $\tau_{1,2}$-cl($\tau_{1,2}$-int(A)). Then (1,2)*-pcl(A) = A ∪ $\tau_{1,2}$-cl($\tau_{1,2}$-int(A)) = A. Thus, (1,2)*-pcl(A) is (1,2)*- g''' -closed in X.

The converses of the statements in the Theorem 4.17 are not true as we see in the following examples.

Example 4.18:

Let X = {a, b, c} with τ = {ϕ, {c}, {b, c}, X}. Then (1,2)*- G''' C(X) = {ϕ, {a}, {a, b}, X}. Then the set A = {b} is not a (1,2)*- g''' -closed set. However (1,2)*-sint(A) = ϕ is a (1,2)*- g''' -closed.

Example 4.19:

Let X = {a, b, c} with τ = {ϕ, {a}, {a, b}, X}. Then (1,2)*- G''' C(X) = {ϕ, {c}, {b, c}, X}. Then the set A = {c} is not regular (1,2)*-open. However A is (1,2)*- g''' -closed and (1,2)*-scl(A) = {c} is a (1,2)*- g''' -closed and (1,2)*-pint(A) = ϕ is also (1,2)*- g''' -closed.

Example 4.20:

In Example 4.19, the set A = {c} is not regular (1,2)*-closed. However A is a (1,2)*- g''' -closed and (1,2)*-pcl(A) = {c} is (1,2)*- g''' -closed.

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