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INTUITIONISTIC FUZZY B* ALGEBRA IN LINEAR TRANSFORMATION

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Abstract:

In this paper, we discussed about the Intuitionistic Fuzzy B* algebra in Linear Transformations (IFB*LT) and shown that set of all linear transformations L (V) defined over an intuitionistic fuzzy vector space V does not from vector space. Here we determine the unique intuitionistic fuzzy B* algebra in Linear Transformation.

Key Words: Fuzzy Set, B Algebra, B* Algebra, Intuitionistic, Intuitionistic Fuzzy Vector Space & Intuitionistic Fuzzy Linear Transformation

Introduction:

Uncertain or imprecise data are inherent and pervasive in many important applications in the areas such as economics, engineering, environment, social science, medical science and business management there have been a great amount of research and applications in the literature concerning some special tools like probability theory, intuitionistic fuzzy set theory, rough set theory, vague set theory and interval mathematics to modeling uncertain data. However, all of these have advantages as well as inherent limitations in dealing with uncertainties.

Initially, fuzzy set theory was proposed by zadeh [2] as a means of representing mathematically any imprecise or vague system of information in the real world in fuzzy set theory, there were no scope to think about the hesitation in the membership degrees which is arise in various real life situations. This situation is over come in 1983 by invention of intuitionistic fuzzy set [1]. Here it is possible to model hesitation and uncertainty by using an additional degree.

Meenakshi and Gandhimathi [2] introduce the concept of thelinear transformation (LT) on intuitionistic fuzzy vector space (IFVS) and studied the several properties of it.

We claim that the set of LTs on IFVSV forms a vector space under addition and multiplication of LTs. But we have shown that, L (V) does not form an IFVS. He we discussed about the solution of the intuitionistic fuzzy B* algebra in LT which is not available in [3].

2. Preliminaries

Definition 2.1: Let X be a non-empty set. A fuzzy set A on X is a mapping A: X →[0, 1] and is defined as A = {x ϵ X/(x, μ(x))}

Definition 2.2: A non-empty set X with a constant 0 and a binary operation * is called a B-algebra. If it satisfies the following axioms.

(i) x * x = 0
(ii) x * 0 = x
(iii) (x*y)*z = x*(z*(0*y)) for all x,y, z ϵ X

Definition 2.3: B* algebra are mathematical structures studied in functional analysis. A B* algebra A is a Banach algebra over the field of complex numbers, together with a map *: A→ A called involution which has the following properties:

(i) (x+y) = x* + y* for all x, y in A
(ii) (λx)* = λ*x* for every λ in c and every X in A; here A* stands of the complex conjugation of λ.
(iii) (xy)* = y*x* for all x, y in A
(iv) (x*)* = x for all in x in A

Definition 2.4: An IFs A is a non-empty set x is an object having the form A = {(x, $\alpha_A(x)$, $\beta_A(x)$)/x ϵ X} where the functions α_A: x→ [0, 1] andβ_A: x→ [0, 1] denote the degree of membership, respectively, and 0 ≤ α_A (x) + β_A (x)≤ ∀ x ϵ X

An IFs A = {(x, $\alpha_A(x)$, β_A (x)/ x ϵ X} in X can be identified to an ordered pair ($\alpha_{A,}\beta_A$) in $I^x \times I^x$. For the sake of simplicity, we shall use the symbol A = (α_A, β_A) for the Ifs A = {(x, $\alpha_A(x)$, β_A (x)/ x ϵ X}

Definition 2.5: Let V and W be two vector spaces over the Intuitionistic Fuzzy algebra (IF). A mapping T of V into W is called a linear transformation if for any x, y ϵ V and αϵ IF.

(i) T (x+y) = T(x) + T(y) and
(ii) T (αx) = α.T(x)

This linear transformation is called IFLT.

Definition 2.6: Let A = (μ_A, V_A) be an intuitionistic fuzzy set of a classical vector space v over F. For any x, y ϵ V and α, βϵ F, if it satisfy $\mu_A(\alpha x + \beta y) \geq \min \{ \mu_A(x), \mu_A(y)\}$ and

$$V_A(\alpha x + \beta y) \leq \max \{V_A(x), V_A(y)\}$$

Then A is called an intuitionistic fuzzy subspace of V.

Let V_n denotes the Let of all n-tuples (<$X_{1\mu}$, X_{1V}>, <$X_{2\mu}$, X_{2V}>…<$X_{n\mu}$, X_{nV}>) over F. An element of V_n is called an intuitionistic fuzzy vector (1FV) of dimension n, where $X_{i\mu}$ and X_{iv} are the membership and non-membership values of the component x_i.

Definition 2.7: Let V and W be two vector spaces over the intuitionistic fuzzy B* algebra. A mapping μ ∗ of V in W is called a linear transformation if for any X, YϵV and αϵIFB*A.

(i) μ*(x+y) = μ*(x)+μ*(y)
(ii) μ*(αx) = αμ*(x)
(iii) (μ ∗) ∗ = μ∀ x, y in V

Example 2.8: Let V_3 and V_2 be vector spaces over IF the mapping $\mu *: v_3 \to v_2$ defined as $\mu^*(x_1, x_2, x_3) = (x_1, x_2)$ is a linear transformation.

Theorem 2.9:

Let V be a vector space over IFB*A and L (V) be the set of all linear transformation defined on V then L (V) is closed under addition and multiplication defined by,

$$(\mu_1^* + \mu_2^*)(x) = \mu_1^*(x) + \mu_2^*(x)$$
$$(\alpha\mu_1 *) = \alpha\mu_1^*(x_1) \text{ for all } \mu_1^*, \mu_2^* \epsilon V\ X\epsilon V \text{ and } \alpha\epsilon IFB^*A$$

Proof:

For X, YϵV

$$\begin{aligned}(\mu_1^* + \mu_2^*)(x+y) &= \mu_1^*(x+y) + \mu_2^*(x+y)\\ &= \mu_1^*(x) + \mu_1^*(y) + \mu_2^*(x) + \mu_2^*(y)\\ &= (\mu_1^* + \mu_2^*)(x) + (\mu_1^* + \mu_2^*)(y) \text{ for all } \mu_1^*, \mu_2^* \epsilon L(V)\end{aligned}$$

Again,

$$\begin{aligned}(\mu_1^* + \mu_2^*)(\alpha x) &= \mu_1^*(\alpha x) + \mu_2^*(\alpha x)\\ &= \alpha\mu_1^*(x) + \alpha\mu_2^*(x)\\ &= \alpha(\mu_1^*(x) + \mu_2^*(x))\\ &= \alpha(\mu_1^* + \mu_2^*)(x) \text{ for all } \mu_1^*, \mu_2^* \epsilon L(V) \text{ and } \alpha \epsilon IFB^*A\end{aligned}$$

Thus,

$\mu_1^* + \mu_2^* \epsilon L(V)$ for all $\mu_1^*, \mu_2^* \epsilon L(V)$

For,

αϵ IFB*A and μ ∗ ϵL(V)

$$\begin{aligned}(\alpha\mu^*)(x+y) &= \alpha(\mu^*(x+y))\\ &= \alpha(\mu^*(x) + \mu^*(y))\\ &= \alpha\mu^*(x) + \alpha\mu^*(y)\end{aligned}$$

And

$$\begin{aligned}(\alpha\mu^*)(\beta x) &= \alpha(\mu^*(\beta x))\\ &= \alpha(\beta(\mu^* x))\\ &= \beta((\alpha\mu^*)(x))\end{aligned}$$

Hence αμ* ϵ L (V)

So L(V) is closed under addition and multiplication.

Theorem 2.10:

For a Intuitionistic fuzzy vector space V over (IFB*A), L(V) is an algebra under multiplication defined by $\mu_1^* \mu_2^*(x) = \mu_1^*(\mu_2^*(x))$ for all $\mu_1^*, \mu_2^* \epsilon L(V)$.

Proof:

L(V) is an Intuitionistic fuzzy vector space follows. We will prove that $\mu_1^*, \mu_2^* \epsilon L(V)$ for all $\mu_1^*, \mu_2^* \epsilon L(V)$.

$$\begin{aligned}(\mu_1^* \mu_2^*)(x+y) &= \mu_1^*(\mu_2^*(x+y))\\ &= \mu_1^*(\mu_2^*(x) + \mu_2^*(y))\\ &= \mu_1^*(\mu_2^*(x)) + \mu_1^*(\mu_2^*(y))\\ &= \mu_1^*\mu_2^*(x) + \mu_1^*\mu_2^*(y)\end{aligned}$$

$$\begin{aligned}(\mu_1^* \mu_2^*)(\alpha x) &= \mu_1^*(\mu_2^*(\alpha x))\\ &= \mu_1^*(\alpha\mu_2^*(x))\\ &= \alpha\mu_1^*(\mu_2^*(x))\\ &= \alpha(\mu_1^*\mu_2^*)(x)\end{aligned}$$

Thus μ_1^*, $\mu_2^* \epsilon L(V)$, Hence the proof.

Theorem 2.11:

For $\mu_1^*, \mu_2^*, \mu_3^* \epsilon L(V)$ and $\alpha, \beta \epsilon IFB^*A$ the following properties are hold.

(i) $\mu_1^* + \mu_2^* = \mu_2^* + \mu_1^*$
(ii) $(\mu_1^* + \mu_2^*) + \mu_3^* = \mu_1^* + (\mu_2^* + \mu_3^*)$
(iii) $(\alpha\beta)\mu_1^* = \alpha(\beta\mu_1^*)$
(iv) $(\alpha + \beta)\mu_1^* = \alpha\mu_1^* + \beta\mu_1^*$
(v) $\alpha(\mu_1^* + \mu_2^*) = \alpha\mu_1^* + \alpha\mu_2^*$
(vi) if $\mu^* = x$ then $\mu^*\mu_1^* = \mu_1^*$ for all $\mu_1^* \epsilon L(V)$
(vii) if $\mu^*(x) = 0$ then $\mu^*\mu_1^* = 0$ for all $\mu_1^* \epsilon L(V)$

Proof:

(i) For any $X \epsilon V$

$$(\mu_1^* + \mu_2^*)(x) = \mu_1^*(x) + \mu_2^*(x) = (\mu_1^* + \mu_2^*)x$$

Hence, $\mu_1^* + \mu_2^* = \mu_2^* + \mu_1^*$

(ii) For any $X \epsilon V$

$$((\mu_1^* + \mu_2^*) + \mu_3^*)(x) = (\mu_1^* + \mu_2^*)(x) + \mu_1^*(x) = \mu_1^*(x) + \mu_2^*(x) + \mu_3^*(x) = \mu_1^*(x) + \mu_2^*(x) + \mu_3^*(x) = \mu_1^* + (\mu_2^* + \mu_3^*)x$$

Hence, $(\mu_1^* + \mu_2^*) + \mu_3^* = \mu_1^* + (\mu_2^* + \mu_3^*)$

(iii) For any $X \epsilon V$

$$((\alpha\beta)\mu_1^*)(x) = \alpha\beta(\mu_1^*(x)) = \alpha(\beta\mu_1^*(x)) = (\alpha(\beta\mu_1^*))(x)$$

Hence, $(\alpha\beta)\mu_1^* = \alpha(\beta\mu_1^*)$

(iv) For $\alpha, \beta \epsilon IFB^*A$

$$((\alpha + \beta)\mu_1^*)(x) = (\alpha + \beta)(\mu_1^*(x)) = \alpha(\mu_1^*(x)) + \beta(\mu_1^*(x)) = \alpha\mu_1^*(x) + \beta\mu_2^*(x) = (\alpha\mu_1^* + \beta\mu_1^*)x, \forall X \epsilon V$$

Hence, $(\alpha + \beta)\mu_1^* = \alpha\mu_1^* + \beta\mu_1^*$

(v) For $\alpha \epsilon IFB^*A$

$$\alpha(\mu_1^* + \mu_2^*)(x) = \alpha((\mu_1^* + \mu_2^*)(x)) = \alpha\mu_1^*(x) + \alpha\mu_2^*(x) = (\alpha\mu_1^* + \alpha\mu_2^*)(x), \forall X \epsilon V$$

Hence, $\alpha(\mu_1^* + \mu_2^*) = \alpha\mu_1^* + \alpha\mu_2^*$

(vi) For the linear transformation

$\mu^*(x) = x$, $\mu^*\mu_1^* = \mu_1^*$ for all $\mu_1^* \epsilon L(V)$

(vii) For the linear transformation

$\mu^*(x) = 0$, $\mu^*\mu_1^* = 0$, for all $\mu_1^* \epsilon L(V)$

Example 2.12: Let $\mu_1^*(x) = <0.6, 0.4> (x)$
$\mu_2^*(x) = <0.5, 0.5> (x) \epsilon L(V2)$ and

$$x = (<0.7, 0.2>, <0.6, 0.3>) \epsilon V2$$

Then it can be verified that $\mu_1^* + \mu_2^* = \mu_2^* + \mu_1^*$. For T3 (x) = $x \epsilon L(V2)$

It can be show that (T1+T2) + T3 = T1+ (T2+T3). Let $\alpha = <0.6, 0.2>$ and $\beta = <0.5, 0.3>$ be two element of IFB*A

Then,

It can also be shown that $(\alpha\beta)\mu_1^* = \alpha(\beta\mu_1^*)$,
$(\alpha + \beta)\mu_1^* = \alpha\mu_1^* + \beta\mu_1^*$ and $\alpha(\mu_1^* + \mu_2^*) = \alpha\mu_1^* + \alpha\mu_2^*$

For $\mu^*(x) = x \epsilon L(V2)$ and
$\mu^*(x) = 0 = (<0, 0>, <0, 0>) \epsilon L(V2)$

It can also be verified that, $\mu^* . \mu_1^* = \mu_1^*$ and

$$\mu^* . \mu_1^* = \mu^*$$

By the following example we show that L(Vn) does not form a vector space,

Let $\mu^*(x) = (<0, 0>, <0, 0>......<0, 0>)$ and
$\mu^*(x) = x \epsilon L(Vn)$ for any $x \epsilon$ Vn.

Then,

$S(\mu^* + \mu_1^*)(x) = \mu^*(x) + \mu_1^*(x)$

= (<0, 0>, <0, 0> ...<0, 0>) + (< $x_{1\mu}$, x_{1V}>, < $x_{2\mu}$, x_{2v}>...< $x_{n\mu}$, x_{nv}>)

= (< $x_{1\mu}$, 0>, $x_{2\mu}$, 0>...< $x_{2\mu}$, 0>)

(μ*(x))* = (μ* <0, 0>...<0, 0>)

= μ<0, 0>...<0, 0>

= μ(x)

So L(Vn) does not form a vector space over the intuitionistic fuzzy B* algebra IFB*A under the addition and fuzzy multiplication

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