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EFFECTS OF INTER-MODAL AND INTRA-MODAL INTERACTIONS IN A THREE-MODE ATOM-MOLECULE BOSE-EINSTEIN CONDENSATES: SQUEEZED STATES

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Abstract:

We theoretically study a three mode atom-molecule Bose-Einstein condensates (BECs) with one atomic BEC, one excited molecular BEC and one stable molecular BEC, considering their intra-modal and inter-modal couplings. The atoms in an atomic BEC are transferred to a stable molecular BEC via an intermediate excited molecular BEC using Bose-Stimulated Raman Adiabatic Passage (STIRAP). We solve analytically the Hamiltonian of the system to study the stability and quantum squeezing of the system. Consistency with the experimental results, our result also confirms the stability of molecules in the stable molecular BEC. Interestingly, in our system, squeezing is present in all three modes whereas if we consider only intermodal interactions, squeezing is present only in atomic BEC.

Key Words: BEC, STIRAP, Squeezing, Sen-Mandal Technique& ETCR

1. Introduction:

The BEC system has great practical significances and can be used in BEC based quantum computing devices [1, 2]. In STIRAP [3, 4], two laser pulses are applied in counterintuitive sequences, One couple the atomic BEC with the excited molecular BEC and the other couple exciter molecular BEC with the stable molecular BEC. The motive of STIRAP is the transfer of atoms in the atomic BEC adiabatically to the stable molecular BEC via an excited molecular BEC with nearly unit efficiency. The time evolution of the system just after the population transfer i.e. just after the passage of laser pulses is not yet exclusively studied. Some works have been done considering only the intra-modal interactions [5, 6]. We recently published a paper considering only the intermodal interactions [7]. But no work not yet did considering all possible interactions of the system i.e. considering both, the inter-modal and intra-modal interactions. In the present work, we take the Hamiltonian of the system just after the passage of the laser pulses. Then the system populated mostly at the stable molecular BEC. Due to the presence of interactions in the system, there is a possibility of the particles in stable molecular BEC mode can return back to the stable atomic BEC mode. So, it is essential to study the stability of the system considering both, the inter-modal and intra-modal interactions. We derive the time evolution of occupation numbers in all three states to estimate the stability of the system after population transfer. We also study the squeezing of all three modes which is essential to improve the quality of quantum communication [8], and to detect weak field [9].In section 2 we construct the Hamiltonian of the system. In sec. 3 we derive the analytical solution for the field operators of the model Hamiltonian. In section 4 we study the time evolution of particle occupation numbers. In section5 we investigated the signature of quantum squeezing in all the three modes and finally we concluded in section 6.

2. The Hamiltonian:

The Hamiltonian of a three mode BECs system (one atomic BEC with two molecular BECs) for exact two photon resonance scan be constructed as [5, 10, 11],

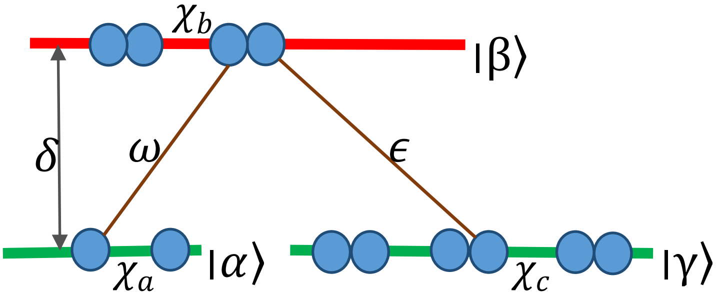


Figure 1: Energy levels and interactions scheme of a three mode atom-molecule BECs.

$$H = \delta b^\dagger b - \frac{\omega}{2}(a^{\dagger 2} b + a^2 b^\dagger) - \frac{\epsilon}{2}(b^\dagger c + b c^\dagger) + \chi_a a^{\dagger 2} a^2 + \chi_b b^{\dagger 2} b^2 + \chi_c c^{\dagger 2} c^2. \quad (1)$$

Where δ is the intermediate detuning, ω is the inter-modal coupling constant between atomic BEC and excited ⋯ molecular BEC mode respectively. χ_a, χ_b and χ_c are the constants of intra-modal interactions of atomic BEC, excited molecular BEC and stable molecular BEC respectively. In the Hamiltonian and also in all following calculations we consider ħ=1. The schematic representation of the system is shown in Fig. 1. Eigen states for atomic BEC, excited molecular BEC and stable molecular BEC are $|\alpha\rangle$, $|\beta\rangle$ and $|\gamma\rangle$ respectively.

3. Solution:

The Heisenberg equation of motion for the time evolution of the field operators' are-

$$\dot{a}(t) = i\omega a^\dagger(t)b(t) - 2i\chi_a a^\dagger(t)a^2(t),$$
$$\dot{b}(t) = -i\delta b(t) + i\frac{\omega}{2}a^2(t) + i\frac{\epsilon}{2}c(t) - 2i\chi_b b^\dagger(t)b^2(t), \quad (2)$$
$$\dot{c}(t) = i\frac{\epsilon}{2}b(t) - 2i\chi_c c^\dagger(t)c^2(t).$$

The above differential equations of annihilation operators contain the coupled nonlinear terms. The exact analytical solution of these operator equations is until impossible. To solve these, we apply Sen-Mandal technique [12] which gives best possible analytic solutions [11]. In these solutions we consider up to 2nd order of interaction constants. Our solutions for the field operators are,

$$a(t) = f_1 a(0) + f_2 a^\dagger(0)b(0) + f_3 a^\dagger(0)a^2(0) + f_4 a^\dagger(0)b(0) + f_5 a^\dagger(0)c(0) + f_6 a^\dagger(0)a^2(0) + f_7 a(0)b^\dagger(0)b(0) + f_8 a^{\dagger 2}(0)a(0)b(0) + f_9 a^\dagger(0)b^\dagger(0)b^2(0) + f_{10} a^3(0)b^\dagger(0) + f_{11} a^{\dagger 2}(0)a^3(0),$$
$$b(t) = g_1 b(0) + g_2 a^2(0) + g_3 c(0) + g_4 b^\dagger(0)b^2(0) + g_5 b(0) + g_6 a^2(0) + g_7 a^\dagger(0)a(0)b(0) + g_8 a^\dagger(0)a^3(0) + g_9 c^\dagger(0)c^2(0) + g_{10} a^{\dagger 2}(0)b^2(0) + g_{11} a^2(0)b^\dagger(0)b(0) + g_{12} b^\dagger(0)b(0)c(0) + g_{13} b^2(0)c^\dagger(0) + g_{14} b^\dagger(0)b^2(0) + g_{15} b^{\dagger 2}(0)b^3(0),$$
$$c(t) = h_1 c(0) + h_2 b(0) + h_3 c^\dagger(0)c^2(0) + h_4 c(0) + h_5 a^2(0) + h_6 b^\dagger(0)b^2(0) + h_7 c^\dagger(0)c^2(0) + h_8 b^\dagger(0)c^2(0) + h_9 b(0)c^\dagger(0)c(0) + h_{10} c^{\dagger 2}(0)c^3(0). \quad (3)$$

Where the time dependent parameters are,

$$f_1 = h_1 = 1,$$
$$f_2 = {g_2}/{2} = \frac{\omega}{\delta}G(t), \quad G(t) = \left(1 - e^{-i\delta t}\right),$$
$$f_3 = -2i\chi_a t,$$
$$f_4 = 2g_6 = g_8 = -\frac{2\chi_a\omega}{\delta^2}[i\delta t - G(t)],$$
$$f_5 = 2h_5 = \frac{\omega\varepsilon}{2\delta^2}[i\delta t - G(t)],$$
$$f_6 = \frac{\omega^2}{2\delta^2}[i\delta t - G(t)] - 2{\chi_a}^2 t^2,$$
$$f_7 = \frac{\omega^2}{\delta^2}[-i\delta t + G(t)],$$
$$f_8 = \frac{2\chi_a\omega}{\delta^2}[3G(t) + i\delta t\{G(t) - 3\}],$$
$$f_9 = g_{11} = \frac{2\chi_b\omega}{\delta^2}\left[i\delta t e^{-i\delta t} - G(t)\right],$$
$$f_{10} = -\frac{2\chi_a\omega}{\delta^2}[i\delta t + G^*(t)],$$
$$f_{11} = -2{\chi_a}^2 t^2,$$
$$g_1 = e^{-i\delta t},$$
$$g_3 = h_2 = \frac{\epsilon}{2\delta}G(t),$$
$$g_4 = -2i\chi_b t e^{-i\delta t},$$
$$g_5 = \frac{(2\omega^2 + \epsilon^2)}{4\delta^2}\left[-i\delta t e^{-i\delta t} + G(t)\right],$$
$$g_7 = \frac{\omega^2}{\delta^2}\left[-i\delta t e^{-i\delta t} + G(t)\right],$$
$$g_9 = \frac{\varepsilon\chi_c}{\delta^2}[-i\delta t + G(t)],$$
$$g_{10} = \frac{\chi_b\omega}{\delta^2}e^{-i\delta t}[i\delta t - G(t)],$$
$$g_{12} = 2h_6 = \frac{2\varepsilon\chi_b}{\delta^2}\left[i\delta t e^{-i\delta t} - G(t)\right],$$
$$g_{13} = \frac{\varepsilon\chi_b}{\delta^2}e^{-i\delta t}[i\delta t - G(t)],$$
$$g_{14} = g_{15} = -2{\chi_b}^2 t^2 e^{-i\delta t},$$

$$h_3 = -2i\chi_c t,$$
$$h_4 = \frac{\epsilon^2}{4\delta^2}[i\delta t - G(t)],$$
$$h_7 = h_{10} = -2{\chi_c}^2 t^2,$$
$$h_8 = -\frac{\chi_c \epsilon}{\delta^2}[i\delta t + G^*(t)],$$
$$h_9 = \frac{2\chi_c\epsilon}{\delta^2}[-i\delta t + G(t)]. \quad (4)$$

Our solutions are not restricted to short time and valid for any time value with the only restriction is that we consider the interactions constant up to the 2nd order. These solutions satisfy the required equal time commutation relations (ETCR) and total particle number conservation law. i.e.

$$[a(t), a^\dagger(t)] = 1, \qquad [b(t), b^\dagger(t)] = 1, \qquad [c(t), c^\dagger(t)] = 1,$$
$$and\, a^\dagger(t)a(t) + 2b^\dagger(t)b(t) + 2c^\dagger(t)c(t) = Constant.$$

4. Particle Occupation Number:

We already derived the time evolution of the annihilation operators. To find out the temporal evolution of the physical properties of the system we consider initially$(t = 0)$all three states are coherent. So, we can represent the composite system initially as,

$$|\psi(0)\rangle = |\alpha\rangle \otimes |\beta\rangle \otimes |\gamma\rangle.$$

The operation of the operators$a(0)$, $b(0)$ and $c(0)$on the composite state will give the corresponding Eigen values α, β and γrespectively. i.e.

$$a(0)|\psi(0)\rangle = \alpha|\alpha\rangle \otimes |\beta\rangle \otimes |\gamma\rangle,$$
$$b(0)|\psi(0)\rangle = \beta|\alpha\rangle \otimes |\beta\rangle \otimes |\gamma\rangle,$$
$$c(0)|\psi(0)\rangle = \gamma|\alpha\rangle \otimes |\beta\rangle \otimes |\gamma\rangle. \quad (5)$$

The particle number at any instant in three states are,

$$N_a(t) = \langle\psi(0)|a^\dagger(t)a(t)|\psi(0)\rangle$$
$$= |\alpha|^2 + |f_2|^2\left(|\beta|^2 + 2|\alpha|^2|\beta|^2 - \frac{1}{2}|\alpha|^4\right)$$
$$+ [(f_2 + f_4 + f_2 {f_3}^*)\alpha^{*2}\beta + f_5\alpha^{*2}\gamma + 2(f_8 + 2{f_{10}}^*)|\alpha|^2\alpha^{*2}\beta + f_9|\beta|^2\alpha^{*2}\beta + \text{c.c.}],$$
$$N_b(t) = \langle\psi(0)|b^\dagger(t)b(t)|\psi(0)\rangle$$
$$= |\beta|^2 + |g_2|^2(|\alpha|^4 - 2|\beta|^2 - 4|\alpha|^2|\beta|^2) + |g_3|^2(|\gamma|^2 - |\beta|^2)$$
$$+ [(g_1 g_2^* + g_1 g_6^*)\alpha^{*2}\beta + g_1 g_3^*\beta\gamma^* + g_2 g_3^*\alpha^2\gamma^* + g_1 g_8^*|\alpha|^2\alpha^{*2}\beta + g_1 g_9^*|\gamma|^2\beta\gamma^*$$
$$+ (g_2 g_4^* - g_1 g_{10}^*)|\beta|^2\alpha^2\beta^* + (g_3 g_4^* - g_1 g_{13}^*)|\beta|^2\beta\gamma + \text{c.c}]$$
$$N_c(t) = \langle\psi(0)|c^\dagger(t)c(t)|\psi(0)\rangle = |\gamma|^2 + |h_2|^2(|\beta|^2 - |\gamma|^2) + [h_2\beta\gamma^* + h_5\alpha^2\gamma^* + h_6|\beta|^2\beta\gamma^* + ({h_2}^* h_3 - h_8)|\gamma|^2\beta^*\gamma + \text{c.c.}]. \quad (6)$$

Just after STIRAP, most of the particles transfer to the stable molecular BEC. So, we take $\alpha = 2, \beta = 2, and \gamma = 10$,the intermediate detuning$\delta = 10\, MHz$, $\omega = \epsilon = 0.1 KHz$and $\chi_a = \chi_b = \chi_c = \chi = 1\, KHz$, $and \tau = \chi t$[6]. We plot the change in particle number from their initial values in Fig. 2 which shows that particle numbers are almost remain same with time. There is a negligible loss in particle number in the stable molecular BEC mode due to presence of inter-modal and intra-modal interactions. So, our result theoretically confirm the stability of the system.

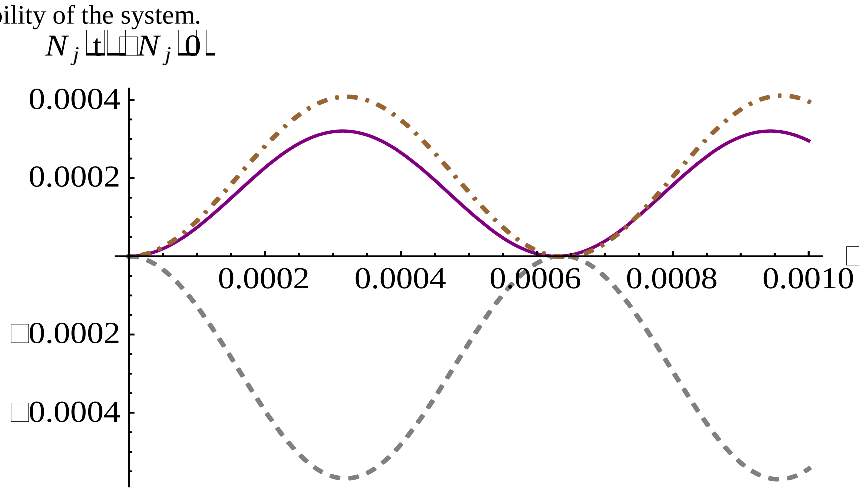


Figure 2: Change in particle number from their initial value. The solid line for atomic BEC mode, the dashed line for excited molecular BEC mode and the dot-dashed line for stable molecular BEC mode

5. Squeezing:

If $j(t)$ is the bosonic annihilation operator of any mode, then the corresponding quadrature operators are defined as,

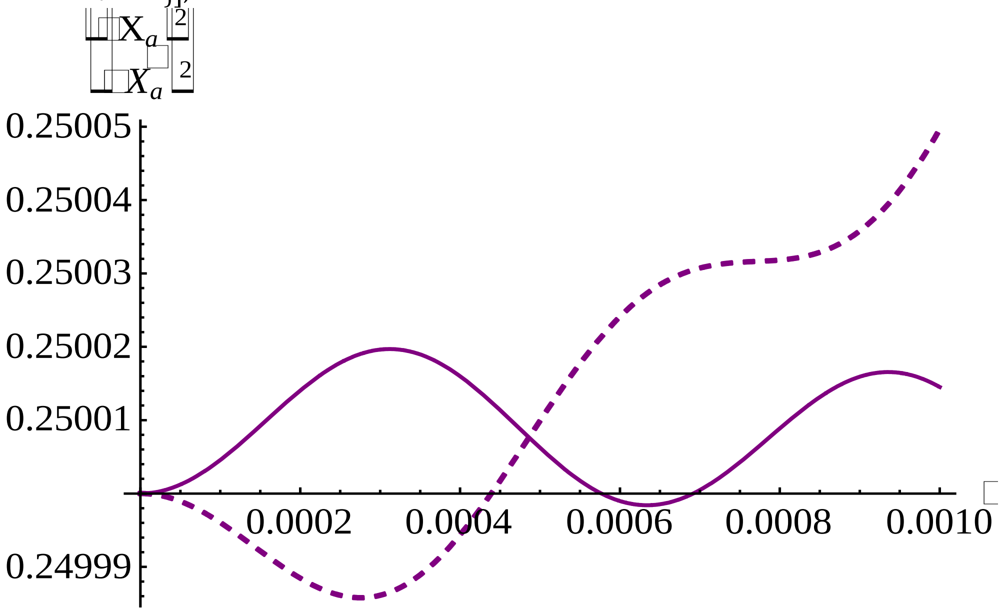
$$X_j = \frac{1}{2}[j(t) + j^{\dagger}(t)],$$
$$\dot{X}_j = \frac{1}{2i}[j(t) - j^{\dagger}(t)].$$

Squeezing is present if one quadrature is shrunk at the expense of the complimentary quadrature from their coherent value. I.e. for squeezing the variances,

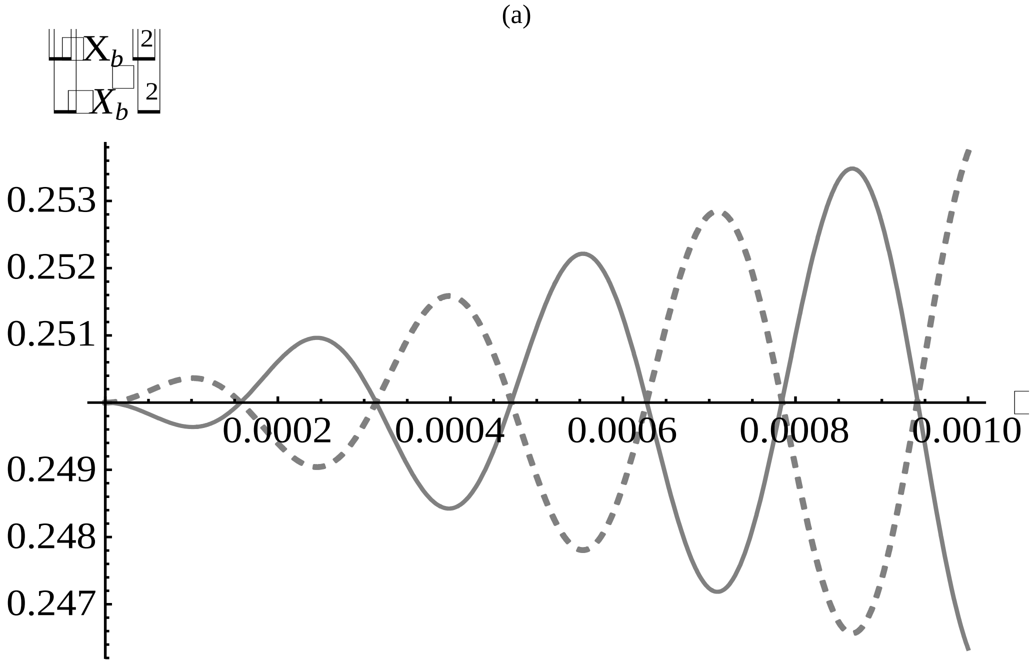
$$\left(\Delta X_j\right)^2 = \langle X_j{}^2\rangle - \langle X_j\rangle^2 < \tfrac{1}{4}, \text{ or} \left(\Delta \dot{X}_j\right)^2 = \langle \dot{X}_j{}^2\rangle - \langle \dot{X}_j\rangle^2 < \tfrac{1}{4}.$$

For our system,

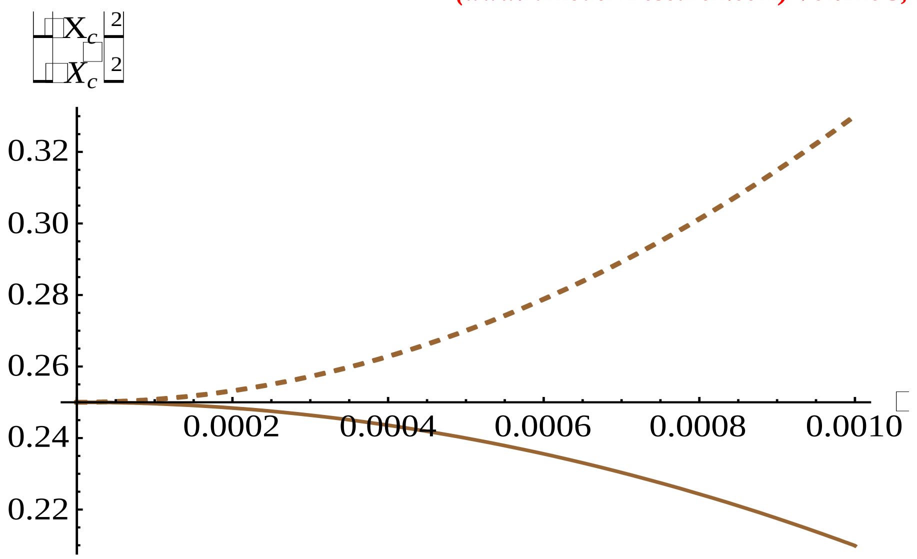
$$\begin{pmatrix}(\Delta X_a)^2\\ (\Delta \dot{X}_a)^2\end{pmatrix} = \frac{1}{4}\big[1 + 2|f_2|^2|\beta|^2 + 2|f_3|^2|\alpha|^4$$
$$+ \{2f_2{}^* f_3\alpha^2\beta^*$$
$$\pm \left(f_2\beta + f_3\alpha^2 + 2f_2f_3|\alpha|^2\beta + f_3{}^2|\alpha|^2\alpha^2 + f_4\beta + f_5c + f_6\alpha^2 + 2f_8|\alpha|^2\beta + f_9|\beta|^2\beta\right)$$
$$+ c.c.\}\big],$$



(a)



(b)



(c)

Figure 3: Variances of quadrature operators in (a) atomic BEC mode, (b) excited molecular BEC mode and (c) stable molecular BEC mode.

$$\begin{pmatrix}(\Delta X_b)^2\\ (\Delta \dot{X}_b)^2\end{pmatrix} = \frac{1}{4}[1+2|g_4|^2|\beta|^4 + \{2g_1g_{10}^*\alpha^2\beta^* + g_1^*g_{11}\alpha^2\beta^* + g_1g_{12}^*\beta\gamma^* + 2g_1^*g_{13}\beta\gamma^* \pm (g_1g_4\beta^2 + g_1g_{11}\alpha^2\beta + g_1g_{12}\beta\gamma + g_1g_{14}\beta^2 + 6g_1g_{15}|\beta|^2\beta^2) + c.c.\}],$$

$$\begin{pmatrix}(\Delta X_c)^2\\ (\Delta \dot{X}_c)^2\end{pmatrix} = \frac{1}{4}[1+2|h_3|^2|\gamma|^4 \pm \{(h_3+h_7)\gamma^2 + h_9\beta\gamma + 6h_{10}|\gamma|^2\gamma^2 + c.c.\}] \quad (7)$$

We plot the above equations with rescaled time in Fig. 3 taking the same value of interaction constants and intermodal detuning as before. Plots show the signature of squeezing in all three modes. It is interesting that if only the inter-modal interaction is consider, squeezing is present only in atomic BEC mode [13], but for our system where the inter-modal as well as intra-modal coupling is consider, squeezing is present in all three modes.

6. Conclusions:

We have study a three mode atom-molecule BEC system where the particles initially in atomic BEC mode transfer efficiently to a stable molecular mode by STIRAP. After the population transfer the laser pulses required for STIRAP go away. So the only interactions are inter-modal and intra-modal interactions. We have taken Hamiltonian of this system and solve it analytically up to the 2nd order of interaction constants.Our result show that the particles numbers in all three modes oscillates with time but the change in particle number is only up to 0.0004% in stable molecular BEC mode. So, the inter-modal and intra-modal interactions do not disturb the stability of the system. Also these interactions give rise squeezing in all three modes, whereas only inter-modal interactions generates squeezing only in atomic BEC mode. So, this system can be used as a BEC based high precession measurement system.

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