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AN ALGORITHM TO SOLVE MULTI OBJECTIVE ASSIGNMENT PROBLEM USING FUZZY PROGRAMMING TECHNIQUE

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Abstract:

In this paper, a new method using a fuzzy programming technique with fuzzy linear membership function is proposed for solving Multi-Objective Assignment problem (MOAP). This method gives an optimum solution is more or less same solution when compared to currently used method**.** To check its feasibility numerical examples are illustrated.

Key Words: Multi-Objective Assignment Problem, Parallel Method & Fuzzy Linear Membership Function

1. Introduction:

A general assignment problem includes N jobs that must assign N workers where each worker has the skill to do all tasks. Due to some personal reason and inability each worker completes the task at different time intervals. The aim is to finish the task at minimum time and using minimum resources to perform the task by allocating it properly to workers. Many researchers have been developed solutions to solve the assignment problem [1-4]. Most of the existing methods for the assignment problem, consider only one-objective situation, such as (a) the minimum cost assignment problem, (b) the minimum finishing time assignment problem. The minimum cost assignment problem focuses on how to assign tasks to workers so that the total operation cost to be minimized. Such problems have been generally discussed and well developed through many operations research.

Using Multiple-Objective method there are many researches also focused to solve the above problems and try to provide the best solution. In the Multi-Objective assignment problem, the objectives alone are considered as fuzzy, Geetha et al. [1993] gave a solution for an assignment problem that minimizes both time and cost. Tsai et al. [1999] studied a multi-objective decision making problem associated with cost, time, and quality by fuzzy concept. Unfortunately, the existing techniques provide only on the 2-objective assignment problem. [Bit et al. [1992] developed an algoritm for solving multi-objective transportation problem using fuzzy programming technique with linear membership function. Bowen et al. [2002] have established a relationship between cost, time, and quality management, and the attainment of client objectives. Afshar et al. [2007] A Multi objective Ant Colony optimization is obtained to analyze the advanced time cost-quality trade-off problem. P.K. De and Bharti Yadav [2011] introduced an algorithm to solve multi-objective assignment problem (MOAP) through interactive fuzzy goal programming approach. Yeola M. C. and Jahav V.A [2016] introduced a new a method for solving Multi-Objective transportation problem using a fuzzy programming technique with linear membership function.Ventepaka et al. [2016] introduced an algorithm for solving Multi objective Assignment problem using Hungarian Algorithm an optimal solution of each objective function by minimizing the resource. This paper is organized as follows: section 2, gives Mathematical model of Multi Objective Assignment problem is discussed. A new method is proposed by an algorithm, to find the solution of MOAP is given in section 4. In section 5, a numerical example is provided to illustrate its feasibility. The conclusion is included in section 6.

2. Mathematical Model of Multi-Objective Assignment Problem:

The linear membership function is used and defined by penalties (cost, delivery time, quality, etc.,) as membership value through defined membership function. As membership value is higher it is closer to the optimal solution. The decision maker would like to minimize the set of P objectives and is given by

$$\mu_k(X^k_{ij}) = \begin{cases} 1 & X^k_{ij} \le L_k \\ \dfrac{U_k - X^k{}_{ij}}{U_k - L_k} & L_k \le X^k_{ij} \le U_k \\ 0 & X^k_{ij} \ge U_k \end{cases} \quad \text{..................(1)}$$

Where $L_k \neq U_k$, k=1, 2.... P. If $L_k = U_k$ then membership value is 1 for any value of k.

Assume that there are *n* jobs and *n* persons. *n* jobs must be performed by *n* persons, where the costs depend on the specific assignments. Each job must be assigned to one and only one person and each person has to perform one and only one job. Let C_{ij} be the cost if the i^{th} person is assigned the j^{th} job, the problem is to find an assignment (which job should be assigned to which person) so that the total cost of performing all jobs is minimized.

Here make an assumption that j^{th} job will be completed by i^{th} person, and let

$$X_{ij} = \begin{cases} 1 & if\ ith\ person\ is\ assigned\ jth\ job \\ 0 & if\ ith\ person\ is\ not\ assigned\ jth\ job \end{cases}$$

Where x_{ij} denotes that j^{th} job is to be assigned to the i^{th} person.

Then, the mathematical model of multi-objective assignment problem is:

$$\text{Min Z} = \sum_{i=1}^{n} \sum_{j=1}^{n} X_{ij}\, C_{ij}$$

Subject to

$\sum_{i=1}^{n} X_{ij}$ =1, j=1, 2....n (only one person should be assigned the j^{th} job)

$\sum_{j=1}^{n} X_{ij}$ =1, i=1, 2....n (only one job is done by the i^{th} person)

Where $Z_k(x)= \{Z_1(x), Z_2(x), Z_3(x), \ldots ,Z_K(x)\}$ is a vector of K objective functions, the subscript on $Z_k(x)$ and superscript on C^k_{ij} are used to identify the number of objective functions(k = 1,2,..K)

4. Proposed Methodology:

In this section, a new algorithm is proposed to solve MOAP using a fuzzy programming technique with linear membership function as discussed by Yeola et al [14].

The proposed method is summarized in the following steps:

Step 1: Consider "m" jobs on "n" machine costs given as a matrix which is a balanced assignment problem, where m=n.

Step 2: Calculate the linear membership value using the equation (1) for each cell and for each objective table.

Step 3: Construct a new table in which each cell having minimum membership value, of all objective tables.

Step 4: For each row identify the largest and smallest membership value. Determine the difference between maximum and minimum membership value for each row. Display them along the side of the table against the respective rows. Similarly, compute the differences for each column and write them below the table against the respective columns.

Step 5: Identify the largest membership value among all the rows and columns. If a tie occurs go to the next step otherwise. Let the greatest difference in the i^{th} row, then find the largest membership value in that row and let be C_{ij}. Allocate the cell and cross out the i^{th} row and j^{th} column. Repeat from step3.

Step 6: (i) If a tie occurs, in the largest difference, then in that row and column find the maximum a membership value. But, Allocate to the next maximum membership value of the cell and cross out that corresponding row and column. Next, go to step 4.

(ii) If a tie occurs in the maximum membership value of the cell, then choose arbitrarily and allocate to it.

Step 7: Repeat the procedure until an optimum solution is attained, then go to Step 8.

Step 8: List out the assignment cost and combination corresponding to sub-problem.

Step 9: Add assignment cost of each sub-problem to obtain the total assignment cost of the main problem which shall be the optimal cost, and also rearrange the combinations.

5. Illustrative Example:

Numerical Illustration 1:

Let us consider the following example [10] to illustrate the proposed method. Assigned cost matrix (ACM). Here cost unit: thousands, time unit: weeks, quality levels: 1, 3, 5, 7, 9.

Jobs/Machine	M_1	M_2	M_3	M_4
J_1	9	7	4	6
J_2	12	5	5	8
J_3	9	9	9	11
J_4	2	7	11	8

Data for Cost (Table 1)

Jobs/Machine	M_1	M_2	M_3	M_4
J_1	2	1	8	2
J_2	9	9	1	8
J_3	8	9	5	6
J_4	1	5	4	9

Data for time (Table 2)

Jobs/Machine	M_1	M_2	M_3	M_4
J_1	1	1	1	5
J_2	7	5	5	9
J_3	1	7	5	7
J_4	1	3	5	3

Data for quality (Table 3)

As the first we calculate membership value for cost, here U_k=12 and L_k=2 then membership value are as follows:

Jobs/Machine	M_1	M_2	M_3	M_4
J_1	0.3	0.5	0.8	0.6
J_2	0	0.7	0.7	0.4
J_3	0.3	0.3	0.3	0.1
J_4	1	0.5	0.1	0.4

Table 4

Next, Membership value of time, here U_k =9 and L_k =1then membership values are as follows:

Jobs/Machine	M_1	M_2	M_3	M_4
J_1	0.88	1	0.13	0.88
J_2	0	0	1	0.13
J_3	0.13	0	0.5	0.38
J_4	1	0.5	0.63	0

Table 5

Membership value for quality, here U_k= 9 and L_k=1then membership values are as follows

Jobs/Machine	M_1	M_2	M_3	M_4
J_1	1	1	1	0.5
J_2	0.25	0.5	0.5	0
J_3	1	0.25	0.5	0.25
J_4	1	0.75	0.5	0.75

Table 6

Next we calculate minimum membership value

Jobs/Machine	M_1	M_2	M_3	M_4
J_1	0.3	0.5	0.13	0.5
J_2	0	0	0.5	0
J_3	0.13	0	0.3	0.1
J_4	1	0.5	0.1	0

Table 7

By step 5, the largest membership value is 1which is in the fourth row. In that corresponding row choose a maximum membership value which is in the cell (J_4, M_1) and allocate to it. Cross out the fourth row and first column.

Repeating the steps 3, 4 and 5 the Assignment schedule is (J_2, M_3), (J_1, M_2), (J_3, M_4).

The Total operation time is 9, total operation cost is 25, and total quality is 14.

Comparison:

Objective Function	Ventepaka Yadaiah and Haragopal V.V[10]	Proposed Method
Cost	25	25
Time	9	9
Quality	14	14

Numerical Illustration 2:

Let us consider the following example [6, 8] to illustrate the proposed method.

Doers/Tasks	1	2	3	4	5	6
1	6	3	5	8	10	6
2	6	4	6	5	9	8
3	11	7	4	8	3	2
4	9	10	8	6	10	4
5	4	6	7	9	8	7
6	3	5	11	10	12	8

Data for Cost (Table 1)

Doers/Tasks	1	2	3	4	5	6
1	4	20	9	3	8	9
2	6	18	8	7	17	8
3	2	8	20	7	15	7
4	12	13	14	6	9	10
5	9	8	7	14	5	9
6	3	9	7	5	3	3

Data for time (Table 2)

Doers/Tasks	1	2	3	4	5	6
1	1	3	1	1	1	5
2	3	5	3	5	7	5
3	1	7	5	3	5	7
4	5	9	3	5	7	3
5	3	9	7	5	3	3
6	3	3	5	7	5	7

Data for quality (Table 3)

As the first we calculate membership value for cost, here U_k=12 and L_k=2 then membership value are as follows

Doers/Tasks	1	2	3	4	5	6
1	0.6	0.9	0.6	0.4	0.2	0.6
2	0.6	0.8	0.6	0.7	0.3	0.4
3	0.1	0.5	0.8	0.4	0.9	0.10
4	0.3	0.2	0.4	0.6	0.2	0.8
5	0.8	0.6	0.5	0.3	0.4	0.5
6	0.9	0.7	0.1	0.2	0	0.4

Table 4

Next, Membership value for time, here U_k =20 and L_k =2 then membership value are as follows:

Doers/Tasks	1	2	3	4	5	6
1	0.89	0	0.61	0.94	0.67	0.61
2	0.78	0.11	0.67	0.72	0.17	0.67
3	1	0.67	0	0.72	0.28	0.72
4	0.44	0.39	0.33	0.78	0.61	0.56
5	0.61	0.67	0.72	0.33	0.83	0.61
6	0.94	0.61	0.72	0.83	0.94	0.94

Table 5

Membership value for quality, here U_k= 9 and L_k=1then membership value are as follows

Doers/Tasks	1	2	3	4	5	6
1	1	0.75	1	1	1	0.5
2	0.75	0.5	0.75	0.5	0.25	0.5
3	1	0.25	0.5	0.75	0.5	0.25
4	0.5	0	0.75	0.38	0.25	0.75
5	0.75	0	0.25	0.5	0.75	0.75
6	0.75	0.75	0.38	0.25	0.5	0.25

Table 6

Next we calculate minimum membership value

Doers/Tasks	1	2	3	4	5	6
1	0.6	0	0.6	0.4	0.2	0.5
2	0.6	0.11	0.6	0.5	0.17	0.4
3	0.1	0.25	0	0.4	0.28	0.10
4	0.3	0	0.33	0.38	0.2	0.56
5	0.61	0	0.25	0.3	0.4	0.5
6	0.75	0.61	0.1	0.2	0	0.25

Table 7

By step 3, the largest membership value is 0.75 which is in the sixth row. In that corresponding row choose a maximum membership value which is in the cell (6, 1) and allocate to it. Cross out the sixth row and first column.

Repeating the steps 3, 4 and 5 the Assignment schedule is (1, 3), (2, 4), (3, 2), (4,6), (5, 5), (6, 1).

The Total operation time is 42, total operation cost is 32, and total quality is 22.

Comparison:

Objective Function	Chiao-Pin Bao et al[8]	Tsai et al[6]	Proposed Method
Cost	41	37	32
Time	42	47	42
Quality	14	16	22

6. Conclusion:

In this paper an algorithm is developed for a MOAP and this approach is used to find the compromise solution of Multi-objective Assignment problem obtained using a fuzzy programming technique with linear membership functions. Obtained results were compared with some of the methods in literature which is illustrated with numerical examples. The Comparison shows that the compromise solution is better. It gives the same optimal solution and also gives more or less cost for both objectives of cost, time and quality.

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