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A STUDY ON IRREDUNDANCE AND INSENSITIVE ARC IN FUZZY GRAPHS

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Abstract:

The aim of this dissertation is to study about the domination, irredundance and insensitive arc in fuzzy graphs and applications of dominations in fuzzy graph.

Introduction:

Elaborated definition of fuzzy graph including fuzzy vertex and fuzzy edges and several fuzzy analogs of graph theoretic concepts such as paths, cycles, connectedness etc was discussed in [1]. In this chapter, we discuss about the basic concepts of fuzzy graph theory such as path, cycle, fuzzy bridge, fuzzy cut node and strong arc. The concept of domination in fuzzy graph by using strong arcs was discussed in [1]. In this chapter, we discuss about dominating sets, minimal dominating sets, domination number, independent sets, maximal independent sets, independence number and relations between dominating sets and independent sets.

Domination in Fuzzy Graphs:

Let G be a fuzzy graph. Let u and v be two nodes of G. We say that v is **dominated** by u if (u, v) is a strong arc.

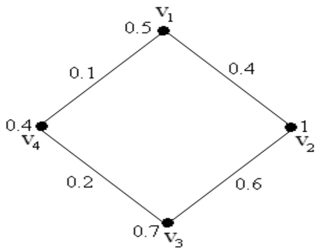

Figure 1

In the above fuzzy graph, v_1 is dominated by v_2, v_2 is dominated by both v_1 and v_3, v_3 is dominated by both v_2 and v_4 and v_4 is dominated by v_3. A subset D of V is called a **dominating set** of G if for every $v \in V-D$, there exists $u \in D$ such that v is dominated by u.

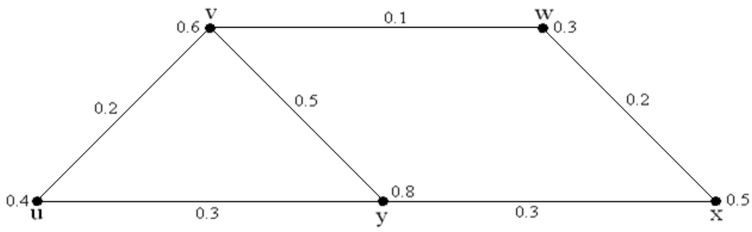

Figure 2

In the above fuzzy graph, **dominating sets** are {u, v, w, x, y}, {u, v, w, x}, {u, v, w, y}, {u, v, x, y}, {u, w, x, y}, {v, w, x, y}, {u, v, w}, {u, v, x}, {u, w, y}, {v, w, y}, {v, x, y}, {w, x, y} and {x, y}. A dominating set D is called a **minimal dominating set** if no proper subset of D is dominating set. In fuzzy graph G of Figure 2, the sets {u, v, w}, {u, v, x}, {u, w, y}, {v, w, y} and {x, y} are minimal dominating sets. A **domination number** $\gamma(G)$ is minimum fuzzy cardinality among all minimal dominating sets of G. For the fuzzy graph G of Figure 2, domination number $\gamma(G) = 1.3$ A dominating set D of fuzzy graph G such that |D| = γ(G) is called **minimum dominating set.** For fuzzy graph G of Figure 2, minimum dominating sets are {u, v, w} and {x, y}.

Theorem:

A dominating set D of G is a minimal dominating set if and only if for each u ∈ D, one of the following two conditions holds

- ✓ u is not a strong neighbor of any node in D
- ✓ There is a node v ∉ D such that $N_S(v) \cap D = \{u\}$.

Proof:

Suppose D is a minimal dominating set of G. Then for each u ∈ D, the set D −{u} is not a dominating set. Thus, there is a node v ∈ V−(D−{u}) which is not dominated by any node in D – {u}. Now either v = u or v ∈ V−D. If v = u, then u is not a strong neighbor of any node in D. If v ∈ V−D and v is not dominated by D−{u}, but it is dominated by D, then u is the only strong neighbor of v. i.e., $N_S(v) \cap D = \{u\}$. Conversely, suppose D is a dominating set and each node u ∈ D, one of the two stated conditions (a), (b) holds. Now we prove that D is a minimal dominating set. Suppose D is not a minimal dominating set. Then there exists a node u ∈ D such that D – {u} is a dominating set. Thus u is a strong neighbor to at least one node in D – {u}. ∴ Condition (a) does not hold. Also if D – {u} is a dominating set , then every node in V – D is a strong neighbor to at least one node in D – {u} . ∴ Condition (b) does not hold. Hence neither condition (a) nor (b) holds, which is a contradiction. ∴ D is minimal dominating set.

Theorem:

Let G be a fuzzy graph without isolated nodes. If D is a minimal dominating set, then V−D is a dominating set.

Proof:

Let D be a minimal dominating set of G. Suppose V−D is not a dominating set. Then there exists a node u ∈ D such that u is not a dominated by any node in V−D. Since G has no isolated nodes, u is strong neighbor of at least one node in D – {u}. Then D – {u} is a dominating set, which is a contradiction that D is a minimal dominating set. Thus every node in D is a strong neighbor of at least one node in V−D. Hence V−D is a dominating set.

Theorem:

For any fuzzy graph G, $\gamma(G) \leq n - \Delta_S(G)$.

Proof:

Let u be a node such that $|N_S(u)| = \Delta_S(G)$. Then $V - N_S(u)$ is a dominating set.

∴ $\gamma(G) \leq |V - N_S(u)| = n - \Delta_S(G)$.

∴ $\gamma(G) \leq n - \Delta_S(G)$.

Independent Sets in Fuzzy Graph:

Two nodes of fuzzy graph are said to be **fuzzy independent** if there is no strong arc between them.

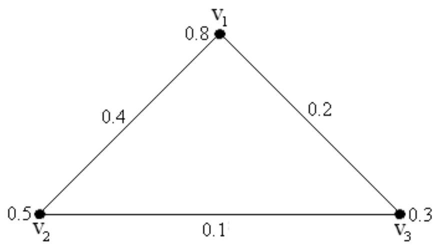

Figure 3

In the above fuzzy graph, v_2 and v_3 are fuzzy independent. A subset S of V is said to be **fuzzy independent set** of G if any two nodes of S are fuzzy independent.

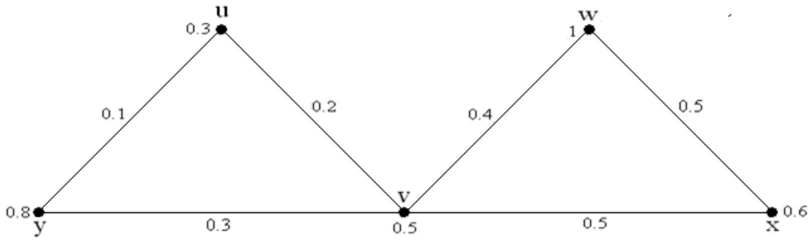

Figure 4 (G)

In the above fuzzy graph G of Fig 2.4, fuzzy independent sets are {u, w, y}, {u, x, y}, {u, w}, {u, x}, {u, y}, {v, w}, {w, y}, {x, y}, {u}, {w}, {v}, {x} and {y}. A fuzzy independent set S is a **maximal fuzzy independent set** if any node set, properly containing S, is not a fuzzy independent set. In the fuzzy graph G of Fig 2.4, maximal fuzzy independent sets are {u, w, y}, {u, x, y} and {v, w}. The **fuzzy independence number** β(G) is the maximum fuzzy cardinality among all maximal fuzzy independent sets of G. For the fuzzy graph G of Fig 2.4**,** fuzzy independence number β(G) = 2.1

Relationship Between Dominating Sets and Independent Sets in Fuzzy Graph:

Theorem:

A fuzzy independent set is a maximal fuzzy independent set if and only if it is fuzzy independent and dominating set.

Proof:

Suppose a fuzzy independent set S is a maximal fuzzy independent set, then for every node u ∈ V−S,the set S ∪{u} is not fuzzy independent set. That is, for every node u ∈ V−S, there is a node v ∈ S such that (u, v) is strong. Thus S is a dominating set. Hence S is both fuzzy independent and dominating set. Conversely, suppose a set S is both fuzzy independent and dominating set. We show that S is a maximal fuzzy independent set. Suppose that S is not a maximal fuzzy independent set. Then there exists a node u ∈ V−S such that S ∪{u} is a fuzzy independent set. But, if S∪{u} is a fuzzy independent set then no node in S is a strong neighbor of u. Thus S is not a dominating set, which is a contradiction. Hence S is maximal fuzzy independent set.

Theorem:

Every maximal fuzzy independent set in a fuzzy graph G is a minimal dominating set in G.

Proof:

Let S be a maximal fuzzy independent set in a fuzzy graph G. By the previous theorem, S is a dominating set. We now show that S is a minimal dominating set. Suppose S is not a minimal dominating set, then there exists a node u ∈ S such that S − {u} is a dominating set. Then there exists at least one node in S − {u} that is a strong neighbor of u. This contradicts that S is a fuzzy independent set of G. Hence S is a minimal dominating set.

Theorem:

For any fuzzy graph G, γ(G) ≤ β(G).

Proof:

Let S be a fuzzy independent set of G such that |S|=β(G). Then G contains no large fuzzy independent set. This means that every node of V−S has at least one strong neighbor in S.

⇒ S is dominating set. ⇒ γ(G) ≤ |S| = β(G) ∴ γ(G) ≤ β(G).

Irredundance in Fuzzy Graphs:

The concept of domination, independent domination and irredundance in fuzzy graph was discussed in [3]. Relations between the parameters of independent domination and irredundance in fuzzy graph were discussed in [4]. In this chapter we discuss about irredundance in fuzzy graphs and some important theorems on irredundance in fuzzy graphs. Let G be a fuzzy graph and S be a set of nodes. A node v is said to be **fuzzy private neighbor** of u ∈ S with respect to S, if N[v] ∩ S = {u}. We define fuzzy private neighborhood of u ∈ S with respect to S, to be P_N[u, S] = {v: N[v] ∩ S = {u}}. In other words, P_N[u, S] = N[u] – N[S-{u}]. Notice that, if u ∈ P_N[u, S] then u is an isolated node in < S > .

S is a minimal fuzzy dominating set if and only if for every node v ∈ S there exists a node w ∈ V – (S – {v}) which is not dominated by S – {v}. Which is equivalent to S is a minimal dominating set if and only if P_N [u, S] ≠ ϕ for some node u ∈ S, that is every node in u ∈ S has at least one private neighbor.

A node u in S ⊆ V is said to be **fuzzy redundant node** if P_N[u, S] = ϕ Equivalently u is redundant in S if N[u] ⊆ N[S – {u}]. Otherwise u is said to be **fuzzy irredundant node**. A set S ⊆ V is said to be **fuzzy irredundant set** in a fuzzy graph G if P_N [u, S] ≠ ϕ for every node in S.

A fuzzy irredundant set S is **maximal fuzzy irredundant set** if for every node v ∈ V – S, the set S ∪ {v} is not fuzzy irredundant set, which means that there exists at least one node w ∈ S ∪ {u} which does not have any private neighbor. Maximum cardinality among all maximal fuzzy irredundant set is called **upper fuzzy irredundance number** and is denoted by IR(G). Minimum cardinality among all maximal fuzzy irredundant set is called **fuzzy irredundance number** and is denoted by ir(G).

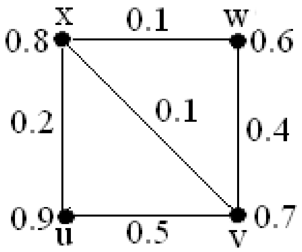

Figure 5

N[u] = {u, v, x}, N[v] = {v, u, w}, N[w]= {w, v} and N[x] = {x, u}

In the above fuzzy graph G, {u, w}, {u, v}, {v, x}, {w, x}, {u}, {v}, {w} & {x} are irredundant sets. IR(G) = 1.6, ir(G) = 1.4 The maximal fuzzy irredundant sets are {u, v }, {u, w}, {v, x} & {w, x}. The minimal fuzzy irredundant sets are {w}, {x}, {u}, and {v}.

Some Theorems on Irredundance of Fuzzy Graphs:

Theorem:

Let G = (σ, μ) be a fuzzy graph and u be a node which is not dominated by the maximal fuzzy irredundant set X. Then for some x ∈ X,

- ✓ P_N[x, X] ⊆ N(u) and
- ✓ For x_1, x_2 ∈ P_N(x, X) such that $x_1 \neq x_2$ either (x_1, x_2) is a strong arc or for i = 1, 2, there exists y_i ∈ X – {x} such that x_i is adjacent to each node of P_N(y_i, X).

Proof:

Since x is a maximal fuzzy irredundant set, some node of {u} ∪ X is fuzzy redundant in {u} ∪ X. Given u is not fuzzy dominated by X

$$\therefore u \in P_N[u, \{u\} \cup X] = N[u] - N[\{u\} \cup X - \{u\}]$$

Hence some x ∈ X is redundant in {x} ∪ X. ∴ P_N[x,{u} U X] = ϕ

$$\Rightarrow N[x] - N[X \cup \{u\} - x] = \phi$$

$\Rightarrow N[x] \subseteq N[X-x] \cup N[u]$ and $P_N[x, X] = N[x] - N[X-\{x\}] \subseteq N[u]$

$\Rightarrow P_N[x, X] \subseteq N[u]$

Since $u \notin P_N[x, X]$

$\therefore P_N[x, X] \subseteq N(u)$.

Let $x_1 \neq x_2$ be two nodes of $P_N[x, X]$ such that (x_1, x_2) is not a strong arc and suppose without loss of generality for all $y_i \in X - \{x\}$, there exists $z_i \in P_N[y_i, X]$ such that (x_i, z_i) is not a strong arc. Consider the set $\{x_1\} \cup X$ and we observe the following result,

$$x_2 \in P_N[x,\{x_1\}\cup X] = N[x] - N[X\cup\{x_1\}-x]$$

$$u \in P_N[x_1, \{x_1\}\cup X] = N[x_1] - N[X\cup\{x_1\}-\{x_1\}]$$

[$\because x_1, x_2 \in P_N[x, X]$ & $P_N[x, X] \subseteq N(u) \Rightarrow x_1, x_2 \in N(u)$] and $z_i \in P_N[y_i, \{x_1\}\cup X]$ for each $y_i \in X - \{x\}$. It follows, $\{x_1\}\cup X$ is a fuzzy irredundant set, which contradicts to the maximality of X. This completes the proof.

Corollary:

If u is not dominated by the maximal fuzzy irredundant set X and x is a node which satisfy the condition, then there exists a node $y \in X-\{x\}$, such that for all $x' \in P_N[x, X]$, $G[u, x', x, y]$ is isomorphic to a strong path P_4.

Proof:

From part (1) of the theorem, $x \notin N(u)$ implies, $x \notin P_N[x, X]$. Therefore for some $y \in X - \{x\}$, (x, y) is a strong arc. [$\because x \notin P_N[x, X] = N[x] - N[X-x]$] By the definition of private neighbor of x with respect to X, (x, x') is a strong arc and (x', y) is not a strong arc. [$\because x' \in P_N[x, X] = N[x] - N[X-x]$] By the theorem (x', u) is a strong arc. Since X does not dominate u, (u, x) and (u, y) are not strong arcs. It follows $G[u, x', x, y]$ is fuzzy path which is isomorphic to a strong path P_4.

Theorem:

Let U the set of nodes of a fuzzy graph $G = (\sigma, \mu)$ that are not dominated by X, a smallest maximal fuzzy irredundant set. For each $u \in U$, define $X_u = \{x\in X : P_N(x, X) \subseteq N(u)\}$. Let $M = \{x_1, x_2,..., x_r\}$ be a subset of X of smallest fuzzy cardinality m such that $X_u \cap M \neq \phi$ for each u in U and $M' = \{x_1', x_2',..., x_r'\}$ be a subset of $V - X$ such that cardinality of M' is also m. Then $\gamma(G) < ir(G) + m$.

Proof:

Let $X = \{x_1, x_2, ..., x_n\}$ be a smallest maximal irredundant set such that $ir(G) = \sum_{x\in X} \sigma(x)$ and let $M = \{x_1, x_2,..., x_r\}$ be a subset of X with fuzzy cardinality m, $0 < m \leq r$. Since $X_u \cap M \neq \phi$ $\therefore$ For each i=1,2, ..., r, $x_i \in X_u$ for some $u \in U$, therefore $P_N(x_i, X) \subseteq N(u)$ Since u is not dominated by X & also $x_i \in X_u = \{x\in X: P_N(x, X) \subseteq N(u)\}$, $x_i \notin N(u)$ and hence $x_i \notin P_N(x_i, X)$. We deduce that, there exists $x_i' \in P_N(x_i, X)$ such that $x_i' \neq x_i$. Let $D = X \cup M'$, where $M' = \{x_1', x_2',..., x_r'\}$ For each $u \in U$, by the definition of M, there exists i, $1\leq i \leq r$ such that $x_i \in X_u = \{x\in X: P_N(x, X) \subseteq N(u)\} \Rightarrow x_i \in \{x\in X: P_N(x, X) \subseteq N(u)\} \Rightarrow P_N(x_i, X) \subseteq N(u) \Rightarrow x_i' \in N(u) \Rightarrow x_i'$ is a strong neighbor of u. Fuzzy cardinality of M' is m and hence D is a dominating set of G of fuzzy cardinality ir(G) + m. However, D contains the maximal irredundant set X, hence D is not a fuzzy irredundant set. D is a minimal dominating set of a fuzzy graph G if and only if D is an irredundant and fuzzy dominating set of G. Therefore D properly contains a minimal dominating set and hence $\gamma(G) < ir(G) + m$.

Corollary:

For any fuzzy graph, $ir(G) > \frac{\gamma(G)}{2}$.

Proof:

By the theorem, $\gamma(G) < ir(G) + m$, for every values of m such that $0 < m < ir(G)$

$\therefore \gamma(G) < ir(G) + ir(G) \Rightarrow \gamma(G) < 2\, ir(G) \therefore ir(G) > \frac{\gamma(G)}{2}$.

Theorem:

Let G = (σ, μ) be a fuzzy graph. If $\gamma(G) = ir(G) + s$, where s is the sum of weight of the nodes $x_1, x_2, \ldots, x_k$ such that $0 < s \le k$, where k is an integer with $k \ge 0$, then G has k+1 induced sub graph isomorphic to P_4 with node sequence (a_i, b_i, c_i, d_i), i = 1,2,…., k+1, where $\cup_{\gamma=1}^{\gamma+1}\{b_i, c_i, d_i\}$ is a set of 3k+3 nodes and for each j = 1,2,3,……k+1, $a_j \notin \cup_{\gamma=1}^{\gamma+1}\{c_i, d_i\}$.

Proof:

Let X be a smallest maximal fuzzy irredundant set and U be the set of nodes, which is not fuzzy dominated by X. Let $X_u = \{x \in X: P_N(x, X) \subseteq N(u)\}$ and $Z = \cup_{\gamma\in 1}\gamma_0$. For each x∈Z choose a node $\gamma_1' \in P_N(x, X)$ such that (u, x_1') is a strong edge. Let B be a strong edge fuzzy bipartite sub graph of G with edge set $\cup_{u\in U}\{(u, x_1'): x \in X_u\}$ and $Q = \{(u_i, x_i'):$ i=1,2,3,….β} be a strong maximum matching in B . We claim that $Y = \{x_1, x_2, \ldots, x_\beta\}$ satisfies $Y \cap X_u \neq \phi$ for each u ∈ U, where fuzzy cardinality of Y is t, $0 < t \le \beta$. Suppose $Y \cap X_{u'} = \phi$ for some u′ in U, then no strong edge incident with u′ in Q and (x', u'), is a strong edge of B not incident with any strong edge of Q which is a contradiction to Q is a maximum matching. Hence, if m is a value defined as in theorem 3.3.3, we have, $\beta \ge t \ge m \ge \gamma(G) - ir(G) = s$, (by hypothesis) which is true for every value of s, 0<s≤k = k

That is, β ≥ k + 1. For each x ∈ Z, there exists u ∈ U such that $P_N(x, X) \subseteq N(u)$. Therefore $x \notin P_N(x, X)$ and there exists $y_x \in X$ such that (x, y_x) is a strong edge. Now, we set, $(a_i, b_i, c_i, d_i) = (y_{x_i}, x_i, x_i', u_i)$, for i = 1, 2, 3, …, k+1. The result now follows from the corollary 3.3.2

Example:

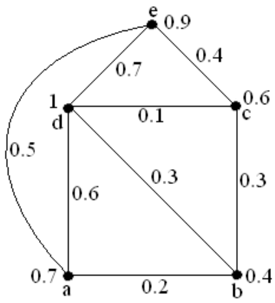

Figure 5

In the above fuzzy graph G, the irredundant sets are {a, c}, {a, e},{b, c}, {b, e}, {b, d}, {c, d}, {c ,e}, {a}, {b}, {c},{ d } and {e}. IR(G) = 1.6 and ir(G) =1

Insensitive Arcs in Fuzzy Graph:

Insensitive arc in domination of fuzzy graph was discussed in [5]. In this chapter we discuss about fuzzy insensitive arcs, minimum arcs in insensitive fuzzy graph and minimum arcs in connected fuzzy graph. The fuzzy graph G is said to be **arc insensitive** if $\gamma(G) = \gamma(G\text{-}e)$ for any arc e of G. We say as γ - insensitive when the domination number is γ. Throughout this chapter we consider the fuzzy graph G to be connected. Let D_1 be the minimum dominating set of G with k nodes where k<n and domination number $\gamma(G)$. Let D_2 be the complement of D_1, i e., $D_2 = V - D_1$.

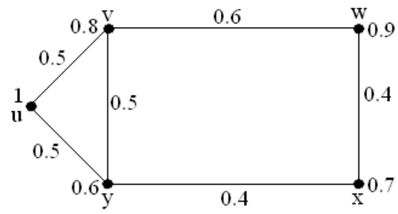

Figure 6

From this figure $D_1 = \{v, y\}$, $D_2 = \{u, w, x\}$, $\gamma = 1.4$ and this fuzzy graph is arc insensitive. i.e., $\gamma(G\text{-}e) = \gamma(G)$.

Theorem:

A fuzzy graph G is arc insensitive if and only if for each e = (u, v) of G and the minimum dominating set D_1 with domination number γ then one of the following conditions is satisfied:

- ✓ $u, v \in D_1$
- ✓ $u, v \in D_2$
- ✓ $u \in D_1$, $v \in D_2$ and N(v) contains at least two nodes in D_1.

Proof:

Consider the fuzzy graph G is arc insensitive. This implies that the removal of any arc e = (u, v) will not affect the dominating set D_1. If the arc e is adjacent to any two nodes u & v of G Also if $u, v \notin D_1$ then $u, v \in V - D_1 = D_2$. If $u, v \notin D_2$ then $u, v \in V - D_2 = D_1$ Another way of possible only if $u \in D_1$, $v \in D_2$ and $N(v) \cap D_1 = \{u\}$. Then the removal of this arc e will affect γ which contradicts to our assumption. Hence N(v) contains at least two nodes in D_1. Converse of the theorem is trivial.

Theorem:

Let G be any complete fuzzy graph such that G* is connected bipartite graph with bipartition D_1 and D_2 in which D_1 and D_2 have at least two nodes then G is arc insensitive.

Proof:

Given G* is connected bipartite graph. ∴ For each edge e = (u, v), $u \in D_1$, $v \in D_2$, $N_S(v)$ contains at least two nodes in D_1 because G is complete fuzzy graph & D_1, D_2 contains at least 2 nodes. Hence by the above theorem, G is arc insensitive.

Minimum Arcs in Insensitive Fuzzy Graph:

In this section we find the minimum number of arcs required to satisfy the arc insensitive property of the fuzzy graph G. Minimum number of arcs is denoted by A_i.

Theorem:

Arc insensitive fuzzy graph exists for $k \geq 2$.

Proof:

Let G be a fuzzy graph with n nodes and order p. Let D_1 be a minimum dominating set with k nodes and domination number γ. Assume G is arc insensitive, i.e., $\gamma(G\text{-}e) = \gamma(G)$. If suppose k = 1, then a single node dominating all other nodes of D_2, which contradicts our assumption. Therefore $k \geq 2$.

Theorem:

Any arc insensitive fuzzy graph G must have $A_i = 2n - 2k$ for $k \geq 2$.

Proof:

Let G be an arc insensitive fuzzy graph with n nodes and order p. Let D_1 be a dominating set with k nodes and D_2 be the complement of D_1 with $n - k$ nodes and cardinality $p - \gamma$. Since by theorem 4.2.3 every node in D_2 has at least two strong neighbors in D_1 and hence the minimum number of arcs required for arc insensitive is twice (n-k). i.e., $A_i = 2(n-k)$.

Minimum Arcs in Connected Fuzzy Graph:

In this section when a fuzzy graph will remain connected even after the removal of any arc and the minimum number of arcs required for such graph where the domination number remains fixed. The minimum number of arcs is denoted by Ac.

Theorem:

Let G be a connected fuzzy graph with n > 3 and let D_1 be a dominating set with a single node then no node in D_2 is pendent node if G remains connected after the removal of any arc, where the domination remains fixed.

Proof:

Let G be a connected fuzzy graph. Given D_1 is a dominating set with k = 1 and domination number γ. Since k = 1, this node dominates all other n−k nodes. Since D_1 is a dominating set.⇒ Every node in D_2 is strong neighbor with D_1. If D_2 has a pendent node then the removal of the arc adjacent to the pendent node will increase the domination number γ. Hence G remains not connected after the removal of any arc.

Theorem:

Let G be a connected fuzzy graph with n > 3 and let D_1 be a dominating set with k = 1. The minimum number of arcs Ac required for the fuzzy graph to remains connected after the removal of any arc is Ac = n−1 + $[\frac{n-1}{2}]$

Proof:

Let G be a connected fuzzy graph with n > 3. Since k = 1, D_1 has a dominating node say {v}; where $0 \leq \sigma(v) \leq 1$. Then {v} dominates remaining (n−1) nodes of D_2. By theorem, D_2 will not have a pendent node; hence every node in D_2 is adjacent to at least one node in D_2 itself. Since D_2 has (n−1) nodes and $[\frac{n-1}{2}]$ edges. ∴ The minimum number of arcs required to have this property is the sum of dominating arcs and arcs between D_2. Hence Ac = (n-1) + $[\frac{n-1}{2}]$.

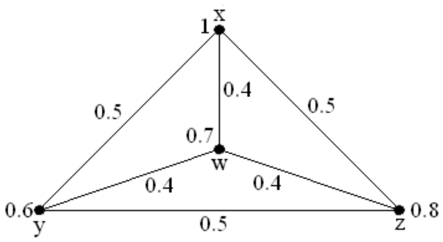

Figure 7

This fuzzy graph remains connected after the removal of any arc. Here n =4, D_1 = {x}, D_2 = {y, w, z}, γ = 1, p = 3.1 & Ac = 5.

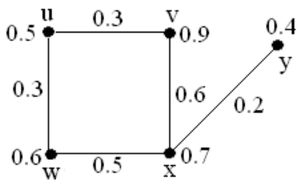

Figure 8

This graph does not hold the connectedness property since it has a pendent node.

Applications of Domination in Fuzzy Graphs:

A number of strategic locations are to be kept under observations. However, it is desired to put radar for observation process only at a few of these locations. How can we determine a set of locations in which to place the radar stations? An acceptable set of locations in which to place radar stations corresponds to a dominating set. Here we would like to find such a set of minimum size. One of the important areas of applications of domination is communication network, where a dominating set represents a set of cities which, acting as transmitting stations, can transmit messages to every city in the network. Another area of application of domination is voting situations. Suppose a group is trying to form a responsive committee with minimum number of persons to represent it. Let each member of the group designate that individual who he feels best understands his needs and would best represent his views. Let the group members be vertices of a digraph and draw an arc from vertex x to vertex y if x was designated by y. Then minimum dominating set would make a representative committee.

Location of Army Posts Problem:

Suppose the commander of the Army Postal services plans to set up a few post offices in an important region with minimum number of post offices to control the whole region. Suppose that the region shown by the large square in figure is divided into the sixteen smaller regions as shown, in Fig 5.1

A_1	A_8	A_9	A_{16}
A_2	A_7	A_{10}	A_{15}
A_3	A_6	A_{11}	A_{14}
A_4	A_5	A_{12}	A_{13}

An Army post located in an area in order to serve not only the square that is located in, but also those areas, which have common borders with that square. Now the problem is to find the smallest possible number of post offices and the locations of these post offices which is sufficient to serve the whole region. We represent the situation by a graph model by introducing each small area by a vertex and edges between two vertices in the corresponding areas have a common border. The graph thus obtained is shown in Fig 5.2

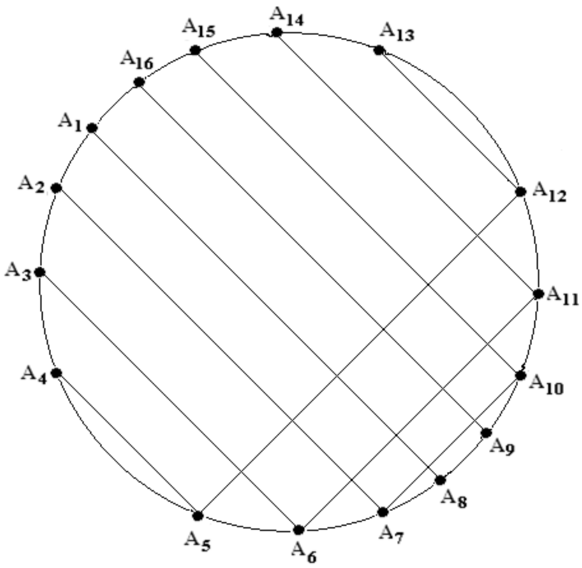

Figure 9

Now, it is enough to find a minimum fuzzy dominating set for graph shown in Figure 9 and its cardinality gives the number of post offices to be opened to cover the whole region. In this case $\gamma(G) = 4$ and the post offices may be located in $\{A_2, A_9, A_{14}, A_5\}$

School Bus Routing Problem:

Now-a-day almost all schools operate school buses for transporting children to and from schools. Among many points, three important points to be noted are

- ✓ The running time of a bus between school and its terminus.
- ✓ Maximum number of students in a bus at any one time and
- ✓ The maximum distance a student has to walk to board a school bus.

Consider a street map of part of a city shown in Fig 5.3, where each edge represents one city block.

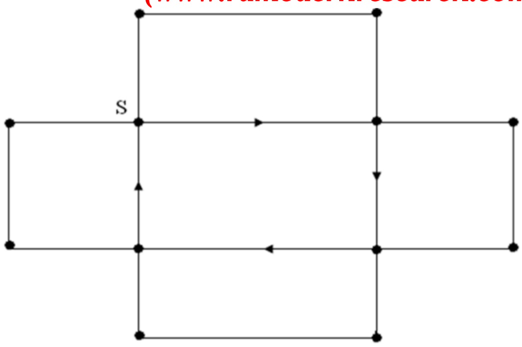

Figure 10

Let us assume that the school is located at the vertex S and the management committee of the school decides that no student shall walk more than one block to board a school bus. Now, the problem is to draw a route for a school bus that leaves the school, gets within one block of every child that uses the school bus and return to the school. One such route is marked by the direction in figure. We observe that those children who reside around the school prefer to walk to the school. Clearly this bus route gives fuzzy dominating set in the sense that the set of all vertices on this bus route is an fuzzy dominating set of the graph.

Four Cube Computer Network Model:

Consider a computer network modeled by a 4-cube. The vertices of the 4-cube represents computers and edges represent direct communication link between two computers. So, in this model we have 16 computers or processers to which it is directly connected. The problem is to collect information from all processors and we like to do it relatively often and relatively fast. So we identify a small set of processors called collecting processors and ask each processor to send its information to one of a small set of collecting processors. We assume that at most a one-unit delay between the time a processor sends its information and time it arrives at a nearest collector is allowed. So, we have to find an dominating set among the set of a processors. A dominating set {u, w, x, z} is shown in the Figure 11.

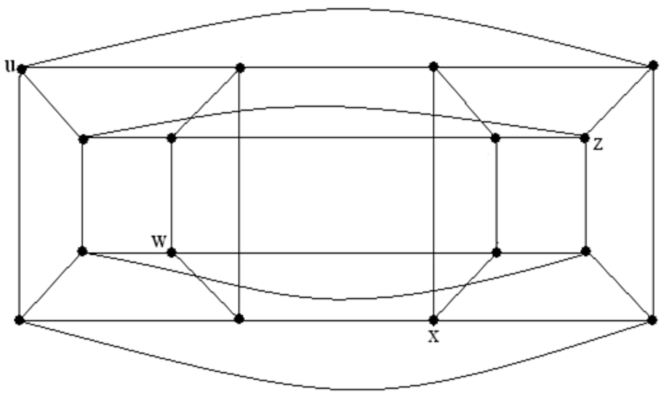

Figure 11

Location Problem:

Consider the problem of locating a single fire station, police station or a similar such service facility to serve the communities. Also, we would like to locate such a service facility in one of these communities and not at an arbitrary point along the road, due to some reasons. Suppose the communities represent vertices of a fuzzy graph and the road link between two communities represents an edge between the corresponding two vertices of the fuzzy graph; let us assume that the weight of an edge c_{ij} between v_i

and v_j represents the travel time between the corresponding two communities and the weights w_i to the vertices may represent the probabilities of the police or fire service being required by the various communities; for example these weights may be taken as proportional to the population of each community. Clearly, the vertex which minimizes the time to reach most distant community is then the out center of the fuzzy graph. Suppose we plan to locate a hospital in one of these communities then we have to minimize the time taken by an ambulance to reach the most distant community and return back to the hospital. In the case of fire station out radius is more important. In the case of Government offices in radius is more important.

Efficient Construction of Connected Dominating Set in Wireless Ad Hoc Networks:

A network is normally modeled as a limited graph G= (V,E) where V&E are limited sets of nodes and edges in the graph. In G, define a vertex u dominates another vertex v if u=v or if u and v are adjacent. A wireless and hoc network is a collection of mobile devices dynamically forming a temporary network without any existing infrastructure. Some examples of the possible uses of wireless ad hoc networks include participants using laptops to discuss some issues in a conference hall, business associates using note books to share information during a meeting and soldiers using wearable computers to exchange situational information on the battlefield. Due to the limited transmission range of wireless network interfaces, when a message is being set from one node to another, intermediated network nodes might be needed to serve as routers to relay this message.

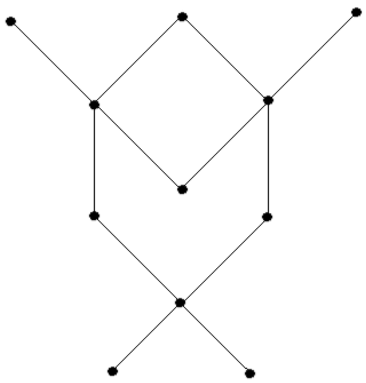

Figure 12

Local Solutions for Global Problems in Wireless Network:

Let P_n be a set of points in general position on the plane. The unit distance graph UDG (P_n) associated to P_n is a graph whose vertex set consists of the elements of $P_{n,}$ two of which are connected if they are at distance at most one. Unit distance graphs are used to model various types of wireless networks, including cellular networks, sensor networks, ad-hoc networks and others in which the nodes represent broadcast stations with a uniform broadcast range we shall refer to networks that can be modeled using unit distance graphs as unit distance wireless networks, abbreviated as UDW networks.

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