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A CASE STUDY ON MASS TRANSFER

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Abstract:

In this paper, the confluent hyper geometric functions and mass transfer is studied. An analysis is considered about the effects of suction/injection and chemical reaction on mass transfer characteristics over a stretching surface. The system of partial differential equations is transformed to a system of ordinary differential equations and the exact analytical solution for the momentum boundary layer problem and the concentration boundary layer equation are obtained.

Introduction:

Transfer of mass from high concentration to low concentration is known as mass transfer. Mass transfer is the phrase commonly used in engineering for physical processes that involve molecular and conductive transport of atoms and molecules within physical systems. Mass transfer includes both fluid flow and separation unit operations. Some common examples of mass transfer processes are the evaporation of water from a pond to the atmosphere, the diffusion of chemical impurities in lakes, rivers, and oceans from natural or artificial point sources. Mass transfer is also responsible for the separation of components in an apparatus such as a distillation column. In HV AC examples of a heat and mass exchangers are cooling towers and evaporative coolers here evaporation of water cools that portion which remains as a liquid, as well as cooling and humidifying the air passing through. The driving force for mass transfer is a difference in concentration; the random motion of molecules causes a net transfer of mass from an area of high concentration to an area of low concentration. The amount of mass transfer can be quantified through the calculation and application of mass transfer coefficients.

Application of Mass Transfer:

- ✓ Mass transfers resistance associated with the biomass are normally neglected.
- ✓ In subtended growth systems such as activated sludge process, biological reaction kinematics normally over shadow. Mass transfer effects because of the relatively small size of biological floe particles, thus the above approach is valid.

Introduction to Confluent Hyper Geometric Function:

Gauss was largely responsible for the systematic study of the hyper geometric function, E.E. Kummer (1810-1893) is the person most associated with developing properties of the related confluent hyper geometric function. Kummer published his work on this function in 1836, and since that time it has been commonly refered to as kummer's function. Like the hyper geometric function, the confluent hyper geometric function is related to a large number of other functions. Kummer's function satisfies a second-order linear differential equation called the confluent hyper geometric equation. A second solution of this differential equation leads to the definition of the confluent hyper geometric function of the second kind, which is also related to many other functions. At the beginning of the twentieth century (1904), Whittaker introduced another pair of confluent hyper geometric functions. The Whittaker functions arise as solutions of the confluent hyper geometric equation after a transformation to Liouville's standard from of the differential equations.

Special Cases and Application:

- ✓ The classic orthogonal polynomials can be expressed as special cases of ${}_2F_1$ with one (or) both a and b being (negative) integrals. Similarly, the Legendre functions are a special case as well.
- ✓ Applications of hyper geometric series include the inversion of elliptic integrals. Elliptic integrals are constructed by taking the ratio of the two linearly independent solutions of the hyper geometric differential equation to form Schwarz-christoffer maps of the fundamental domain to the complex projective line (or) Riemann sphere.
- ✓ The kummer function ${}_1F_1(a,b;z)$ is known as the confluent hyper geometric function.
- ✓ The function ${}_2F_1$ has several integral representations, including the Euler hyper geometric integral.
- ✓ The hyper geometric series ${}_2F_1$ is closely related to the legandre polynomials, and when used in the form of spherical harmonics, it expresses, in a certain sense.

History and Generalization:

- ✓ Studies in the nineteenth century included those of Ernst kummer, and the fundamental characterization by Bernhard Riemann of the F-function by means of the differential equation.
- ✓ Riemann showed that the second order differential equation (inz) for F, examined in the complex plane, could be characterized (on the Riemann sphere) by its three regular singularities that effectively the entire algorithmic side of the theory was a consequence of basic facts and the use of Mobius transformation as a symmetry group.
- ✓ The hyper geometric series were generalized to several variables, for example by Paul Email Appell. But a comparable general theory took long to emerge.
- ✓ Many identities were found, some quite remarkable.
- ✓ Generalizations, the q-series analogues, called the basic hyper geometric series, were given by Eduard Heine in the late nineteenth century.
- ✓ The ratio of successive terms, instead of being a rational function of n, are considered to be a rational function of q^n
- ✓ Another generalization, the elliptic hyper geometric series, are those series where the ratio of terms is an elliptic function (a doubly periodic geomorphic function) of n.
- ✓ During the twentieth century this was a fruitful area of combinatorial mathematics, with numerous connections to other fields.
- ✓ There are a number of new definitions of hyper geometric series, by Aomoto, Israel Gelfand and others and applications for example to the combinatory of arranging a number of hyper planes in complex N-space.
- ✓ Hyper geometric series can be developed on Riemannian Symmetric spaces and semi-simple lie groups.
- ✓ A number of hyper geometric function dentties were discovered in the nineteenth and twentieth century's. One classical list of such identities is Bailey's list.
- ✓ It is currently understood that there is a very large number of such identities, and several algorithms are now known to generate and prove these identities.

- ✓ The actual value of the resulting number is in a sense less important than the various patterns that emerge; and so it is with hyper geometric identities as well.

Introduction to Chemical Reaction:

A chemical reaction is a process that always results in the inter conversion of chemical substances. The substances or substances initially involved in a chemical reaction are called reactants. Chemical reactions are usually characterized by a chemical change, and they yield one or more products, which usually have properties different from the reactants. Chemical reactants encompass changes that strictly involve the motion of electrons in the forming and breaking of chemical bonds. The general concept of a chemical reaction, in particular the notion of a chemical equation is applicable to transformations of elementary particles, as well as nuclear reactions. Different chemical reactions are used in combination in chemical synthesis in order to get a desired product. Chemical changes are a result of chemical reactions. All chemical reactions involved a change in substances and a change in energy. Neither matter nor energy is created or destroyed in a chemical reaction they may be changed. Chemical reactions are classified as

- ✓ Synthesis Reaction
- ✓ Decomposition Reaction
- ✓ Single replacement Reaction
- ✓ Double replacement Reaction

Mass Transfer Over Stretching Surface With Chemical Reaction and Suction / Injection:

The Mass and momentum transport in laminar boundary on moving, stationary and linearly stretching surface has important applications in polymer industry and electrochemistry. The purpose of this investigation is to study the effects of suction/injection and chemical reaction on mass transfer over a stretching surface.

Mathematical Analysis:

Consider a steady, laminar, incompressible and viscous fluid on a continuous stretching surface with mass transfer, chemical reaction and suction\injection. The fluid properties are assumed to be constant in a limited temperature range. The concentration of diffusing species is very small in comparison to other chemical species; the concentration of species far from the surface, $C\infty$ is infinitesimally very small. The chemical reactions are taking place in the flow and all physical properties are assumed to be constant. The x-axis runs along the continuous surface in the direction of the motion and the y-axis is perpendicular to it. The basic boundary layer equations for the steady flow of Boussinesq type of fluid are;

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2} \qquad (2)$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-k_1 C \qquad (3)$$

The constant k_1 is the first order chemical reaction rate (when $k_1<0$ generating reactant and $k_1>0$ destructive reactant). The boundary conditions for the present problem are:

$$u = U_0 x,\ v = v_W,\ C = C_W \quad \text{at} \quad y = 0 \qquad (4)$$

$$u \to 0,\ C \to C_\infty, \qquad \text{as} \quad y \to \infty$$

The suitable similarity variables, for the problem under consideration, are:

$$\eta = \sqrt{\frac{U_0}{V}}\, y \quad , \quad \Psi(x,y) = \sqrt{vU_0}\, x f(\eta) \qquad (5)$$

$$C = C_\infty + cx^{\gamma}\Phi(\eta)$$

The stream function $\Psi(x,y)$ is defined as

$$u = \frac{\partial \Psi}{\partial y} \quad , \quad v = -\frac{\partial \Psi}{\partial x} \qquad (6)$$

The wall concentration on the stretching surface C_W is:

$$C(x, y=0) - C_\infty = C_W - C_\infty = cx^{\gamma_1}\phi(0) \qquad (7)$$

By, using equations (5) and (6) in equations (2) & (3), the two equations are

$$f''' + ff'' - (f')^2 = 0 \qquad (8)$$

$$\phi'' - L\phi + Sc(f\phi' - \gamma_1 f'\phi) = 0 \qquad (9)$$

The transformed boundary conditions are given by:

$$f(0) = -\frac{v_W}{\sqrt{vU_0}} = f_W \quad , \quad f'(0) = 1 \;,\; \varphi(0) = 1, \text{ at } \eta = 0 \qquad (10)$$

$$f'(\infty) = 0 \;,\; \phi(\infty) = 0 \text{ at } \eta \to \infty$$

Materials and Methods:

Exact Solution of the Momentum Boundary Layer Problem:

A solution of non-linear differential equation (8) in the form:

$$f(\eta) = f_W + \frac{1 - e^{-\alpha\eta}}{\alpha} \;.\; \alpha > 0 \qquad (11)$$

Which is satisfied by the following boundary conditions.

$$f(0) = f_W \quad , \quad f'(0) = 1 \qquad \text{at } \eta = 0$$

$$f'(\infty) = 0 \text{ at } \eta \to \infty \qquad (12)$$

Substitute equation (11) into equation (.8) and using boundary conditions equations (12) we get

$$\alpha = \frac{f_W \pm \sqrt{f_w^{\,2} + 4}}{2}$$

The velocity components take the form:

$$u = U_0 x (e^{-\alpha\eta}) \qquad (13)$$

And $v = -\sqrt{vU_0}\,(f_W + \frac{1 - e^{-\alpha\eta}}{\alpha})$ (14)

Thus, from equation (11), we get a simple exact analytical solution of the boundary value problem (28) with boundary conditions (12).

Exact Solution of the Concentration Boundary Layer Equation:

On substituting equation (11) in the ordinary differential equation (9) the new ordinary differential equation is

$$\phi'' + Sc\left(f_W + \frac{1 - e^{-\alpha\eta}}{\alpha} \right)\phi' - (Sc\gamma e^{-\alpha\eta} + L)\phi = 0 \qquad (15)$$

Corresponding boundary conditions are given by:

$$\phi(0) = 1 \quad , \quad \phi(\infty) = 0 \qquad (16)$$

Equation (15) is a homogenous linear ordinary differential of second order with variable coefficients. To find the solution of boundary value problem (15): (16) introduce the change of variable:

$$\frac{Sc}{\alpha^2}e^{-\alpha\eta} = \xi \qquad (17)$$

Equation (15) can be written as:

$$\xi\phi'' + \left[1-(\alpha f_W + 1)\frac{Sc}{\alpha^2} + \xi\right]\phi'' - (\gamma + \frac{1}{\alpha^2}\xi^{-1})\phi = 0 \qquad (18)$$

With the boundary conditions:

$$\phi(\xi = \frac{Sc}{\alpha^2}) = 1 \text{ and } \phi(\infty) = 0 \qquad (19)$$

Equation (18) is similar to Kummer's differential equation, Hence, the solution is in the form of kummer's confluent hyper geometric function. Therefore, the solution of equation (18) satisfying (19) in terms of Kummer confluent hyper geometric function F_1 is given by:

$$\phi(\xi) = \left(\frac{\alpha^2}{Sc}\xi\right)^{(k_1+k_2)} \frac{{}_1F_1(k_1+k_2-\gamma;1+2k_2;-\xi)}{{}_1F_1\left(k_1+k_2-\gamma;1+2k_2;-\frac{Sc}{\alpha^2}\right)} \qquad (20)$$

$$k_1 = \frac{Sc}{2\alpha^2}(1+\alpha f_W)$$

$$k_2 = \sqrt{\frac{4L\alpha^2 + Sc^2(1+\alpha f_W)^2}{2\alpha^2}}$$

The solution of (20) in terms of η is written

$$\phi(\eta) = (e)^{-(k_1+k_2)\alpha\eta} \frac{{}_1F_1\left(k_1+k_2-\gamma;1+2k_2;-\frac{Sc}{\alpha^2}e^{-\alpha\eta}\right)}{{}_1F_1\left(k_1+k_2-\gamma;1+2k_2;-\frac{Sc}{\alpha^2}\right)} \qquad (21)$$

Mass Transfer Coefficient:

The mass transfer coefficient (ie) the local surface mass flux, with equation (21), can be expressed as:

$$M_W = D(\frac{\partial C}{\partial y})_{y=0}$$

$$= -Ddx^{\gamma}\phi'(0)\sqrt{\frac{U_0}{v}}$$

$$= -Ddx^{\gamma}\sqrt{\frac{U_0}{v}}$$

$$= \left[\frac{-\alpha(k_1+k_2-\gamma)Sc(k_1+k_2-\gamma)_1F_1(1+k_1+k_2-\gamma,2(1+k_2);-\frac{Sc}{\alpha^2})}{\alpha(1+2k_2)_1F_1(1+k_1+k_2-\gamma,1+2k_2;-\frac{Sc}{\alpha^2})}\right] \qquad (22)$$

Conclusion:

The effect of suction/injection and chemical reaction on mass transfer characteristics over a stretching surface is analyzed. The transformation of a system of partially differential equation to a system of ordinary differential equation is executed. The exact analytical solutions for the momentum boundary layer problem and the concentration boundary layer equations are obtained.

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