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A BRIEF REVIEW ON GRAPH LABELINGS

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Introduction:

Graph theory has a very wide range of applications in engineering, in physical, social, and biological sciences, in linguistics, and in numerous other areas. A graph can be used to represent almost any physical situation involving discrete objects and a relationship among them. A labeling (or valuation) of a graph is a map that carries graph elements to numbers (usually to the positive or non-negative integers). The most common choices of domain are the set of all vertices and edges (such labeling are called total labeling), the vertex-set alone (vertex- labeling) or the edge-set alone (edge-labeling). It is interesting to consider the sum of all labels associated with a graph element. This will be called the weight of the element. Various authors have introduced labeling that generalizes the idea of magic square. Sedlacek defined a graph to be magic if it has an edge labeling with range, the real numbers, such that the sum of the labels around any vertex equals some constant, independent of the choice of vertex. It was called super magic if the labels are consecutive integers. Several others have studied these labeling. Let G be a simple graph with order p and size q.

The function f : V(G)→{0,1,…,q} is called a graceful labeling if, (i) f is 1 – 1, (ii) The edges receive all the labels (number) from 1 to q, where the label of an edge is computed to be the absolute value of the difference between the vertex labels at its ends; and the edge labels are all different. (i.e) if e = (x,y) then the label of e is │f(x)-f(y)│. An harmonious labeling of a given graph *G* with *q* edges is an assignment to each vertex $x \in V(G)$, a distinct element $\lambda(x)$, of the group of integers modulo *q*, so that when each edge *xy* is labelled $\lambda(x) + \lambda(y)$, the edge labels are all distinct. In the case of a tree with *q* edges, one element of the group is assigned to two vertices while all others are used precisely once. A graph which admits an harmonious labelling is said to be harmonious. An edge magic total labeling of a graph G is a one-to-one map λ from $V(G)$ on to the integers $1,2,3,\ldots v+e$ where $v=|V(G)|$ and $e=|E(G)|$ with the property that for any edge (xy), $\lambda(x)+\lambda(xy)+\lambda(y)=k$ for some constant *k*. In other words, if $wt(xy)=k$ for any choice of edge xy, then *k* is called magic sum of G. Any graph with an edge magic total labeling will be called edge magic.

Graceful Graphs:

Theorem:

Let G be a graceful, even (simple) graph having edges, and let έ and ó denote the sums of the even edge labels and odd edge labels in a graceful labeling of G, respectively.

Case1:

If $\frac{e(e+1)}{2} \equiv 0 \pmod 4$ then
1. έ ≡0 (mod 4)
2. O ≡0 (mod 4)
3. $\frac{(e+1)}{2} \equiv 0 \pmod 2$

4. $\frac{(e+2)}{4} \equiv 0 \pmod 2$

Case 2:

If $\frac{e(e+1)}{2} \equiv 2 \pmod 4$ then

1. $\acute{\varepsilon} \equiv 2 \pmod 4$
2. $O \equiv 0 \pmod 4$
3. $\frac{(e+1)}{2} \equiv 0 \pmod 2$
4. $\frac{(e+2)}{4} \equiv 1 \pmod 2$

Proof:

Suppose G is a graceful, even (simple) graph (i.e., it is Eulerian) having edges. Then there exists a graceful labeling L_G of G. Since G is graceful and Eulerian, the sum of the resulting edge labels of G, which equals $\sum_{i=1}^{e} i = \frac{(e+1)}{2}$, In other words, $\frac{(e+1)}{2} \equiv 2k \pmod 4$, where k=0(Case 1) or k=1(Case 2).

If k=0 (Case 1), then $\frac{(e+1)}{2} \equiv 0 \pmod{} \equiv \acute{\varepsilon} + O.$

If k=1 (Case 2), then $\frac{(e+1)}{2} \equiv 2 \pmod 4 \equiv \acute{\varepsilon} + O.$

By analyzing partial sums of sequences of consecutive even integers and consecutive odd integers respectively, we observe that $\acute{\varepsilon} \equiv 0 \pmod 4$ or $2 \pmod 4$, and $O \equiv 0 \pmod 4$. If L_G has an even number of odd edge labels an $\equiv 1 \pmod 4$. If L_G has an odd number of odd edge labels. Thus the only possibility for Case 1(k=0) is $\frac{(e+1)}{2} \equiv 0 \pmod 4 \equiv \acute{\varepsilon} + O \;\; 0 \pmod 4 + 0 \pmod 4$, and the only possibility for Case 2 (k=1) is $\frac{(e+1)}{2} \equiv 2 \pmod 4 \equiv \acute{\varepsilon} + O \;\; 2 \pmod 4 + 0 \pmod 4$. We have that $\frac{(e+1)}{2} \equiv 0 \pmod 2$. By inspection, we also see that L_G has $\frac{(e+2)}{4}$ edge labels that Are$\equiv 2 \pmod 4$. It follows that in Case 1, Where k=0and $\acute{\varepsilon} \equiv 0 \pmod 4$, We have the number of edge labels that are $\equiv 2 \pmod 4 = \frac{(e+2)}{4} \equiv 0 \pmod 2$. It also follows that in Case 2, where k=1and $\acute{\varepsilon} \equiv 2 \pmod 4$, we have the number of edge labels that are $\equiv 2 \pmod 4 = \frac{(e+2)}{4} \equiv 1 \pmod 2$. We have therefore proven properties 1, 2, 3, and 4 for graphs in Case 1, and have also proven properties 1, 2, 3, and 4 for graphs in Case 2. This concludes the proof of the theorem.

Theorem:

All generalized Petersen graphs P(n,1)=P(n,n-1) are graceful.

Proof:

By the definitions of generalized Petersen graphs and prisms, we have P (n,1)= D_n. Since all prisms D_n are graceful [5], it follows that all generalized Petersen graphs of the form P(n,1) (and equivalently, P(n,n-1)are also graceful.

Theorem:

The graphs generalized Petersen graphs P(n,k) are graceful for n=5, 6, 7, 8, 9, and 10.

Proof:

To prove the gracefulness of the generalized Petersen graphs P(n,k)for n=5, 6, 7, 8, 9, and 10, it suffices to provide graceful labelings for these graphs. If we order the 2n vertices of P(n,k) as $\{a_0, a_1, \ldots, a_{n-1}, b_0, b_1, \ldots, b_{n-1}\}$ with edges set $\{a_i a_{i+1} |$ i=0,1,...,n-1$\}$ $\bigcup\{ a_i b_i |$i=0,1,...,n-1$\}\bigcup\{ b_i b_{i+k} |$i=0,1,...,n-1$\}$ where all subscripts are taken modulo n, Then the following vertex labelings are graceful labelings for these graphs:

P(5,1):$\{ x_1,\ldots, x_{10}\}$=0,1,6,2,15,14,7,4,12,3

P(5,2):$\{ x_1,\ldots, x_{10}\}$=0,1,3,6,14,15,13,10,2,4

P(6,1): $\{ x_1,\ldots, x_{12}\}$=0,1,3,9,4,18,17,513,6,15,2

P(6,2):$\{ x_1,\ldots, x_{12}\}$=0,1,4,14,3,18,17,5,10,6,5,2

P(6,3):$\{ x_1,\ldots, x_{12}\}$=0,1,3,11,4,15,14,13,12,8,9,2

P(7,1):$\{ x_1,\ldots, x_{14}\}$=0,1,3,6,15,4,20,21,9,16,10,5,19,2

P(7,2):$\{ x_1,\ldots, x_{14}\}$=0,1,3,12,5,10,21,20,19,6,418,14,2

P(7,3):$\{ x_1,\ldots, x_{14}\}$=0,1,3,7,12,4,20,21,19,17,18,2,10,5

P(8,1):$\{ x_1,\ldots, x_{16}\}$=0,1,3,6,11,20,4,23,24,7,18,14,21,8,22,2

P(8,2):$\{ x_1,\ldots, x_{16}\}$=0,1,3,6,20,4,17,24,23,22,8,12,16,21,5,2

P(8,3):$\{ x_1,\ldots, x_{16}\}$=0,1,3,10,5,21,11,23,24,22,20,4,18,2,19,9

P(8,4):$\{ x_1,\ldots, x_{16}\}$=0,1,3,9,14,17,2,20,19,18,11,16,5,7,15,4

P(9,1):$\{ x_1,\ldots, x_{18}\}$=0,1,3,6,11,7,25,4,27,26,12,22,13,5,20,8,24,2

P(9,2):$\{ x_1,\ldots, x_{18}\}$=0,1,3,6,19,9,4,11,27,26,25,22,14,7,23,24,5,2

P(9,3):$\{ x_1,\ldots, x_{18}\}$=0,1,3,6,13,7,23,4,27,26,25,24,21,5,20,9,14,2

P(9,4):$\{ x_1,\ldots, x_{18}\}$=0,1,3,6,10,18,24,4,26,27,25,22,23,20,2,13,9,5

P(10,1):$\{ x_1,\ldots, x_{20}\}$=0,1,3,6,10,15,5,27,4,29,30,16,24,12,21,8,25,9,28,2

P(10,2):$\{ x_1,\ldots, x_{20}\}$=0,1,3,6,13,24,8,12,25,30,29,28,23,18,5,9,27,26,4,2

P(10,3):$\{ x_1,\ldots, x_{20}\}$=0,1,3,6,10,24,5,26,4,30,29,28,27,12,21,15,25,11,20,2

P(10,4):$\{ x_1,\ldots, x_{20}\}$=0,1,3,6,10,23,11,26,12,19,30,28,27,25,4,18,2,5,20,7

P(10,5):$\{ x_1,\ldots, x_{20}\}$=0,1,3,8,12,21,9,20,2,25,24,23,17,14,19,5,6,7,22,4

Double Cones:

The class of double cones $C_n+\overline{K_2}$, where n=1,2,..,∞ is among the classes of graphs that are the joins of other graphs. $C_n+\overline{K_2}$ has n+2vertices and 3n edges. By definition, it is clear that the double cone C_1 is the graph that joins two vertices to a loop. Being a non simple graph, it cannot be gracefully labeled, for reasons discussed above in Section. Similarly, The double cone $C_2+\overline{K_2}$ is not graceful, as it too is a non simple graph. However, some double cones have been found to be graceful, As illustrated by the following proposition.

Theorem:

The double cones $C_n + \overline{K}_2$ are graceful for n=3, 4, 5, 7, 8, 9, 11, and 12.

Proof:

If we order the n+2 vertices of $C_n + \overline{K}_2$ as $\{x_1,..., x_{n+2}\}=\{a_0, a_1, b_0, b_1,..., b_{n-1}$ with edge set $\{a_0 b_i$ |i=0,1,...,n-1} $\cup$ $\{a_1 b_i$ |i=0,1,...,n-1}$\cup$ $\{b_i b_{i+1}$ |i=0,1,...,n-1}$\cup$ $\{b_0 b_{n-1}\}$ then the following vertex labelings are graceful labelings for these graphs:

$C_3 + \overline{K}_2 : \{x_1,..., x_5\}$=0,1,2,6,9

$C_4 + \overline{K}_2 : \{x_1,..., x_6\}$=0,3,6,10,12,11

$C_5 + \overline{K}_2 : \{x_1,..., x_7\}$=0,2,4,10,15,14,11

$C_7 + \overline{K}_2 : \{x_1,..., x_9\}$=0,1,2,5,19,13,21,10,17

$C_8 + \overline{K}_2 : \{x_1,..., x_{10}\}$=0,1,2,9,22,12,15,20,14,18

$C_9 + \overline{K}_2 : \{x_1,..., x_{11}\}$=0,2,3,9,26,16,27,22,4,23,15

$C_{11} + \overline{K}_2 : \{x_1,..., x_{13}\}$=0,1,2,5,16,29,7,27,10,31,19,33,25

$C_{12} + \overline{K}_2 : \{x_1,..., x_{14}\}$=0,1,2,5,16,34,10,27,7,32,13,36,22,30

As we observed with the class of generalized Petersen graphs, there exists an infinite subclass of non graceful double cones. The proof of the following proposition is a direct result of Rosa's parity condition.

Theorem:

All double cones $C_n + \overline{K}_2$ where n ≡ 2 (mod 4), are not graceful.

Proof:

By definition, we see that double cones $C_n + \overline{K}_2$ where n ≡ 2 (mod 4), are simple, even graphs with e edges, with e≡2 (mod 4). Such graphs, by Rosa's parity condition, cannot be gracefully labeled.

A Class of Product Graphs:

Product graphs of the form $K_4 \times P_n$ where n=1,2,..,∞ have 4n vertices and 10n-4 edges. A graceful labeling of $K_4 \times P_1$ which is just K_4 is provided by Golomb along with his proof that complete graphs K_n are graceful only for n ≤ 4 $K_1 \times P_n$, $K_2 \times P_n$ and $K_3 \times P_n$ are all graceful.

Theorem:

Product graphs of the form $K_4 \times P_n$ are graceful for n = 1, 2, 3, 4, and 5.

Proof:

If we order the 4n vertices of $K_4 \times P_n$ as $\{x_1,..., x_{4n}\}$ where the n copies of K_4 have vertices $\{x_1,..., x_4\}$ (1st copy of K_4), $\{x_5,..., x_8\}$

2nd copy of K_4),...,{χ_{4n-3},..., χ_{4n}} nth copy of K_4 and are connected to each other by edges{$\chi_i\ \chi_{i+4}$ | I=1,2,...,4n-4}, then the following vertex labelings are graceful labelings for these graphs:

$K_4 \times P_1$:{χ_1,..., χ_4}=0,1,4,6

$K_4 \times P_2$:{χ_1,..., χ_8}=0,1,5,16,6,15,13,3

$K_4 \times P_3$:{χ_1,..., χ_{12}}=0,1,4,26,24,18,13,6,10,2,23,25

$K_4 \times P_4$:{χ_1,..., χ_{16}}=0,1,3,36,24,32,7,2,13,6,22,34,27,33,4,14

$K_4 \times P_5$:{χ_1,..., χ_{20}}=0,1,3,46,19,42,7,2,6,14,38,44,45,34,24,8,12,5,19,3,0

Harmonious Labelings of Trees:

An harmonious labeling of a given a graph *G* with *q* edges is an assignment to each vertex $x \in V(G)$, a distinct element $\lambda(x)$, of the group of integers modulo *q*, so that when each edge *xy* is labelled $\lambda(x) + \lambda(y)$, the edge labels are all distinct. In the case of a tree with *q* edges, one element of the group is assigned to two vertices while all others are used precisely once. A graph which admits an harmonious labelling is said to be harmonious. In the same paper it is verified that all trees on at most 10 vertices are harmonious and it is conjectured that all trees are harmonious. Employing a computer search similar to that used to find graceful labellings for trees we have established the following theorem.

Theorem:

All trees on at most 26 vertices are harmonious

Proof:

For a given tree *T* and labelling *L* of the vertices, let z(T,L) be the number of distinct edge labels. For n = —V(T) —, the aim is to find L such that z(T,L) = n-1. If *L* is a labelling and *v;w* $\in$ *V* (*T*), deone *Sw*(*L*; *v;w*) to be the labelling got from *L* by swapping the labels on *v* and *w*. The method is like this, using a parameter *M*: Start with any labelling of *V* (*T*).

1. If *z*(*T,L*) = *n* - 1, stop.
2. For each pair *{v;w}*, replace *L* by L^1 = *Sw*(*L*; *v;w*) if *z*(*T,* L^1) >*z*(*T,L*).
3. If step 3 finishes with *L* unchanged, replace *L* by *Sw*(*L*; *v;w*),
4. Where {*v;w}* is chosen at random from the set of all *{v;w}* such that
   (a) *{v;w}* has not been chosen during the most recent *M* times this step has been executed.
   (b) *Sw*(*L*; *v;w*) is maximal subject to (a).
5. Repeat from step 2.

This method can be described as a combination of hill-climbing and tabu search. Sometimes it appears to "get stuck" and needs to be restarted from step 1 with a new labelling chosen at random. A value of *M* = 10 seems ok for small trees, but a slightly larger value seems to be needed for larger trees. The purpose of *M* is to prevent the algorithm from repeatedly cycling around within some small set of labellings. Since the generation algorithm produces trees in an order such that most trees are very similar to the previous tree, it proved advantageous to use the graceful/harmonious labelling of each tree as the starting point for the next tree.

Edge Magic Graphs:

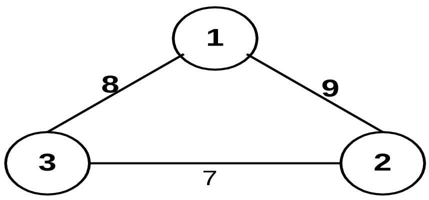
Theorem:

The complete bipartite graph $K_{m,n}$ is magic for any m and n.

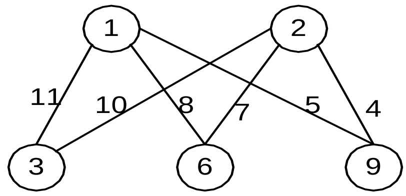
Proof:

The proof is depends on the two set s_1 and s_2 .We have two partition in which s_1 has *n* number of vertices and s_2 has *m* number of vertices. The two set & s_2 has the label of the form $s_1 = \{n+1, 2n+2, ..., m(n+1)\}$ $s_2 = \{1, 2, ...n\}$ and we define a magic labeling with $k = (m+2)(n+1)$.

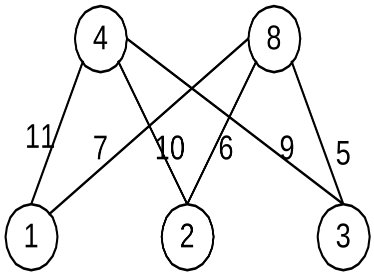
Examples:



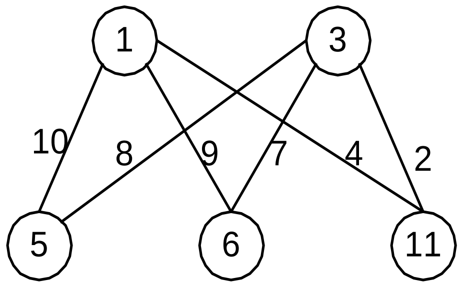
$K_{1,2}$ with magic sum k =12



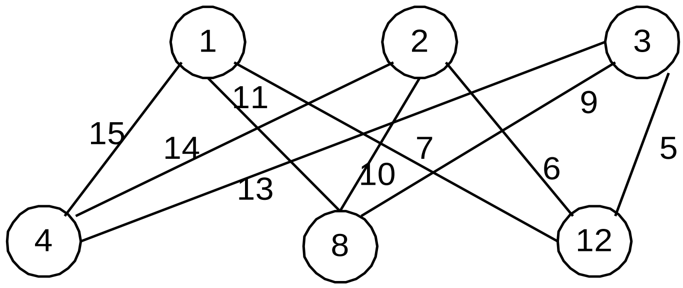
$K_{2,3}$ with magic sum k =15



$K_{2,3}$ with magic sum k =16



$K_{2,3}$ with magic sum k =16



$K_{3,3}$ with magic sum k =20

Lemma:

In any magic labeling of a star, the center receives label 1, *n*+1,or 2*n*+1.

Proof:

Let us assume that the center receives label *x*. Then

$$kn = (1+2+3+....+(2n+1)) + (n-1)x$$

$$kn = \frac{(2n+2)(2n+1)}{2} + (n-1)x$$

$$kn = \frac{(2n+2)}{2} + nx - x$$

x=-kn+nx+(2n+2/2)(mod n)

$$x \equiv 0 + 0 + \frac{(2n+2)(2n+1)}{2} (\text{mod}\, n)$$
$$x \equiv \frac{2(n+1)(2n+1)}{2} (\text{mod}\, n)$$
$$x \equiv n(2n+1) + 1(2n+1)(\text{mod}\, n)$$
$$x \equiv 2n^2 + n + 2n + 1(\text{mod}\, n)$$
$$x \equiv n(2n+1+2) + 1(\text{mod}\, n)$$
$$x \equiv 1(\text{mod}\, n)$$

Hence the lemma

Theorem:

There are 3. 2^n magic labeling of $K_{1,n}$ up to equivalence.

Proof:

Denote the center of a $K_{1,n}$ by c the leaves by $v_1, v_2, \ldots. v_n$ and edge (c, v_i) by e_i. From the above theorem we know that the possible cases for an edge-magic total labeling are when $\lambda(c) = 1$ then we have the equation

$$kn = \frac{2(n+1)(2n+1)}{2} + (n-1)1$$
$$k = \frac{(n+1)(2n+1)}{n} + 1 - \frac{1}{n}$$
$$k = \frac{(n+1)(2n+1) + n - 1}{n}$$
$$k = \frac{2n^2 + 2n + n + 1 + n - 1}{n}$$
$$k = \frac{2n^2 + 4n}{n}$$
$$k = 2n + 4$$

When $\lambda(c) = n+1$ then

$$kn = \frac{(2n+1)(2n+2)}{2} + (n-1)(n+1)$$
$$kn = \frac{(2n+1)2(n+1)}{2} + n^2 + n - n - 1$$
$$kn = (2n+1)(n+1) + n^2 - 1$$
$$kn = 2n^2 + 2n + n + 1 + n^2 - 1$$
$$kn = 3n^2 + 3n$$
$$k = \frac{3n^2 + 3n}{n}$$
$$k = \frac{3n(n+1)}{n}$$
$$k = 3(n+1)$$

When $\lambda(c) = 2n+1$ then

$$kn = \frac{(2n+1)(2n+2)}{2} + (n-1)(2n+1)$$
$$kn = \frac{(2n+1)2(n+1)}{2} + 2n^2 + n - 2n - 1$$
$$kn = 2n^2 + 2n + n + 1 + 2n^2 - n - 1$$
$$kn = 4n^2 + 2n$$
$$k = \frac{4n^2 + 2n}{n}$$
$$k = 4n + 2$$

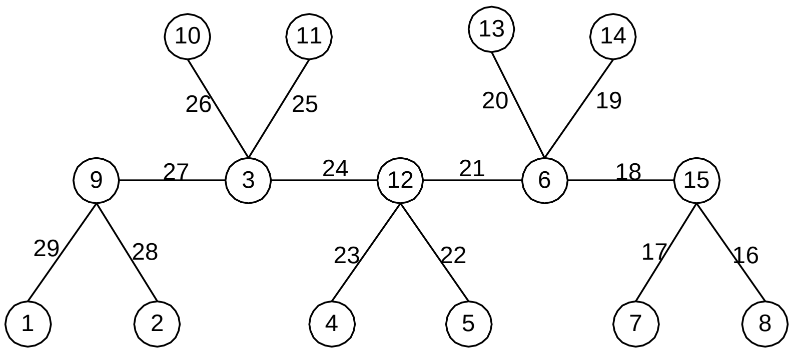
As the labeling is magic, the sums $\lambda(v_i) + \lambda(e_i)$ must all be equal to $M = k - \lambda(c)$ (so $M = 2n+3, 2n+2$ or $2n+1$). Then in each case there is exactly one way to partition the $2n+1$ integers $1,2,\ldots,2n+1$ into $n+1$ sets $\{\lambda(c)\}$, $\{a_1,b_1\}$, $\{a_2,b_2\},\ldots,$ $\{a_n,b_n\}$. Where every $a_i + b_i = M$ For convenience choose the labels so that $a_i < b_i$ for every i and $a_1 < a_2 < \ldots. < a_n$. Then up to isomorphism one can assume that $\{\lambda(v_i), \lambda(e_i)\} = \{a_i, b_i\}$. Each of these n equations provides two choices according to whether $\lambda(v_i) = a_i$ or b_i so each of three values of $\lambda(c)$ gives 2^n edge-magic total labeling of $K_{1,n}$.

Theorem:

All caterpillars are edge-magic.

Proof:

We describe an edge magic total labeling λ of the caterpillar G described above. First the stars are ordered $s^1, s^3, s^5, \ldots, s^2, s^4, s^6, \ldots$ and then the leaves of the stars are labeled with the smallest integers, starting from one as a label on s^1 and ascending. When the leaves of s^i are labeled, c_{i-1} receives the smallest label (except when $i = 1$) and c_{i+1} the largest one.



Labeling of a Caterpillar

So the leaves of s^1 receive $1,2,3\ldots e_1$ with $\lambda(c_2) = e_1$, then the vertices of s_3 receive $v_1, v_1+1,\ldots, e_1+e_3-1$, With $\lambda(c_2) = e_1$ and $\lambda(c_2) = e_1 + e_3 - 1$, and so on. This uses labels $1,2,3,\ldots,= \sum_1^n e_i - n + 2 = v$. Then the edges are labeled. The smallest available labels, namely $v_1, v_1+1, \dots, v+e_n$, are applied to the edges of S^n, then the next e_{n-1} to the edges of S^{n-1}, and so on until the edges of S^1 are labeled. In each star, the smallest label is given to the edge whose perimeter vertex has the largest label, and so on. The labeling is illustrated in figure.

Complete Graphs and Duality of Labelings:

In this section we discuss about edge magic total labeling of complete graphs and we also study about duality of graphs.

Theorem:

Suppose K_v has an edge-magic total labeling of with magic sum *k*. The number p of vertices that receive even labels satisfies the following condition:

If $v \equiv 0$ or $3(\bmod 4)$ and k is even then $p = \frac{1}{2}\left(v-1 \pm \sqrt{v+1}\right)$.

If $v \equiv 1$ or $2(\bmod 4)$ and k is even then $p = \frac{1}{2}\left(v-1 \pm \sqrt{v-1}\right)$

If $v \equiv 0$ or $3(\bmod 4)$ and k is odd then $p = \frac{1}{2}\left(v+1 \pm \sqrt{v+1}\right)$

If $v \equiv 1$or $2(\bmod 4)$ and k is odd then $p = \frac{1}{2}\left(v+1 \pm \sqrt{v+3}\right)$

Proof:

Suppose λ is an edge magic total labeling of K_v with magic sum k. Let V_e denote the set of all vertices x such that $\lambda(x)$ is even , and V_0 the set of a vertices x with $\lambda(x)$ odd; define p to be the number of elements of V_e Write E_1 for the set of edges with both end points in the same set, either V_0 or V_e and E_2 for the set of edges joining the two vertex-sets, so that

$$|E_1| = \binom{p}{2} + \binom{v-p}{2} \text{ and } |E_2| = p(v-p).$$

If k is even, then $\lambda(yz)$ is even whenever yz is an edge in E_1 and odd whenever yz is in E_2 so there are precisely $p + \binom{p}{2} + \binom{v-p}{2}$ even labels. But these labels must be even integers from 1 to $\binom{v+1}{2}$, So $p + \binom{p}{2} + \binom{v-p}{2} = \left\lfloor \frac{1}{2}\binom{v+1}{2} \right\rfloor$

If $\binom{v+1}{2}$ even, then $p + \frac{p(p-1)}{2} + \frac{(v-p)(v-p-1)}{2} = \frac{1}{2}\binom{v+1}{2}$

$$\Rightarrow \quad 2p + 2p^2 - 2vp + v^2 - v = \frac{v^2+v}{4}$$

$$\Rightarrow \quad p + p^2 - vp = \frac{v^2+v}{4} - \frac{v^2}{2} + \frac{v}{2}$$

$$\Rightarrow \quad p^2 + (1-v)p = \frac{v^2 + v - 2v^2 + 2v}{4}$$

$$\Rightarrow \quad p^2 + (1-v)p = \frac{-v^2 + 3v}{4}$$

$$\Rightarrow \quad p^2 + (1-v)p + \frac{(v^2 - 3v)}{4} = 0$$

$$\Rightarrow \quad p=\frac{-(1-v)\pm\sqrt{(1-v)^2-4(1)\left(\frac{v^2-3v}{4}\right)}}{2(1)}$$

$$\Rightarrow \quad p=\frac{(v-1)\pm\sqrt{1+v^2-2v-v^2+3v}}{2}$$

$$\Rightarrow \quad p=\frac{1}{2}\left(v-1\pm\sqrt{v+1}\right).$$

This equation has solution $p=\frac{1}{2}\left(v-1\pm\sqrt{v+1}\right)$. Similarly if $\binom{v+1}{2}$ Odd gives solution $p=\frac{1}{2}\left(v-1\pm\sqrt{v-1}\right)$. If k is odd, the edges in E_1 are those that receive the odd labels, and instead of above equation we have

$$p+p(v-p)=\left\lfloor\frac{1}{2}\binom{v+1}{2}\right\rfloor$$

$$p+pv-p^2=\frac{1}{2}\frac{(v+1)v}{2}$$

$$\Rightarrow \quad -p^2+(1+v)p-\frac{v^2+v}{4}=0$$

$$\Rightarrow \quad p+pv-p^2=\frac{v^2+v}{4}$$

$$\Rightarrow \quad p=\frac{-(1+v)\pm\sqrt{(1+v)^2-4(-1)(-1)\frac{v^2+v}{4}}}{2(1)}$$

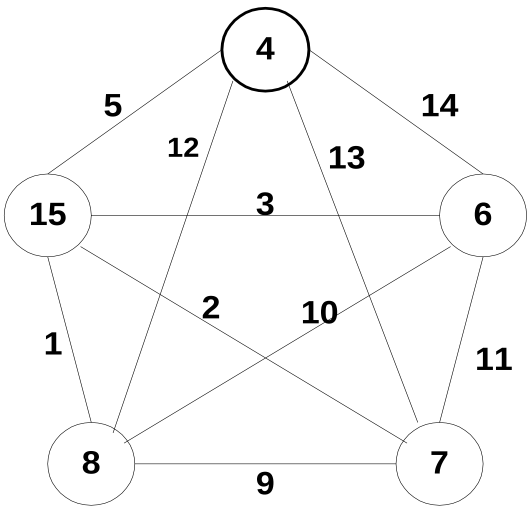
$$\Rightarrow \quad p=\frac{1}{2}(v+1)\pm\sqrt{1+v^2+2v-v^2-v}$$

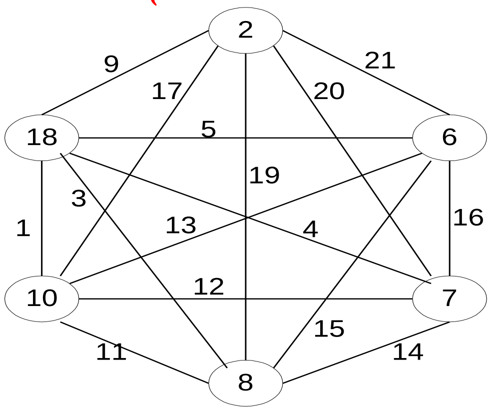
$$\Rightarrow \quad p=\frac{1}{2}\left(v+1\pm\sqrt{v+1}\right)$$ This equation is when $\binom{v+1}{2}$ is even.

Similarly if $\binom{v+1}{2}$ Odd then this has the solution $p=\frac{1}{2}\left(v+1\pm\sqrt{v+3}\right)$ Using the fact that $\binom{v+1}{2}$ is even when $v\equiv 0$ or $3(\bmod 4)$ and odd otherwise, we have the result. Now p must be an integer, so the functions whose roots are taken must always be perfect squares.

Example:

K_5 with magic sum k =24





K_6 with magic sum k =29

Conclusion:

While the labeling of graphs is perceived to be a primarily theoretical subject in the field of graph theory and discrete mathematics, labeled graphs often serve as models in a wide range of applications. Such applications include coding theory and communication network addressing. In this project we have described graceful labeling, harmonious labeling, edge magic total labeling, dual labeling.

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