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ANALYTICAL EXPRESSIONS OF RADIATIVE EFFECTS ON STEADY FREE CONVECTIVE FLOW

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Abstract:

In this research article we investigate the steady state radiative effects on steady state free convection heat transfer past a hot vertical plate. The dimensionless steady state temperature is derived by using the direct integration. Using this steady state temperature, we can also obtain the analytical expression for dimensionless steady state velocity. The shear stress functions for velocity profiles are also discussed analytically. The analytical expression for temperature gradient is also derived.

Keywords: Porous, Free Convection, Radiation, Steady State Non-Linear Boundary Value Problems & Shear Stress.

1. Introduction:

In this study we solve analytically the process of heat transfer in an isotropic, homogenous porous medium adjacent to a hot vertical plate using steady state non-liner boundary value problem. This study plays an important role in the area of porous media convection studies. The non-linear boundary value problem in thermal stability of boundary layer flows with adiabatic free surface along an inclined heated plate in discussed. The purpose of this work is to study the effects of transverse sinusoidal suction velocity on the flow and mass transfer on free convective optically thin grey fluid over a porous vertical plate in the presence of radiation. The governing equation s has been transformed to ordinary differential equations. Numerical solutions are obtained for different values of radiation parameter, Grashof number. It is found that dimensionless velocity decreases with increase of radiation parameter, increase, and decrease with increase of dimensionless temperature decreases with the increase of radiation parameter. Finally the effects of various parameters of the flow quantities are studied with the help of graphs. The study of heat transfer with chemical reaction is of great practical importance to engineers and scientists because of its almost universal occurrence in many branches of science and engineering reaction is said to be first-order of the rate of reaction is directly proportional to the concentration itself. Several studies have appeared recently analyzing the effects of thermal radiation in convection flows in porous media. Radiative-conduction heat transfer flows find numerous applications in glass manufacturing, furnace technology, high temperature aerodynamics, fire dynamics and spacecraft re-entry [1]. Many studies have appeared concerning the interaction of radiative flux with thermal convection flows. For example Chang et.al. [2] Studied the effect of radiation heat transfer on free convection regimes in enclosures, with applications in geophysics and geothermal reservoirs. Mudan [3] studied thermal radiation heat transfer from liquid pool fires.

2. Mathematical Formulation of the Problem:

Here we are going to consider the steady flow of a viscous incompressible fluid occupying a semi-infinite region of the space past an infinite hot vertical plate moving upwards with constant velocity embedded in a porous medium, by using steady state non-linear boundary value problem which is fig 1.when we select the co-ordinate system as the x' - axis is directed along the plate from the leading edge in the plate in the vertically upward direction and the y'-axis is normal to the plate. Except the influence of density variation in the body force term all fluid properties are considered as a constant. The radiation heat flux in the x' - direction is considered negligible in comparison to the y'-direction. The fluid is gray. Absorbing-emitting but non-Scattering. Gravity acts in the opposite direction to the positive x' - axis. The equation can be shown as:

$$\frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} = 0 \tag{1}$$

$$\frac{\partial u'}{\partial t'} = \nu \frac{\partial^2 u'}{\partial y'^2} + g\beta\left(T' - T'_\infty\right) - \frac{\nu u'}{K'} \tag{2}$$

$$\frac{\partial T'}{\partial t'} = \frac{k_1}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2} - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial y'}. \tag{3}$$

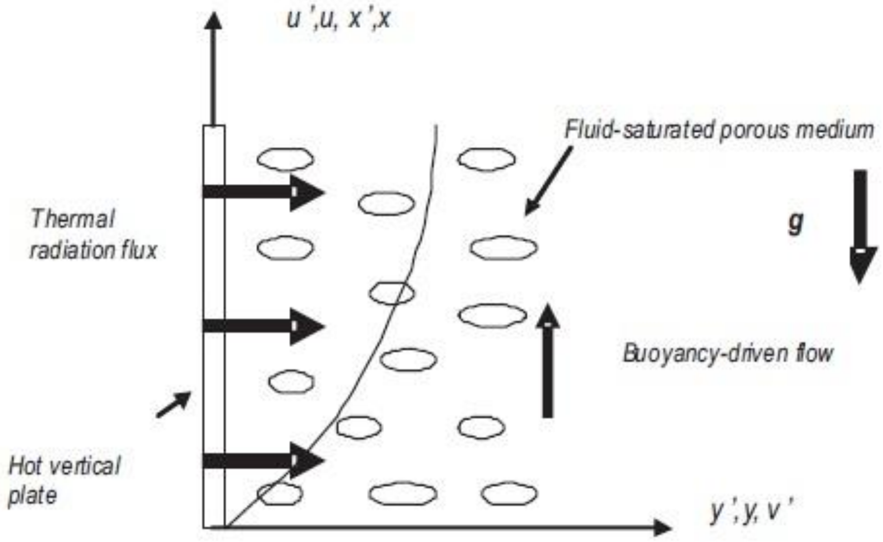

Fig. 1. Physical model and co-ordinate system.

The boundary conditions at the wall and in the free stream are:

$$\left.\begin{array}{llll} u' = 0, & T' = T'_\infty & for & y' \geq 0, \quad t' \leq 0, \\ u' = U, & T' = T'_w & for & y' = 0, \quad t' > 0, \\ u' = 0, & T' \to T'_\infty & for & y' \to \infty, \end{array}\right\} \tag{4}$$

Here $u', v', t', \nu, g, \beta, T', T'_\infty, k_1, C_p, \rho, q_r$ and K are denotes the velocity component along the plate, the velocity component normal to the plate, dimensional time, the kinematic coefficient of viscosity, The gravitational acceleration, the coefficient of thermal expansions, the temperature of the fluid, the temperature of the fluid far away from the plate(in the stream), The thermal conductivity, the specific heat at constant pressure, the density of the fluid, the radiative heat flux and the permeability of the porous medium (dimension, m^2) respectively. T'_w is the temperature at the plate U and is the velocity of the moving plate. The radiation flux on the basis of the Rosseland diffusion model for radiation heat transfer is expressed as:

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T'^4}{\partial y'}, \tag{5}$$

Where σ^* and k^* are Stefan-Boltzmann constant and the spectral mean absorption coefficient of the medium. It is assumed that the temperature differences within the flow are sufficiently small such that T'^4 may be expressed as linear function of the temperature. It can be established by expanding T'^4 in a Taylor series about T'_∞ and neglecting higher order term, that T'^4 can be expressed in the following way:

$$T'^4 = 4T'^3_\infty T' - 3T'^4_\infty \tag{6}$$

Implementing equation (5) and (6) in equation (3) we arrive at the modified energy conservation equation:

$$\frac{\partial T'}{\partial t'} = \frac{k_1}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2} + \frac{1}{\rho C_p}\frac{4\sigma^*}{3k^*}\frac{\partial^2 T'^4}{\partial y'^2} \tag{7}$$

To present solutions which are independent of the geometry of the flow regime, we introduce a series of non-dimensional transformations, expressed as:

$u = \frac{u'}{U}$ which represents dimensionless velocity, $y = \frac{y'U}{v}$ which represents dimensionless distance, $t = \frac{t'U^2}{v}$ which represents dimensionless time, $\theta = \frac{T' - T'_\infty}{T'_w - T'_\infty}$ which represents dimensionless temperature, $pr = \frac{\rho v C_p}{k_1}$ which represents prandtl number, $Gr = \frac{g\beta v(T'_w - T'_\infty)}{U^3}$ which represents Grashof number, $K_r = \frac{16\sigma^* T'^3_\infty}{3k^* k_1}$ which represents radiation-conduction parameter, and $K_p^2 = \frac{v^2}{KU^2}$ which represents inverse permeability parameter for the porous medium.

The continuity equation is satisfied and equations (2) and (7) are thereby transformed to:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + Gr\theta - K_p^2 u, \tag{9}$$

$$(1+K_r)\frac{\partial^2 \theta}{\partial y^2} - \Pr\frac{\partial \theta}{\partial t} = 0. \tag{10}$$

The corresponding boundary conditions are:

$$\left.\begin{array}{llll} u = 0, & \theta = 0 & for & y \ge 0, \ t \le 0, \\ u = 1, & \theta = 1 & for & y = 0, \ t > 0, \\ u = 0, & \theta \to 0 & for & y \to \infty, \end{array}\right\} \tag{11}$$

3. Solution of the steady state non-linear boundary value problem:

Linear and non-linear phenomena are of fundamental importance in various fields of science and engineering. Most models of real-life problems are still very difficult to solve. Therefore, approximate analytical solutions such as Homotopy perturbation method (HPM)

[8-22] were introduced. This method is the most effective and convenient ones for both linear and non-linear equations. Perturbation method is based on assuming a small parameter. The majority of non-linear problems, especially those having strong non-linearity, have no small parameters at all and the approximate solutions obtained by the perturbation methods, in most cases, are valid only for small values of the small parameter. Generally, the perturbation solutions are uniformly valid as long as a scientific system parameter is small. However, we cannot rely fully on the approximations, because there is no criterion on which the small parameter should exit. Thus it is essential to check the validity of the approximations numerically and/or experimentally. To overcome these difficulties, HPM have been proposed recently.

Many authors have applied the Homotopy perturbation method (HPM) to solve the non-linear boundary value Problem in physics and engineering science [8, 9, 15, 16]. This method is also used to solve some of the non-linear problem in Physical sciences [11-16].This method is a combination of Homotopy in topology and classic perturbation techniques. Ji-Huan He used to solve the Light hill equation [11], the Diffusion equation [12] and the Blasius equation [13]. The HPM is unique in its applicability, accuracy and efficiency. The HPM uses the imbedding parameter p as a small parameter and only a few iterations are needed to search for an asymptotic solution.

The steady state simultaneous non-linear boundary value problem of the equation (9)-(11) can be expresses are as follows:

$$\frac{d^2u}{dy^2} + Gr\theta - K_p{}^2 u = 0 \tag{12}$$

$$\left(\frac{(1+K_r)}{\Pr}\right)\frac{d^2\theta}{dy^2} = 0 \tag{13}$$

The corresponding boundary conditions are as follows:

$$u = 1, \quad \theta = 1 \qquad for \quad y = 0 \tag{14}$$

$$u = 1, \quad \theta = 1 \qquad for \quad y = 0 \tag{15}$$

Solving the equations (13)-(14) and (15), we get the dimensionless temperature θ is given by

$$\theta = e^{-\left(\frac{1+K_r}{pr}\right)y} \tag{16}$$

$$\frac{d^2u}{dy^2} + Gr\theta - K_p^2 u = 0 \tag{17}$$

$$\frac{d^2u}{dy^2} - K_p{}^2 u = -Gr\theta \tag{18}$$

$$\frac{d^2u}{dy^2} - K_p{}^2 u = -Gr\, e^{-\frac{(1+K_r)}{\Pr}y} \tag{19}$$

Now solving the equations (18) and (19)

$$u = \left\{1 + \frac{Gr\ pr}{\left[(1+K_r)^2 - K_p{}^2 pr^2\right]}\right\} e^{-K_p y} - \frac{Gr\ pr\ e^{-(1+K_r)y}}{\left[(1+K_r)^2 - K_p{}^2 pr^2\right]} \tag{20}$$

The shear stress function is given by

$$\left(\frac{du}{dy}\right)_{y=0} = (-K_p)\left(1+\frac{Gr\ pr}{\left[(1+K_r)^2 - K_p{}^2 pr^2\right]}\right) + \frac{Gr\ pr\ (1+K_r)}{(1+K_r)^2 - K_p{}^2 pr^2} \quad (21)$$

$$\left(\frac{d\theta}{dy}\right)_{y=0} = -\left(\frac{1+K_r}{\Pr}\right) \quad (22)$$

4. Results and Discussion:

Figure 1, represents schematic diagram for physical model and co-ordinate system. Figure 2-4, represents the dimensionless distance velocity $u(y)$ versus the dimensionless distance (y). From Fig.2, it is evident that when inverse permeability parameter (K_p) increases the corresponding velocity distribution decreases in some fixed values of the other dimensionless parameter $Gr, \Pr$ and K_r. From Fig.3, it is inferred that, when the dimensionless parameter radiation-conduction parameter (K_r) increases, the corresponding dimensionless velocity decreases in some fixed values of the other dimensionless parameter $Gr, \Pr$ and K_P. From Fig.4, it is observed that, when the Grashof number (Gr) increases, the corresponding dimensionless velocity distribution also increases in some fixed values for other dimensionless parameter $\Pr, K_p$ and K_r.

Figure 5 represents the dimensionless temperature profile θ versus the dimensionless velocity y. From this Fig., it is observed that, when the radiation conduction parameter (K_r) increases corresponding dimensionless profile θ decreased in some fixed other parameters. Figure 6 represents the shear stress function versus the inverse permeability parameter (K_p),from this Fig., it is evident that, when the dimensionless parameter (K_r) increase, the corresponding shear stress function also increases in some fixed values of other dimensionless parameters Gr and $\Pr$. Figure 7 represents the shear stress function versus the Grashof number (Gr). From this Fig., it is noted that, when the dimensionless parameter (K_r) increases. The corresponding shear stress function also increases in some fixed values of other dimensionless parameters $\Pr$ and K_p.

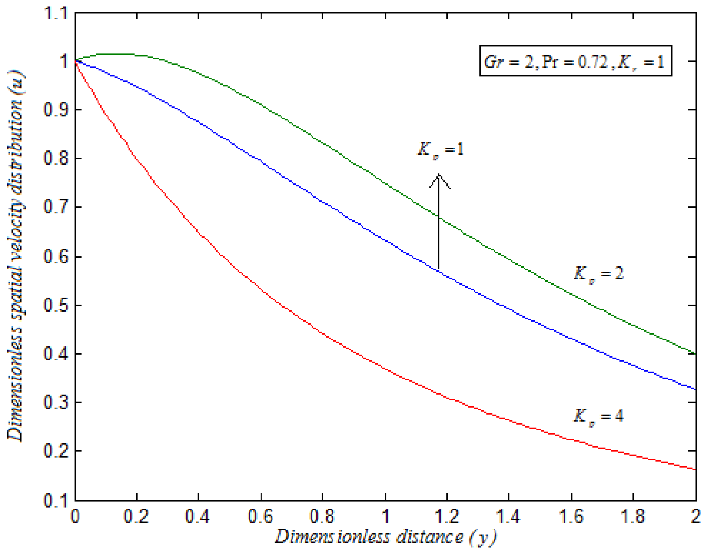

Fig.2: Dimensionless distance y versus the dimensionless spatial velocity distribution $u\ (y)$ The curves are plotted for various values of the dimensionless parameter K_p and in some fixed values of the other dimensionless parameters Gr, Pr and K_r using the eqn, (20).

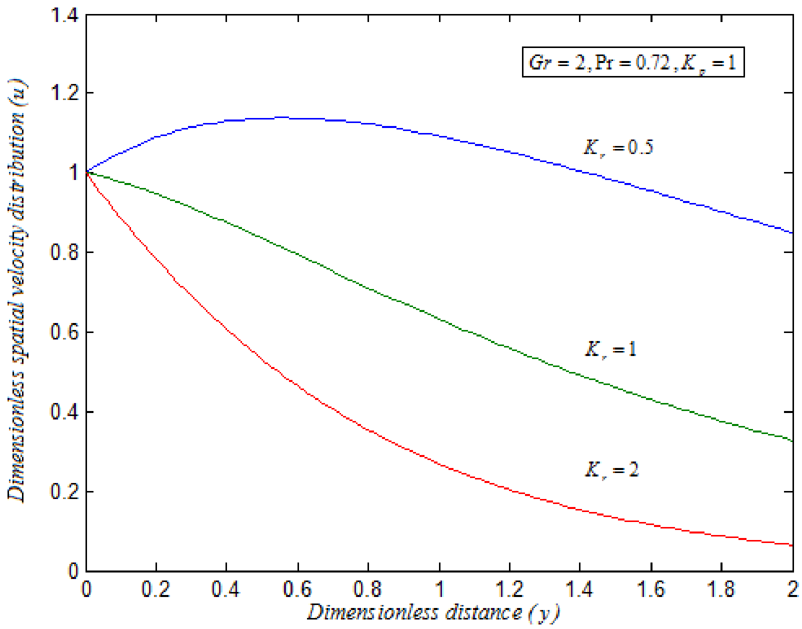

Fig.3: Dimensionless distance y versus the dimensionless spatial velocity distribution $u\ (y)$. the curves are plotted for various values of the dimensionless parameter K_r and in some fixed values of the other dimensionless parameters Gr, Pr and K_p using the eqn .(20).

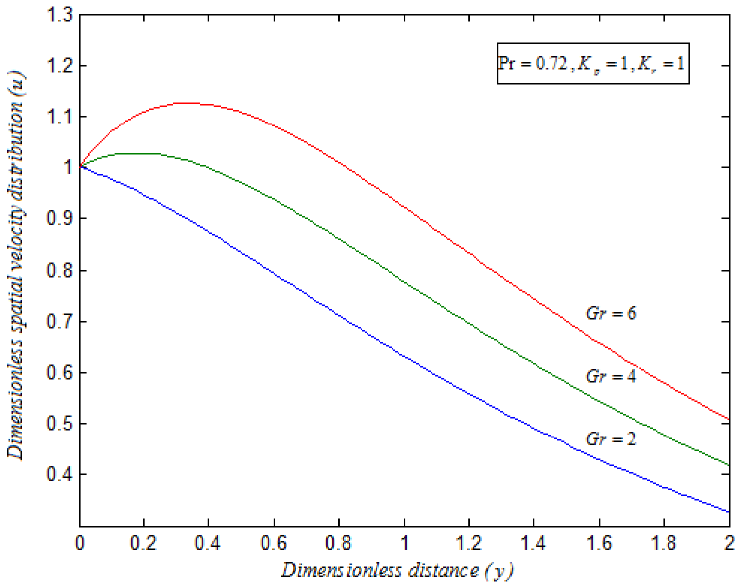

Fig.4: Dimensionless distance y versus the dimensionless spatial velocity distribution $u(y)$. The curves are plotted for various values of the dimensionless parameter Gr and in some fixed values of the other dimensionless parameters Pr, K_r and K_p using the eqn. (20).

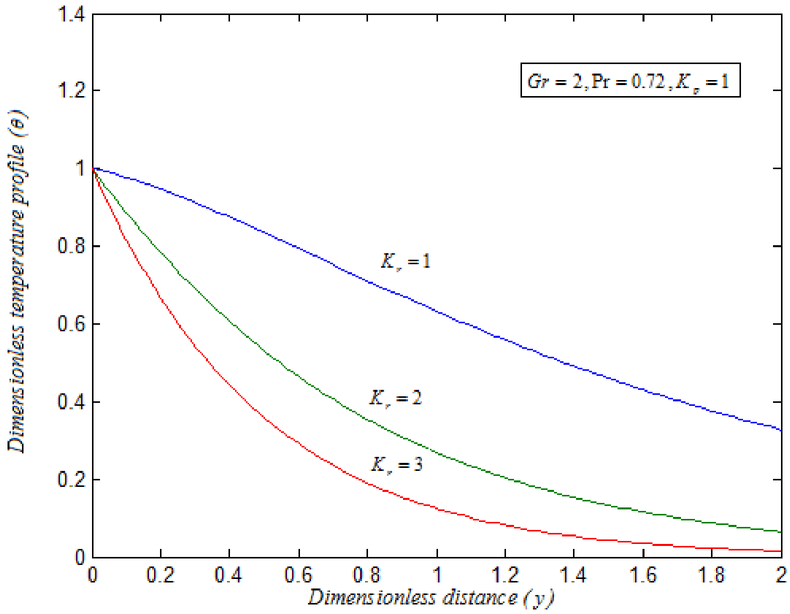

Fig.5: Dimensionless distance y versus the dimensionless temperature profile $\theta(y)$. The curves are plotted for various values of the dimensionless parameter K_r and in some fixed values of the other dimensionless parameters $Gr, \Pr$ and K_p using the eqn .(20).

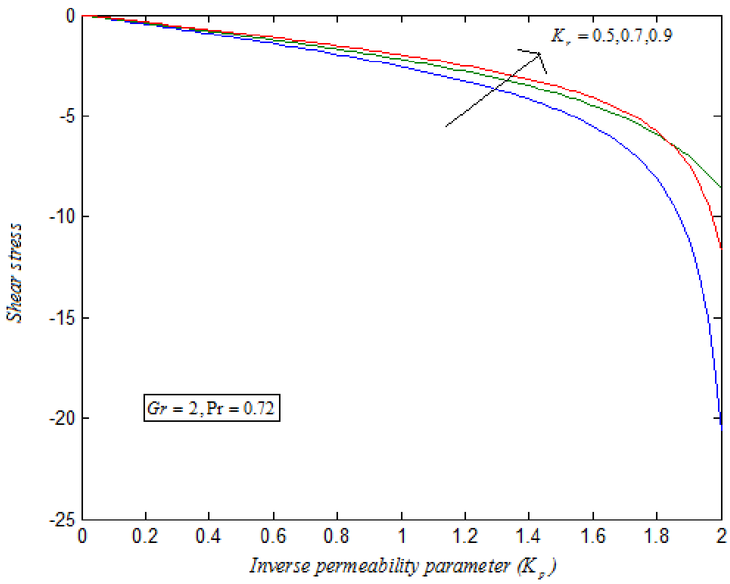

Fig.6: Inverse permeability parameter K_p versus the shear stress *of* K_p. the curves are plotted for various values of the dimensionless parameter K_r and in some fixed values of the other dimensionless parameters Gr and $\Pr$ using the eqn .(21).

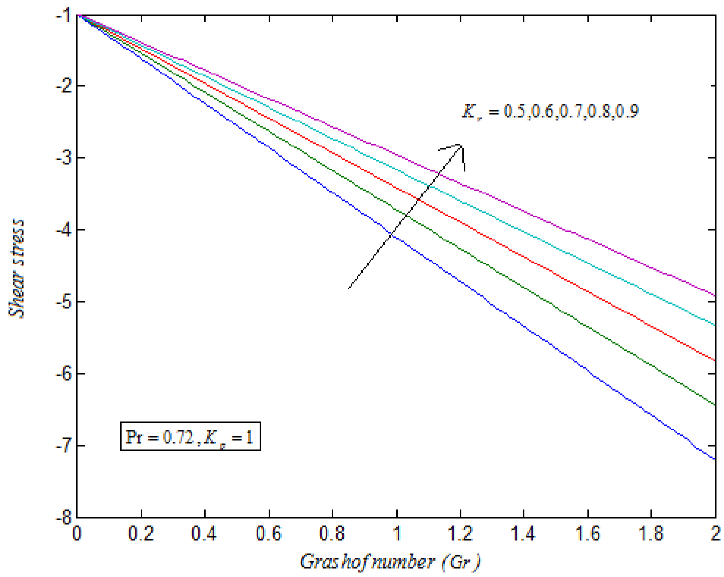

Fig.7: Grashof number Gr versus the shear stress $of \ Gr$. The curves are plotted for various values of the dimensionless parameter K_r and in some fixed values of the other dimensionless parameters Pr and K_p using the eqn.(21).

5. Conclusion:

The mathematical analysis for the steady state convection heat transfer from a vertical translating plate adjacent to a Darien porous medium in the presence of significant thermal radiation has been investigated. The approximate analytical expressions of the steady state dimensionless velocity profile and dimensionless temperature profile has been derived by using the direction integration. The analytical expressions of shear stress function and the temperature gradient are also derived by using the analytical expressions for the dimensionless velocity and dimensionless temperature.

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Appendix: A

Nomenclature:

Symbol	Meaning
U	Dimensionless velocity
y	Non-dimensional distance
t	Dimensionless time
θ	Dimensionless time
Pr	Prandtl number

Gr	Grashof number
K_r	Radiation-conduction parameter
$K_p{}^2$	Inverse permeability parameter for the porous medium