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ON PRIME LABELING OF THETA GRAPH

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Abstract:

A graph G with vertex set V is said to have a prime labeling if its vertices are labeled with distinct integers $1, 2, 3, \dots |V|$ *such that for each edge xy the labels assigned to x and y are relatively prime. A graph which admits prime labeling is called a prime graph. In this paper; we investigate prime labeling of Theta graph. We also discuss prime labeling in the context of some graph operations namely Fusion, Duplication, Switching and Path union*

Key Words: Prime Labeling, Fusion, Duplication, Switching & Path union

Introduction:

In this paper, we consider only finite simple undirected graph. The graph G has vertex set $V = V(G)$ and the edge set $E = E(G)$. The set of vertices adjacent to a vertex u of G is denoted by $N(u)$. For notations and terminology we refer to J.A. Bondy and U.S.R. Murthy [1]. In the present work. T_a denotes the Theta graph with 7 vertices and 8 edges. We give brief summary of definitions which are useful for the present investigation. Enough literatures available in printed as well as electronics form on different types of graph labeling and more than 1000 research papers have been published so far in past four decades. A current survey of various graphs labeling problem can be found in [7] (Gallian J, 2009)

Following are the common features of any graph labeling problem.

- ✓ A set of numbers from which vertex labels are assigned.
- ✓ A rule that assigns value to each edge.
- ✓ A condition that these values must satisfy.

The notion of prime labeling was introduced by Roger Entringer and was discussed in a paper by A. Tout (1982 P 365-368) [2]. Many researchers have studied prime graph for example in H.C. Fu (1994 P 181-186) [5] have proved that path P_n on n vertices is a prime graph.

T.O. Dertsky (1991 P 359-369) [4] have proved that the cycle C_n on n vertices is a prime graph. S.M. Lee (1998 P 59 -67) [3] have proved that wheel W_n is a prime graph iff n is even. Around 1980 Roger Entringer conjectured that all tress have prime labeling, which is not settled till today. The prime labeling for planner grid is investigated by M. Sundaram (2006 P205-209) [6]. In [8] S.K. Vaidhya and K.K. Kanmani) have proved that the prime labeling for some cycle related graphs. In [9] S. Meena and K. Vaithilingam, Prime Labeling for some Helm related graphs. We will provide brief summary of definitions and other information which are necessary for the the present investigations.

Definition 1: If the vertices of the graph are assigned values subject to certain conditions then it is known as (vertex) graph labeling.

Definition 2: Let $G = (V(G), E(G))$ be a graph with n vertices. A bijection $f: V(G) \to \{1, 2, \dots n\}$ is called a Prime labeling if for each edge $e = uv$, $gcd(f(u), f(v)) = 1$. A graph which admits prime labeling is called a prime graph.

Definition 3: An independent set of vertices in a graph G is a set of mutually non-adjacent vertices.

Definition 4: A Theta graph is a block with two non-adjacent vertices of degree 3 and all other vertices of degree 2 is called a Theta graph.

Definition 5: Let u and v be two distinct vertices of a graph G. A new graph G_1 is constructed by fusing (identifying) two vertices u and v by a single vertex x in G_1 such that every edge which was incident with either u (or) v in G now incident with x in G_1.

Definition 6: Duplication of a vertex v_k of a graph G produces a new graph G_1 by adding a vertex $v_k^{'}$ with $N(v_k^{'}) = N(v_k)$. In other words, a vertex $v_k^{'}$ is said to be a duplication of v_k if all the vertices which are adjacent to v_k are now adjacent to $v_k^{'}$.

Definition 7: A vertex switching G_v of a graph G is obtained by taking a vertex v of G, removing the entire edges incident with v and adding edges joining v to every vertex which are not adjacent to v in G.

Definition 8: Let $G_{1,}G_2, G_3, \dots G_n,\ n \geq 2$ be n copies of a fixed graph G. The graph obtained by adding an edge between G_i and G_{i+1} for $i = 1, 2, \dots n-1$ is called the path union of G.

Proposition 1:

The Theta graph T_a is a prime graph

Proof:

Let T_a be a Theta graph with centre v_0.

$$V(T_a) = \{v_0, v_1, v_2, \dots \dots . v_6\}$$
$$E(T_a) = \{v_i v_{i+1} / 1 \leq i \leq 5\} \cup \{v_0 v_1, v_0 v_4\} \cup \{v_1 v_6\}$$

Then $|V(T_a)| = 7$ and $|E(T_a)| = 8$

Define a label $f: V(T_a) \to \{1, 2, 3 \dots \dots . .7\}$

Such that $f(v_0) = 7$ where v_0 is the centre vertex of T_a.

$f(v_i) = i$ for $1 \leq i \leq 6$

(i.e.) $f(v_1) = 1$

$f(v_2) = 2$

$f(v_3) = 3$

$f(v_4) = 4$

$f(v_5) = 5$

and $f(v_6) = 6$

For each edge, $e = v_i v_{i+1} \in T_a$ for $1 \leq i \leq 5$

$\gcd\left(f(v_i), f(v_j)\right) = 1$ and for the edge $v_0 v_1, v_0 v_4 \in T_a$

$\gcd(f(v_0), f(v_1)) = 1$ and $\gcd(f(v_0), f(v_4)) = 1$

Hence T_a admits prime labeling.

Example:

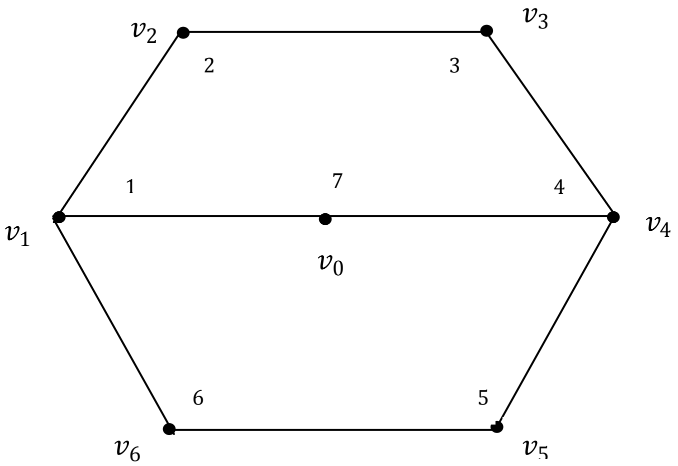


Figure 1: Theta graph T_a is a prime graph

Proposition 2:

The Fusion of any two vertices in the cycle of T_a is a prime graph.

Proof:

Let T_a be a Theta graph with centre v_0.

$V(T_a) = \{v_0, v_1, v_2, \dots \dots . v_6\}$

$E(T_a) = \{v_i v_{i+1} / 1 \leq i \leq 5\} \cup \{v_0 v_1, v_0 v_4\} \cup \{v_1 v_6\}$

Then $|V(T_a)| = 7$ and $|E(T_a)| = 8$.

Let G be a graph obtained by fusion of two vertices v_i and v_{i+1} in the cycle of T_a.

Then $|V(G)| = 6$

Define a label $f: V(G) \to \{1, 2, \dots .6\}$

such that $f(v_0) = 6$ where v_0 be the centre vertex

$f(v_1) = 1$

$f(v_4) = 5$

$f(v_2) = 2$ where v_2 and v_3 are the are fussed by single vertex v .

(i.e.) $v_2 = v_3 = v$.

and $f(v_5) = 3$

Finally, $f(v_6) = 4$ For each edge $e = v_i v_j \in G$, $\gcd\left(f(v_i), f(v_j)\right) = 1$

Hence G admits prime labeling.

Example:

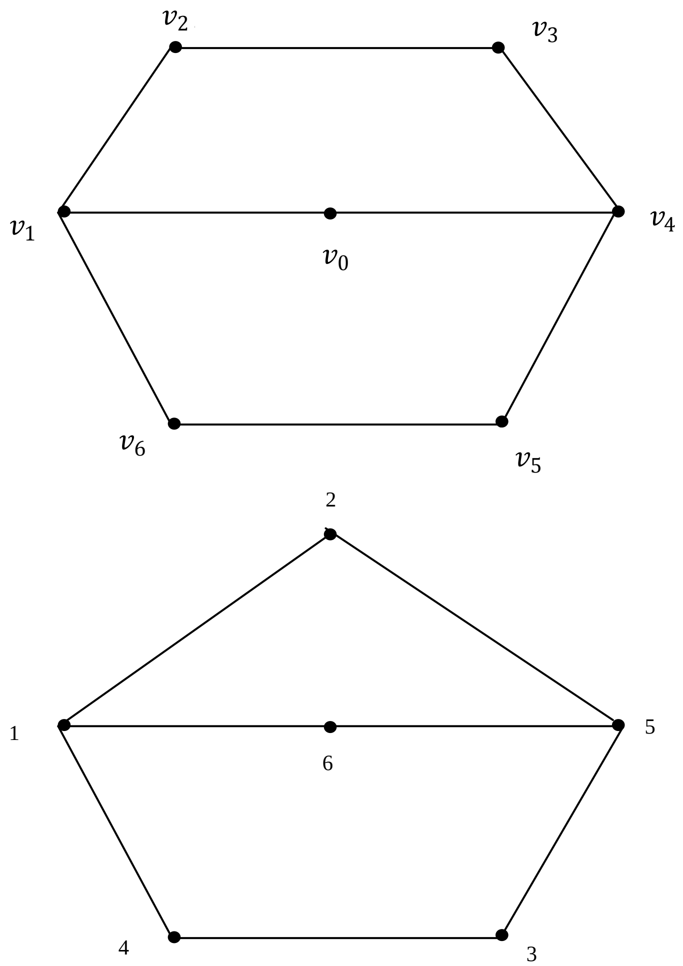


Figure 2: The fusion of v_2 and v_4 in T_a is a prime graph

Proposition 3:

The Duplication of any vertex v_i in the cycle of T_a is a prime graph.

Proof:

Let T_a be a Theta graph with centre v_0

$$V(T_a) = \{v_0, v_1, v_2, \dots \dots . v_6\}$$

and $$E(T_a) = \{v_i v_{i+1} / 1 \le i \le 5\} \cup \{v_0 v_1, v_0 v_4\} \cup \{v_1 v_6\}$$

Then $|V(T_a)| = 7$ and $|E(T_a)| = 8$.

Let G_i be a graph obtained from T_a after duplication vertex of the vertex v_i in T_a and $v_i^{'}$ be the duplication vertex of the vertex v_i in T_a. Clearly $|V(G_i)| = 8$.

Define a label $f: V(G_i) \to \{1, 2, \dots \dots 8\}$, Such that $f(v_0) = 7$,

$f(v_4^{'}) = 8$ where $v_4^{'}$ is the duplicating vertex of v_4

$f(v_i) = i$ for $1 \le i \le 6$

(i.e) $f(v_1) = 1$

$f(v_2) = 2$

$f(v_3) = 3$

$f(v_4) = 4$

$f(v_5) = 5$

Finally, $f(v_6) = 6$

Now, for each edge $e = v_i v_j \in G_i$, $\gcd(f(v_i), f(v_j)) = 1$,

The graph G_i admits prime labeling.

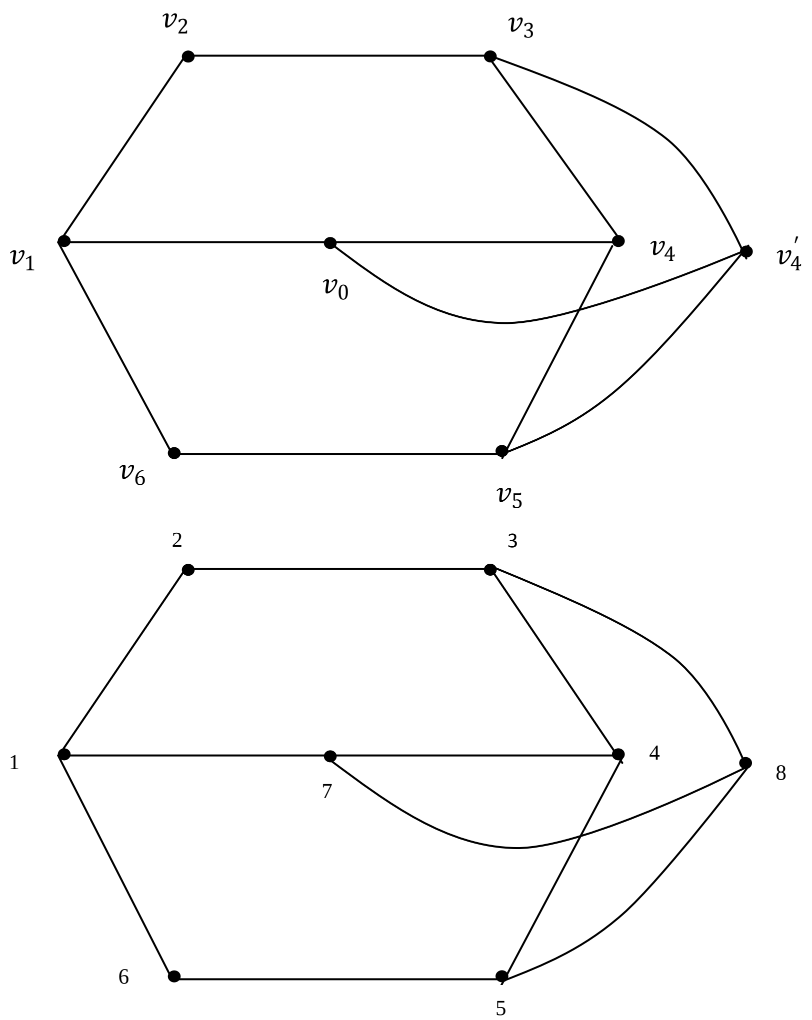


Figure 3: The Duplication of the vertex v_4 in T_a is a prime graph

Proposition 4:

The switching of arbitrary vertex in the Theta graph T_a is a prime graph.

Proof:

Let T_a be a Theta graph with centre v_0.

$V(T_a) = \{v_0, v_1, v_2, \ldots \ldots . v_6\}$

And $E(T_a) = \{v_i v_{i+1} / 1 \leq i \leq 5\} \cup \{v_0 v_1, v_0 v_4\} \cup \{v_1 v_6\}$

Then $|V(T_a)| = 7$ and $|E(T_a)| = 8$

Let G_s be the graph obtained from T_a after switching the centre vertex v_0 of T_a.

Define a label $f: V(G_s) \rightarrow \{1, 2, \ldots \ldots 7\}$

Such that $f(v_0) = 7$ where v_o is the switching vertex.

$f(v_i) = i$ for $1 \leq i \leq 6$

(i.e) $f(v_1) = 1$

$f(v_2) = 2$

$f(v_3) = 3$

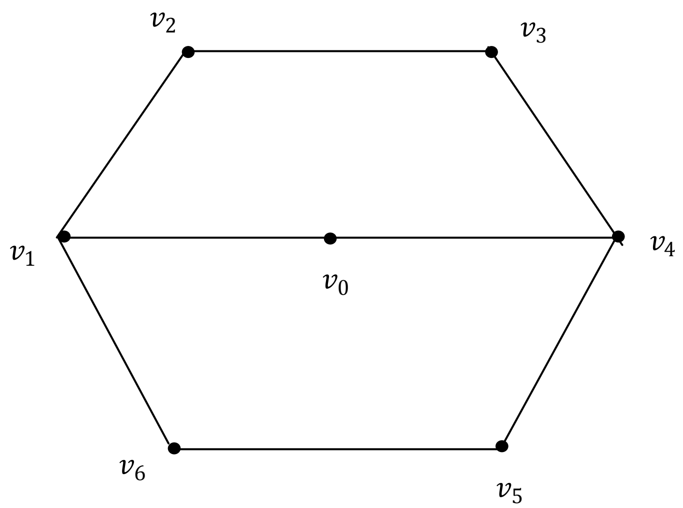
$f(v_4) = 4$

$f(v_5) = 5$

Finally, $f(v_6) = 6$

Clearly, for each edge $e = v_i v_j \in G_s$, $\gcd\left(f(v_i), f(v_j)\right) = 1$

$\therefore$ The graph G_s admits prime labeling.



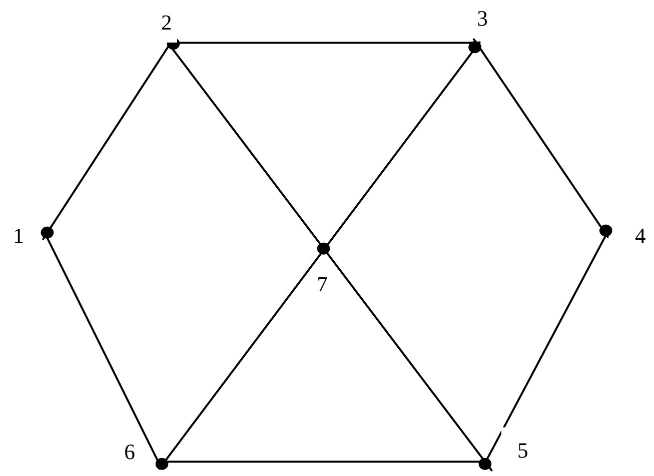


Figure 4: The switching of the centre vertex v_0 in T_a is a prime graph

Proposition 5:

The graph obtained by path union of two pieces of Theta graph T_a is a prime graph.

Proof:

Consider, two copies of Theta graphs T_a and T_a^* respectively.

Then $V(T_a) = \{u_0, u_1, u_2, u_3, u_4, u_5, u_6\}$

$E(T_a) = \{u_i u_{i+1} / 1 \leq i \leq 5\} \cup \{u_0 u_1, u_0 u_4\} \cup \{v_0 v_6\}$

And $V(T_a^*) = \{v_0, v_1, v_2, \ldots . v_6\}$

$E(T_a^*) = \{v_i v_{i+1} / 1 \leq i \leq 5\} \cup \{v_0 v_1, v_0 v_4\} \cup \{v_0 v_6\}$

Let G_k be the graph obtained by the path union of two pieces of Theta graphs T_a and T_a^*. $V(G_k) = V(T_a) \cup V(T_a^*)$

$E(G_k) = E(T_a) \cup E(T_a^*) \cup \{u_k v_k\}$

Then $|V(G_k)| = 14$

We assign the labels $1, 2, \ldots \ldots \ldots 6, 7$ for T_a and $9, 10, \ldots \ldots \ldots 15$ for T_a^*

Define a label $f: V(G_k) \rightarrow \{1, 2, \ldots \ldots 7,\ 9, 10, 11 \ldots \ldots .15\}$

Labeling the Theta graph T_a

$f(u_0) = 7$

$f(u_i) = i$ for $1 \leq i \leq 6$

$f(u_1) = 1$

$f(u_2) = 2$

$f(u_3) = 3$

$f(u_4) = 4$

$f(u_5) = 5$

$f(u_6) = 6$

Labeling the Theta graph T_a^*

$f(v_0) = 15$

$f(v_1) = 11$

$f(v_2) = 10$

$f(v_3) = 9$

$f(v_4) = 14$, v_4 is adjacent to v_0

$f(v_5) = 13$

$f(v_6) = 12$

Then for each edge $e = u_i u_j \in T_a$ and for the edge $e = v_i v_j \in T_a^*$ and $u_k v_k \in G_k$

$\gcd\left(f(u_i), f(u_j)\right) = 1$; $\gcd(f(v_i), f(v_j)) = 1$ and $\gcd(f(u_k), f(v_k)) = 1$.

Hence G_k admits prime labeling

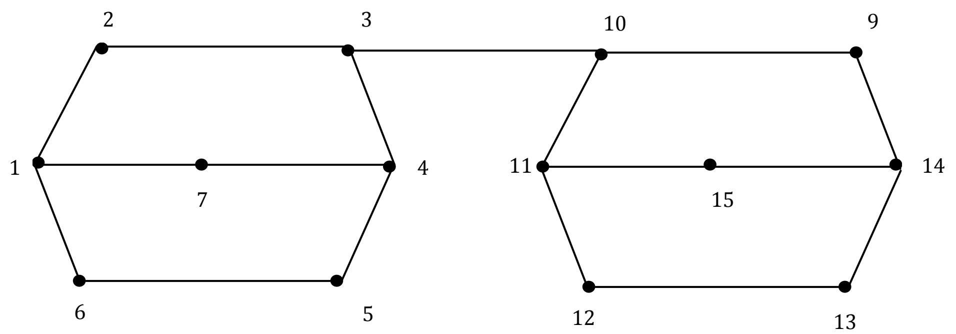


Figure 5: The path union of T_a and T_a^* is a prime graph

Conclusion:

Here, we have investigated five results corresponding to prime labeling on some special graph, namely Theta graph. Analogous work can be carried out for other families and in the context of different types of graph labeling techniques.

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