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G_δ-COMPACTNESS ON FUZZY CHAOTIC STRUCTURE SPACES

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Abstract:

The purpose of this paper is to introduce the concepts of fuzzy orbit open set, fuzzy periodic open set, fuzzy chaotic set, fuzzy chaos space, fuzzy chaotic structure space,fuzzy chaotic G_δ-compact spaces, fuzzy chaotic G_δ-Lindelöf spaces with some interesting properties are established.

Key Words: Fuzzy Orbit Open Set, Fuzzy Periodic Open Set, Fuzzy Chaotic Set, Fuzzy Chaos Space, Fuzzy Chaotic Structure Space & Fuzzy Chaotic G_δ-Lindelöf spaces.

1. Introduction:

The concept of fuzzy sets was introduced by Zadeh [6] in 1965. Since then fuzzy sets have applications in many fields such as information [4] and control [5]. Subsequently, Chang [2] defined the notion of fuzzy topological space in 1980. The concept of chaotic function in general metric space was introduced by R. L. Devaney [3]. It has many applications in traffic forecasting, animation, computer graphics, medical field, image processing, etc. In 1991, Bin Shahna [1] defined fuzzy compact spaces. In this paper, fuzzy chaos space and fuzzy chaotic structure space are introduced. The concepts of fuzzy orbit open set, fuzzy periodic open set, fuzzy chaotic set are also introduced.The concepts of fuzzy chaotic G_δ-compact spaces, fuzzy chaotic G_δ-Lindelöf spaces and fuzzy chaotic G_δ-normal spaces are introduced. Some interesting properties of these spaces are also discussed.

2. Preliminaries:

Definition 2.1 [6] A fuzzy set in X is a function with domain X and values in I,that is an element of I^X.

Definition 2.5 [2] Let $T \subset I^X$ satisfying the following conditions:

(i) $0, 1 \in T$,

(ii) if $\mu_1, \mu_2 \in T$, then $\mu_1 \wedge \mu_2 \in T$,

(iii) if$\{ \mu_j : j \in J\} \subset T$, then $\vee_{j\in J}\ \mu_j \in T$.

T is called a fuzzy topology on X and (X, T) a fuzzy topological space (or f. t. s.).The elements of T are called fuzzy open sets. A fuzzy setv is called fuzzy closed set if $1 — v \in T$. We denote T^c the collection of all fuzzy closed sets in this fuzzy topologicalspace.

Definition 2.7 [5] Let (X, T) be a fuzzy topological space and $\lambda \in I^X$is called fuzzydense in X if there exists no fuzzy closed set μ in (X, T) such that $\lambda < \mu < 1$. That iscl(λ) = 1.

Definition 2.8 [2] Let f: (X, T) → (Y, U) be a mapping from a fuzzy topologicalspace X to fuzzy topological space Y. f is called fuzzy continuous if the inverseimageof every fuzzy closed set in (Y, S) is fuzzy closed in (X, T).

Definition 2.12 [3] Orbit of a point x in X under the mapping f is$O_f(x) = \{x, f(x), f^2(x),\dots\}$.

Definition 2.13 [3] x in X is called a periodic point of f if $f^n(x) = x$, for somen $\in Z_+$. Smallest of these n is called period of x.

Definition 2.14 [3] f is sensitive if for each δ>0 there exists

(i) $\varepsilon > 0$,

(ii) $y \in X$,

(iii) $n \in Z_+$such that $d(x, y) < \delta$ and $d(f^n(x), f^n(y)) > \varepsilon$.

Notation 2.1 Let $F \subseteq X$ and S(F) = {f| f is sensitive onF}.

Definition 2.15 [3] Let (X, τ) be a topological space and F∈ K(X). Let f : F → Fbe continuous. Thenf is chaotic on F if

(i) $O_f(x) = F$, for some $x \in F$

(ii) periodic points of f are dense in F and

(iii) f∈ S(F).

Notation 2.2 (i) C(F) = {f : F → F \ f is chaotic on F},

(ii) CH(X) = {f : K(X) — C(F) $\neq \phi$}, where K(X) is a collection of all nonzero fuzzy compact subsets of X.

3. Fuzzy Chaotic G_δ-Compact Spaces:

Definition 3.1 Let X be a nonempty set and let f : X→ X be any mapping. Let λ be any fuzzy set in X. The fuzzy orbit $O_f(\lambda)$ of λ under the mapping f is defined as $O_f(\lambda) = \{\lambda, f(\lambda).\ f^2(\lambda),..\}$.

Definition 3.2 Let X be a nonempty set and let f : X→ X be any mapping. The fuzzy orbit set of λ under the mapping f is defined as $FO_f(\lambda) = \{\lambda \wedge f(\lambda) \wedge f^2(\lambda) \wedge\dots\}$ the intersection of all members of $O_f(\lambda)$.

Example 3.1 Let X = {a, b, c}. Define a fuzzy orbit set λ: X → [0,1] as follows λ(a) = 0.5, λ(b) = 0.6, λ(c) = 0.7. Define f : X → X as f (a) = b, f(b) = c, f(c) = a. The fuzzy orbit set of   under the the mapping f is defined as $FO_f(\lambda) = \lambda \wedge f(\lambda) \wedge f^2(\lambda) \wedge\dots$ $FO_f(\lambda)(a) = 0.5$, $FO_f(\lambda)(b) = 0.5$, $FO_f(\lambda)(c) = 0.5$.

Definition 3.3 Let (X, T) be a fuzzy topological space. Let f : X → X be any mapping. The fuzzy orbit set under the mapping f which is in fuzzy topology T is called fuzzy orbit open set under the mapping f. Its complement is called a fuzzy orbit closed set under the mapping f.

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Example 3.2 Let X = {a, b, c}. Define T = {0, 1, λ, Υ} where λ, Υ: X → [0,1] are defined as λ(a) = 0.3, λ (b) = 0.3, λ (c) = 0.1, Υ(a) = 0.3, Υ(b) = 0, Υ(c) = 0. Define f : X → X as f(a) = a, f(b) = a, f(c) = a. The fuzzy orbit set of λ under the mapping f is defined as $FO_f(\lambda) = \lambda \wedge f(\lambda) \wedge f^2(\lambda) \wedge \ldots$. $FO_f(\lambda) = \Upsilon$. Therefore Υ is a fuzzy orbit open set under the mapping f.

Definition 3.4 Let X be a nonempty set and let f : X → X be any mapping. Then a fuzzy set of X is called fuzzy periodic set with respect to f if $f^n(\Upsilon) = \Upsilon$, for some n ∈ Z+. smallest of these n is called fuzzy periodic of X.

Definition 3.5 Let (X, T) be a fuzzy topological space. Let f : X → X be any mapping. The fuzzy periodic set with respect to f which is in fuzzy topology T is called fuzzy periodic open set with respect to f. Its complement is called a fuzzy periodic closed set with respect to f.

Notation 3.1 P = ∧{fuzzy periodic open sets with respect to f }.

Definition 3.6 Let (X, T) be a fuzzy topological space and λ ∈ KF(X) (Where KF(X) is a collection of all nonempty fuzzy compact subsets of X). Let f : X → X be any mapping. Then f is fuzzy chaotic with respect to λ if

(i) cl $FO_f(\lambda) = 1$,

(ii) P is fuzzy dense.

Example 3.3 Let X = {a, b, c}. Define T = {0, 1, μ_1, μ_2, μ_3, μ_4} where $\mu_1, \mu_2, \mu_3, \mu_4$: X → [0,1] are defined as μ_1 (a) =0.4, μ_1 (b) = 0.8, μ_1 (c) = 0.4, μ_2 (a) = 0.4, μ_2(b) = 0.8, μ_2 (c) = 0.5, μ_3 (a) = 0.8, μ_3 (b) = 0.8, μ_3 (c) = 0.6, μ_4 (a) = 0.9, μ_4 (b) = 0.8, μ_4 (c) = 0.9. Letλ: X → I be defined as λ (a) = 0.3, (b) = 0.6, λ(c) = 0.3. Define f : X → X as f(a) = b, f(b) = c, f(c) = a. The fuzzy orbit set of λ under the mapping f is defined as $FO_f(\lambda) = \lambda \wedge f(\lambda) \wedge f^2(\lambda) \wedge \ldots$

$FO_f(\lambda)$ (a) = 0.3, $FO_f(\lambda)$(b) = 0.3, $FO_f(\lambda)$ (c) = 0.3. Therefore cl $FO_f(\lambda) = 1$. Here P(a) = 0.4, P(b) = 0.8, P(c) = 0.4 and cl (P) is fuzzy dense. Hence f is fuzzy chaotic with respect to λ.

Notation 3.2 (i) FC (λ) = {f: X → X / f is fuzzy chaotic with respect to λ}.

(ii) FCH(λ) = {λ∈ KF(X) / FC(λ) ≠ ϕ}.

Definition 3.7 A fuzzy topological space (X, T) is called a fuzzy chaos space if FCH (λ) ≠ ϕ. If (X, T) is fuzzy chaos space then the element of the FCH(X) are called chaotic sets in X.

Definition 3.8 Let (X, T) be a fuzzy chaos space. Let C be the collection of fuzzy chaotic sets in X satisfying the following conditions:

(i) 0, 1 ∈C,

(ii) if $\mu_1, \mu_2 \in$ C, then $\mu_1 \wedge \mu_2$ ∈C,

(iii) if{ μ_j : j ∈ J} ⊂$\mathfrak{C}$, then $\vee_{j\in J}\ \mu_j$ ∈C

Then C is called the fuzzy chaotic structure in X. The triple (X, T, C) is called fuzzy chaotic structure space. The elements of $\mathfrak{C}$ are called fuzzy open chaotic sets. The complement of fuzzy open chaotic set is called fuzzy closed chaotic set.

Definition 3.9 Let (X, T, C) be a fuzzy chaotic structure space and let λ be any fuzzy open chaotic set in (X, T, C). We define

(i) CInt (λ) = {μ : μ ≤ λ, μ is a fuzzy open chaotic set} is called fuzzy chaotic interior of λ.

(ii) CCl(λ) = {μ : μ ≥ λ, μ is a fuzzy closed chaotic set} is called fuzzy chaotic closure of λ.

Definition 3.10 Let (X, T, C) be a fuzzy chaotic structure space and let λ be any fuzzy open chaotic set in X. λ is called a fuzzy chaotic G_δ-set if $\lambda = \wedge_{i=1}^{\infty} \lambda_i$ where λ_i are fuzzy open chaotic sets, for i = 1 to ∞.

Definition 3.11 Let (X, T, C) be a fuzzy chaotic structure space and let λ be any fuzzy open chaotic set in X. λ is called a fuzzy chaotic F_σ-set if $\lambda = \vee_{i=1}^{\infty} \lambda_i$ where λ_i are fuzzy open chaotic sets, for i = 1 to ∞.

Definition 3.12 Let λ be any fuzzy set in the fuzzy chaotic structure space (X, T, C). Then we define

$CCl_\sigma(\lambda)$ = fuzzy chaotic σ-closure of λ.

= the smallest fuzzy chaotic F_σ-set containing λ.

$CInt_\sigma(\lambda)$= fuzzy chaotic σ-interior of λ.

= the greatest fuzzy chaotic G_δ-set contained in λ.

Definition 3.13 Let (X, T, C_1) and (Y, S, C_2) be fuzzy chaotic structure spaces. Let f: (X, T, C_1) → (Y, S, C_2) be a function.f is called M-fuzzy chaotic G_δ-continuous if the inverse image of every fuzzy chaotic G_δ-set in (Y, S, C_2) is fuzzy chaotic G_δ in (X, T, C_1).

Definition 3.14 Let (X, T, C_1) and (Y, S, C_2) be **f**uzzy chaotic structure spaces. Let f: (X, T, C_1) → (Y, S, C_2) be a function.f is called M-fuzzy chaotic F_σ-map if for every fuzzy chaotic F_σ-set λ in (X, T, C_1), f(λ) is fuzzy chaotic F_σ-set λ in (Y, S, C_2).

Definition 3.15 A fuzzy chaotic structure space (X, T,C) is called fuzzy chaotic G_δ-space if every fuzzy chaotic G_δ-set of (X, T,C) is fuzzy open chaotic.

Remark 3.1 Let (X, T, C_1) and (Y, S, C_2) be fuzzy chaotic structure spaces.f: (X, T, C_1) → (Y, S, C_2) is M-fuzzy chaotic G_δ-continuous ⇔ for every fuzzy chaotic F_σ-set λ of (Y, S, C_2),$f^{-1}(\lambda)$ is fuzzy chaotic $F_\sigma$set in (X, T,C_1).

Proposition 3.1 For a fuzzy open chaotic set λof afuzzy chaotic structure space (X, T, C) the following hold.

(a) $1 - CCl_\sigma(\lambda) = CInt_\sigma(1 - \lambda)$.

(b) $1 - CInt_\sigma(\lambda) = CCl_\sigma(1 - \lambda)$.

Proposition 3.2 Let (X, T, C_1) and (Y, S, C_2) be a fuzzy chaotic structure spaces. Let f: (X, T, C_1)→ (Y, S, C_2) be fuzzy open chaotic and injective. Then f is fuzzy chaotic G_δ-map.

Proof: Suppose λ is fuzzy chaotic $G_\delta$in (X, T, C_1). Then $\lambda = \wedge_{i=1}^{\infty} \boldsymbol{\lambda_i}$ where each λ_i (i ∈ I) is fuzzy open chaotic in (X, T, C).Now by hypothesis on f, we have $f(\lambda) = f(\wedge_{i=1}^{\infty} \boldsymbol{\lambda_i}) = \wedge_{i=1}^{\infty} \mathbf{f}(\boldsymbol{\lambda_i})$which implies that f(λ) is fuzzy chaotic G_δ.

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Proposition 3.3 Let (X, T, C_1) and (Y, S, C_2) be fuzzy chaotic structure spaces. Let f: (X, T, C_1) → (Y, S, C_2) befuzzy continuous and injective. Then f is M-fuzzy chaotic G_δ- continuous.

Proposition 3.4 Let (X, T, C_1), (Y, S, C_2) and (Z, R,C_3)be any three fuzzy chaotic structure spaces. Suppose f :(X, T, C_1)→(Y, S, C_2)and g : (Y, S, C_2)→(Z, R,C_3) are both M-fuzzy chaotic G_δ-continuous. Then g ∘f :(X, T, C_1)→(Z, R,C_3) is M-fuzzy chaotic G_δ-continuous.

Notation: Let (X, T, C_1) be a fuzzy chaotic structure space P_{2X} denotes the projection of X × Z onto Z, where (Z, R, C3) is any fuzzy chaotic structure space.

Definition 3.16A fuzzy chaotic structure space(X, T, C_1) is said to be fuzzy chaotic G_δ-compact if the projection p_{2X}: X × Z → Z is fuzzy chaotic F_σ for any fuzzy chaotic structure space(Z, R,C_3).

Proposition 3.5A M-fuzzy chaotic G_δ-continuous image of a fuzzy chaotic G_δ-compact space is fuzzy chaotic G_δ-compact.

Definition 3.17 Let (X, T, C_1) and (Y, S, C_2) be any two fuzzy chaotic structure spaces. A M-fuzzy chaotic G_δ-continuous map f :(X, T, C_1)→(Y, S, C_2) is fuzzy chaotic G_δ-perfect iff f × I_Z is fuzzy chaotic F_σ for every fuzzy chaotic structure space (Z, R,C_3).

Proposition 3.6 If f : (X, T, C_1)→(Y, S, C_2) is fuzzy chaotic G_δ-perfect mapping of a fuzzy chaotic structure space (X, T, C_1) onto a fuzzy chaotic G_δ-compact space (Y, S, C_2), then (X, T, C_1) is fuzzy chaotic G_δ-compact.

Proof: Since f is fuzzy chaotic G_δ-perfect, f × I_Z : X × Z → Y × Z is fuzzy chaotic F_σ for any fuzzy chaotic structure space (Z, R,C_3). For (X, T, C_1) to be fuzzy chaotic G_δ-compact we show that p_{2X}: X × Z →Z is upper fuzzy chaotic F_σ. Noting that p_{2X} is the composition of two fuzzy chaotic F_σ-mappings f × $I_Z$and p_{2Y}: Y × Z →Zthe result follows.

Definition 3.18 Let (X, T, C_1)and (Y, S, C_2) be any two fuzzy chaotic structure spaces. (X, T, C_1) and (Y, S, C_2) are said to be fuzzy chaotic G_δ-homeomorphiciff there exists f :(X, T, C_1)→(Y, S, C_2) such that f is one-to-one, onto, M-fuzzy chaotic G_δ-continuous and fuzzy chaotic G_δ. Such an f is called M-fuzzy chaotic G_δ-homeomorphism.

Proposition 3.7 Let (X, T, C_1)and (Y, S, C_2) be any two fuzzy chaotic structure spaces. If f :(X, T, C_1)→(Y, S, C_2) is fuzzy chaotic G_δ-homeomorphism, then f($CCl_\sigma(\lambda)$) = CCl_σ(f(λ)).

Proposition 3.8 Let (X, T, C_1) be afuzzy chaotic structure space and (Y, S, C_2)be a fuzzy chaotic G_δ-compact space. Then the projection p : X × Y → X is fuzzy chaotic G_δ-perfect mapping.

Proof: We show that p × I_Z : (X × Y) × Z → X × Z is a fuzzy chaotic F_σ-mapping for any fuzzy chaotic structure space (Z, R,C_3). Since (Y, S, C_2)is fuzzy chaotic G_δ-compact,p_{2Y}: Y × (X × Z) → X × Z is fuzzy chaotic F_σ. That is, p × I_Z is fuzzy chaotic F_σ follows by noting that it is the composition p_{2Y} ∘h, where h :(X × Y) × Z → Y × (X × Z) is a fuzzy chaotic G_δ-homeomorphism.

Proposition 3.9 The product of two fuzzy chaotic G_δ-compact spaces is fuzzy chaotic G_δ-compact.

Proof : Let (X, T, C_1) and (Y, S, C_2) be two fuzzy chaotic G_δ-compact spaces. Then p_{2X} : X × (Y × Z) → (Y × Z) and p_{2Y}: (Y × Z) → Z are fuzzy chaotic F_σ-mappings for any fuzzy chaotic structure space(Z, R,C_3). We show that $p_{2(X \times Y)}$:(X × Y) × Z → Z is fuzzy chaotic F_σ. Since $p_{2(X \times Y)}$ = p_{2Y} ∘p_{2X}[X× (Y × Z) and (X × Y) × Z are fuzzy homeomorphic], being a composition of two fuzzy chaotic F_σ-mappings, is fuzzy chaotic F_σ and hence X × Y is fuzzy chaotic G_δ-compact.

Definition 3.19 AM-fuzzy chaotic G_δ-continuous mapping f :(X, T, C_1)→(Y, S, C_2) of afuzzy chaotic structure space (X, T, C_1) into afuzzy chaotic structure space (Y, S, C_2) is called a fuzzy chaotic G_δ-quasi perfect mapping if f × I_Z : X × Z → Y × Z is fuzzy chaotic F_σ for any fuzzy chaotic G_δ-space (Z, R,C_3).

Proposition 3.10 Let (X_1, T_1, C_1), (X_2, T_2, C_2) and (X_3, T_3, C_3) be any three fuzzy chaotic structure spaces. The composition g∘f : (X_1, T_1, C_1) → (X_3, T_3, C_3) of fuzzy chaotic G_δ-quasi perfect mappings f : (X_1, T_1, C_1) → (X_2, T_2, C_2) and g : (X_2, T_2, C_2)→ (X_3, T_3, C_3) is fuzzy chaotic G_δ-quasi perfect.

Proof: Let (Z, R, C_3) be a fuzzy chaotic G_δ-space. Then (g ∘f) × I_Z : X_1 × Z → X_3 × Zis fuzzy chaotic F_σ follows from the identity (g ∘f) × I_Z = (g × I_Z) ∘(f × I_Z) by noting that f and g are fuzzy chaotic G_δ-quasi perfect mappings.

Proposition 3.11 Let (X_1, T_1, C_1), (X_2, T_2, C_2) and (X_3, T_3, C_3) be any three fuzzy chaotic structure spaces. Let the mappings f : (X_1, T_1, C_1) → (X_2, T_2, C_2) and g : (X_2, T_2, C_2) → (X_3, T_3, C_3) be M-fuzzy chaotic G_δ-continuous mappings. Then

(a) if g ∘f is fuzzy chaotic G_δ-quasi perfect and f is surjective, then g is fuzzy chaotic G_δ-quasi perfect,

(b) if g ∘f is fuzzy chaotic G_δ-quasi perfect and g is injective, then f is fuzzy chaotic G_δ-quasi perfect.

Proof: (a) We show that g ×I_Z : X_2 × Z → X_3 × Z is fuzzy chaotic F_σ for any fuzzy chaotic G_δ-space (Z, R, C_3). Let μ be any fuzzy chaotic F_σ-set of X_2 × Z. Then $(f \times I_Z)^{-1}$ (μ) is a fuzzy chaotic F_σ-set of X_1 × Z. Since (g ∘f) × I_Z is fuzzy chaotic F_σ and ((g ∘f) × I_Z)($(f \times I_Z)^{-1}$ (μ)) = (g × I_Z)(μ), it follows that (g × I_Z)(μ) is fuzzy chaotic F_σ-set of X_3 × Z.

(b) We show that f ×I_Z : X_1 × Z → X_2 × Z is fuzzy chaotic F_σ for any fuzzy chaotic G_δ-space (Z, R, C_3). Let μ be any fuzzy chaotic F_σ-set of X_1× Z. Then ((g ∘f) × I_Z)(μ) is a fuzzy chaotic F_σ-set of X_3 × Z. Since g × I_Z is M-fuzzy chaotic G_δ-continuous and $(g \times I_Z)^{-1}$ (((g ∘f) × I_Z)(μ)) = (f × I_Z)(μ), it follows that (f × I_Z)(μ) is fuzzy chaotic F_σ-set of X_2 × Z.

Definition 3.20 A fuzzy chaotic structure space (X, T, C_1) is called fuzzy chaotic G_δ-Lindelöf if the projection p_{2X}:X × Z → Z is a fuzzy chaotic F_σ-mapping for any fuzzy chaotic G_δ-space (Z, R, C_3).

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Proposition 3.12 A M-fuzzy chaotic G_δ-continuous image of a fuzzy chaotic G_δ-Lindelöf space is fuzzy chaotic G_δ-Lindelöf.

Proof : Let f be a M-fuzzy chaotic G_δ-continuous mapping from a fuzzy chaotic G_δ-Lindelöf space (X, T, C_1) onto a fuzzy chaotic structure space (Y, S, C_2) and I_Z be the identity mapping onto a fuzzy chaotic G_δ-space (Z, R, C_3). Then $f \times I_Z : X \times Z \to Y \times Z$ is fuzzy chaotic G_δ-continuous. Let μ be any fuzzy chaotic F_σ-set of $Y \times Z$. Then $(f \times I_Z)^{-1}(\mu)$ is fuzzy chaotic F_σ-set of $X \times Z$. Since $p_{2X} : X \times Z \to Z$ is fuzzy chaotic F_σ, $p_{2X}((f \times I_Z)^{-1}(\mu)) = p_{2Y}(\mu)$ is a fuzzy chaotic F_σ-set of (Z, R, C_3) showing that p_{2Y} is a fuzzy chaotic F_σ-mapping. Hence, (Y, S, C_2) is fuzzy chaotic G_δ-Lindelöf.

Proposition 3.13 If $f : (X, T, C_1) \to (Y, S, C_2)$ is a fuzzy chaotic G_δ-quasi perfect mapping of a fuzzy chaotic structure space (X, T, C_1) onto a fuzzy chaotic G_δ-Lindelöf space (Y, S, C_2), then (X, T, C_1) is fuzzy chaotic G_δ-Lindelöf.

Proof: Since f is fuzzy chaotic G_δ-quasi perfect, $f \times I_Z : X \times Z \to Y \times Z$ is fuzzy chaotic F_σ for any fuzzy chaotic G_δ-space (Z, R, C_3). For (X, T, C_1) to be fuzzy chaotic G_δ-Lindelöf, we show that $p_{2X} : X \times Z \to Z$ is fuzzy chaotic F_σ. Noting that p_{2X} is the composition of two fuzzy chaotic F_σ-mappings $f \times I_Z$ and $p_{2Y} : Y \times Z \to Z$, the result follows.

Proposition 3.14 Let (X, T, C_1) be a fuzzy chaotic G_δ-space and (Y, S, C_2) be a fuzzy chaotic G_δ-Lindelöf space. Then the projection $p : X \times Z \to X$ is an fuzzy chaotic G_δ-quasi perfect mapping.

Proof: We show that $p \times I_Z : (X \times Y) \times Z \to X \times Z$ is a fuzzy chaotic G_δ-quasi perfect mapping for a fuzzy chaotic G_δ-space (Z, R, C_3). Since (Y, S, C_2) is fuzzy chaotic G_δ-Lindelöf, $p_{2Y} : Y \times (X \times Z) \to (X \times Z)$ is fuzzy chaotic F_σ-mapping. That is, $p \times I_Z$ is fuzzy chaotic F_σ follows by noting that it is the composition $p_{2Y} \circ h$, where $h : (X \times Y) \times Z \to Y \times (X \times Z)$ is a fuzzy chaotic G_δ-homeomorphism.

Proposition 3.15 Let (X, T, C_1)be a fuzzy chaotic G_δ-compact space. If (Y, S, C_2) is a fuzzy chaotic G_δ-Lindelöf space, then $X \times Y$ is a fuzzy chaotic G_δ-Lindelöf.

Proof: Let (Z, R, C_3) be a fuzzy chaotic G_δ-space. Since (X, T, C_1) is fuzzy chaotic G_δ-compact and (Y, S, C_2) is fuzzy chaotic G_δ-Lindelöf, $p_{2X} : X \times (Y \times Z) \to (Y \times Z)$ and $p_{2Y} : Y \times Z \to Z$ are fuzzy chaotic F_σ-mappings. We show that $p_{2(X \times Y)} : (X \times Y) \times Z \to Z$ is fuzzy chaotic F_σ. Since $p_{2(X \times Y)} = p_{2Y} \circ p_{2X}$ [$X \times (Y \times Z)$ and $(X \times Y) \times Z$ are fuzzy homeomorphic], being a composition of two fuzzy chaotic F_σ-mappings, is fuzzy chaotic F_σ and hence $X \times Y$ is fuzzy chaotic G_δ-Lindelöf.

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