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ON b-SEPARATED M-SETS

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Abstract:

In this chapter the concepts of b-open M-sets, b-closed M-sets and b-separated M-sets are studied. Also, some of their properties and characterizations are discussed.

Key Words: b-open M-sets, b-closed M-sets & b-separated M-sets

1. Introduction:

Since then the introduction of M-topological spaces by Gitish and Sunil Jacob [6], various authors [1,2 and 6] studied the many interesting topological properties in M- topological spaces. Andrijevic [3] studied b-open sets. EI-Atik A A et al[4]. Studied the applications of b-connectedness. In this article, the concept of b-open M-sets, b-closed M-sets and b-separated M-sets are studied. Also, some of their properties and characterizations are discussed.

2. Preliminaries:

Definition 2.1 [6]: An M-set M drawn from the set X is represented by a function Count M or C_M defined as $C_M : X \to W$ where W represents the set of whole numbers. Here is $C_M(x)$ the number of occurrences of the element x in the M-set M. We represent the M-set M drawn from the set $X = \{x_1, x_2 \ldots x_n\}$ as $M = \left\{\frac{m_1}{x_1}, \frac{m_2}{x_2} \ldots\ldots \frac{m_n}{x_n}\right\}$ where m_i is the number of occurrences of the element x_i, i = 1,2,...,n in the M-set M. Those elements which are not included in the M-set have zero count. Since the count of each element in an M-set is always a non-negative integer so, W is taken as the range space instead of N.

Definition 2.2 [6]: Let $M \in [X]^w$ and $\tau \subseteq P^*(M)$. Then τ is called a Multiset topology of M if τ satisfies the following properties.

- ✓ The M-set M and the empty M-set ϕ are in τ.
- ✓ The M-set union of the elements of any sub collection of τ is in τ.
- ✓ The M-set intersection of the elements of any finite sub collection of τ is in τ.

Definition 2.3 [1]: A subset A of a Topological space (X, τ) is called a b-open set if A ⊆ cl(int(A)) ∪ int(cl(A)).

3. On b-Separated M-Sets:

Throughout this chapter X denotes a non-empty set and C_M: X ⟶ W where W represents the set of whole numbers.

Definition 3.1: Let (M, τ) be an M-topological space. Any sub M-set A of M is said to be a b-open M-set if $A \subseteq cl(int(A)) \cup int(cl(A))$ with $C_A(x) \leq C_{cl(int(A)) \cup int(cl(A))}(x)$, for all $x \in X$. The collection of all b-open M-sets in (M, τ) is denoted BO(M). The complement of a b-open M-set is a b-closed M-set.

Example 3.1: Let X = { a, b, c }, w = 2 and M = { 1/a, 1/b, 2/c }. Let τ = {M, ϕ, { 1/a }, {1/b}, {1/a, 1/b}}. Then τ is an M-topology and (M, τ) is an M-topological space. Also τ^c = {ϕ, M, {1/a, 2/c}, {1/b, 2/c}, {2/c}}. Let A = {1/a}. Clearly, $A \subseteq cl(int(A)) \cup int(cl(A))$ and $C_A(x) \leq C_{cl(int(A)) \cup int(cl(A))}(x)$, for all $x \in X$. Hence A = {1/a} is a b-open M-set.

Example 3.2: Let X = { a, b, c }, w = 2 and M = {1/a, 2/b, 1/c }. Then τ is an M-topology and (M, τ) is an M-topological space. Also τ^c = {ϕ, M, {2/b, 1/c}, {1/a, 2/b }, {2 /b}} . Let A = {1/a, 1/b}. Clearly, $int(cl(A)) \cap cl(int(A)) \subseteq A$ and $C_{int(cl(A)) \cap cl(int(A))}(x) \leq C_A(x)$, for all $x \in X$. Hence A= {1/a, 1/b} is a b-closed M-set.

Definition 3.2: Let (M, τ) be an M-topological space and A be any sub M-set of M. The b-closure of A and b-interior of A respectively denoted and defined by bcl(A) = ∩ {B : B ⊇ A, each B ⊆ M is a b-open M-set} with $C_{bcl(A)}(x)$ = min { $C_B(x)$: B ⊇ A each B ⊆ M is a b-open M-set}, for all $x \in X$. bint(A) = ∪ {B: B ⊆ A, each B ⊆ M is a b-closed M-set } with $C_{bint(A)}(x)$ = max {$C_B(x)$: B ⊆ A each B ⊆ M is a b-closed M-set }, for all $x \in X$.

Example 3.3: Let X = { a, b, c }, w = 2 and M = { 1/a, 2/b, 1/c }. Then τ is an M-topology and (M, τ) is an M-topological space. Also τ^c = { ϕ, M, { 1/a, 2/b }, { 2/b, 1/c }, { 2/b } }. The collection of all b-open M-sets is { { M, ϕ, { 2/b, 1/c }, { 1/a, 2/b }, { 1/b, 1/c }, { 1/c }, { 1/a, 1/b }, { 1/a } }. Let A = { 1/b } be a sub M-set of M. Then bint(A) = ϕ with $C_{bint(A)}(x)$ = max { $C_B(x)$: B ⊆ A each B ⊆ M is a b-open M-set }, for all $x \in X$. The collection of all b-closed M-sets is { ϕ, M, { 1/a }, { 1/c }, { 1/a, 1/b }, {1/a, 2/b }, { 1/b, 1/c }, { 2/b, 1/c } }. Let A = { 1/a, 2/b } be a sub M-set of M. Then bcl(A) = { 1/a, 2/b } with $C_{bcl(A)}(x)$ = min { $C_B(x)$: B ⊇ A each B ⊆ M is a b-closed M-set }, for all $x \in X$.

Definition 3.3: Any M-topological space (M, τ) is said to be a b-connected M-space if M cannot be expressed as the union of two disjoint nonempty b-open M-sets of M.

Definition 3.4: Let (M, τ) be an M-topological space. Any sub M-set A ⊆ M is called a b-neighborhood M-set of a point $x \in^m M$ if there exists a b-open M-set U ⊆ A such that $x \in^m U \subseteq A$ with $C_{\{m/x\}} \leq C_{U(x)} \leq C_{A(x)}$, for all $x \in X$.

Definition 3.5: Let (M, τ) be an M-topological space. Two sub M-sets A and B of M are said to b-separated M-set if and only if A ∩ bcl(B) = ϕ with $C_{A \cap bcl(B)}(x)$ = 0 and bcl(A) ∩ B = ϕ with $C_{bcl(A) \cap B}(x) = 0$, for all $x \in X$.

Example 3.4: Let X = { a, b, c }, w = 2 and M = { 1/a, 1/b, 2/c }. Let τ = { M, ϕ, { 1/a }, { 1/b }, { 1/a, 1/b } }. Then τ is an M-topology and (M, τ) is an M-topological space. Also τ^c = { ϕ, M, { 1/a, 2/c }, { 1/b, 2/c }, { 2/c } }. The collection of all b-open M-sets is { { 1/a }, { 1/b }, { 1/a, 1/b }, { 1/a, 1/c }, { 1/a, 2/c }, { 1/b, 1/c }, { 1/b, 2/c }, { 1/a, 1/b, 1/c } }.Hence the collection of all b-closed M-sets is { { M, ϕ, { 1/b, 2/c }, { 1/a, 2/c }, { 2/c }, { 1/b, 1/c }, { 1/b }, { 1/a, 1/c }, { 1/a } }. Let A = { 1/a } and B = { 2/c }. Then, bcl(B) = { 2/c } with $C_{bcl(B)}(a) = 0$, $C_{bcl(B)}(b) = 0$, $C_{bcl(B)}(c) = 2$. Therefore,A ∩ bcl(B) = { 1/a } ∩ { 2/c } = ϕ with $C_{A \cap bcl(B)}(a) = 0$, $C_{A \cap bcl(B)}(b) = 0$, $C_{A \cap bcl(B)}(c) = 0$. Then bcl(A) = { 1/a } with $C_{bcl(A)}(a) = 1$, $C_{bcl(A)}(b) = 0$, $C_{bcl(A)}(c) = 0$ and B = { 2/c }. But bcl(A) ∩ B = { 1/a } ∩ { 2/c } = ϕ with $C_{bcl(A) \cap B}(a) = 0$, $C_{bcl(A) \cap B}(b) = 0$, $C_{bcl(A) \cap B}(c) = 0$. Hence, A and B are b-Separated M-sets.

Proposition 3.1: Let (M, τ) be an M-topological space and A and B be any two nonempty sub M-sets of M. Then the following statements hold:

- ✓ For any two sub M-sets A_1 and B_1 of M, if A and B are b-Separated M-sets such that A_1 ⊆ A with $C_{A1}(x) \leq C_A(x)$, for all $x \in X$ and B_1 ⊆ B with $C_{B1}(x) \leq C_B(x)$, for all $x \in X$. Then A_1 and B_1 are also b-separeted M-sets.
- ✓ If each of A and B are both b-closed M-sets and b-open M-sets such that, A ∩ B = ϕ with $C_{A \cap B}(x) = 0$, for all $x \in X$, then A and B are b–separated M-sets.
- ✓ If each of A and B are both b-closed M-sets and b-open M-sets and if H = A ∩ (M ⊖ B) with $C_H(x) = C_{A \cap (M \ominus B)}(x)$, for all $x \in X$ and G = B ∩ (M ⊖ A) with $C_G(x) = C_{B \cap (M \ominus B)}(x)$, for all $x \in X$, then H and G are b–separated M- sets.

Proposition 3.2: Let (M, τ) be an M-topological space. The sub M-sets A and B of M are b–separated M-sets if and only if there exist U and V in BO(M) such that A ⊆ U with $C_A(x) \leq C_U(x)$, for all $x \in X$, B ⊆ V with $C_B(x) \leq C_V(x)$, for all $x \in X$ and A ∩ V = ϕ with $C_{A \cap V}(x) = 0$, for all $x \in X$, B ∩ U = ϕ with $C_{B \cap U}(x) = 0$, for all $x \in X$.

Definition 3.6: Let (M, τ) be an M-topological space. Let A be any sub M-set of M. A point $x \in^m M$ is called a b-limit point of A if every b-open M-set U ⊆ M with $C_U(x) < C_M(x)$ containing m/x contains a point of A other than m/x.

Proposition 3.3: Let (M, τ) be an M-topological space. Let A and B be any two nonempty disjoint sub M-sets of M and E = A∪B with $C_E(x) = C_{A \cup B}(x)$, for all $x \in X$. Then A and B are b-separated M-set if and only if each of A and B is nb-closed M-set (b-open M-set) in E.

Definition 3.7: Let (M, τ) be an M–topological space and A and B be any two sub M-sets. Any sub M-set S of M is said to be b-connected relative to M if there do not exist two b-separated sub M-sets A and B relative to M and S = A ∪ B with $C_S(x) = C_{A \cup B}(x)$, for all $x \in X$. Otherwise, S is said to be a b-disconnected M-set.

Proposition 3.4: Let (M, τ) be an M-topological space. Let E be any sub M-set of M. If E is a b-connected M-set, then bcl(E) is a b-connected M-set.

Proposition 3.5: Let (M, τ) be an M-topological space. Let A ⊆ B ∪ C with $C_A(x) \leq C_{B \cup C}(x)$ such that A is a nonempty b-connected M-set in (M, τ) and B, C are b–separated M-sets. Then only one of the following conditions holds

- ✓ A ⊆ B and A∩C = ϕ with $C_A(x) \leq C_B(x)$ and $C_{A \cap C}(x) = 0$, for all $x \in X$.
- ✓ A ⊆ C and A∩B = ϕ with $C_A(x) \leq C_C(x)$ and $C_{A \cap B}(x) = 0$, for all $x \in X$.

Definition 3.8:

Let (M, τ_1) and (N, τ_2) be any two M-topological spaces. Then any M-set function f : (M, τ_1) $\longrightarrow$ (M, τ_1) is said to be a

- ✓ b-continuous M-set function if for each b-open sub M-set V of N, the M-set f^{-1}(V) is a b-open sub M-set of (M, τ_1)
- ✓ b-open M-set if the image of each open M-set in (M, τ_1) is a b-open M-set in (N, τ_2)
- ✓ b-closed M-set if the image of each closed M-set in (M, τ_1) is a b-closed M-set in (N, τ_2).

Proposition 3.6: Let (M, τ_1) and (N, τ_2) be any two M-topological spaces. Let f : (M, τ_1) $\rightarrow$ (N, τ_2) be a b-continuous M-set function. Then $bcl(f^{-1}(B)) \subseteq f^{-1}(cl(B))$ with $C_{bcl\,(f^{-1}(B))} \leq C_{f^{-1}(cl(B))}$, for all $x \in X$ for each $B \subseteq Y$ with $C_B(x) \leq C_Y(x)$, for all $x \in X$.

Proposition 3.7: Let (M, τ_1) and (N, τ_2) be any two M-topological spaces. Let f : (M, τ_1) $\rightarrow$ (N, τ_2) be a b-continuous M-set function. and if K is b-connected M-set in (M, τ_1) then f(K) is a connected M-set in (N, τ_2).

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