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DEGREE BASED TOPOLOGICAL INDICES OF BANANA TREE GRAPH

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Abstract:

Banana tree, is defined by a graph obtained by connecting one leaf of each of p copies of q-star graphs with a single root vertex that is distinct from all the stars, $p \geq 2, q \geq 4$.The degree based indices like Randic Index, Geometric-Arithmetic Index, Sum-Connectivity Index, Harmonic Index, First Zagreb, Second Zagreb, Second Modified Zagreb, Inverse sum, Albertson, Atom Bond connectivity, Symmetric Division Index and Augmented Zagreb index of B(p,q) are computed and derived as a closed formula in two variables p and q and the results are tabulated as a ready reckoner.

Key Words: Star Graph, Banana Tree, M-Polynomial, Topological Indices

Introduction:

In chemical graph theory, a molecular graph is a simple graph (having no loops and multiple edges) in which atoms and chemical bonds between them are represented by vertices and edges respectively. A graph *B(V,E),* with vertex set *V (B)* and edge set *E(B)* is connected if there exists a connection between any pair of vertices in *G.* A network is simply a connected graph having no multiple edges and loops. The degree of a vertex is the number of vertices which are connected to that fixed vertex by the edges. The distance between two vertices *u* and *v* is denoted as $d(u, v) = d_B(u, v)$ and is defined as the length of shortest path between *u* and *v* in graph *G.* The number of vertices of *G,* adjacent to a given vertex *v,* is the "degree" of this vertex, and will be denoted by $d_v(B)$ or, if misunderstanding is not possible, simply by *dv*. For any edge in E (B) with end vertices u and v, d(u) is denoted by 'i' and d(v) is denoted by 'j' in the topological indices formulae. The concept of degree is somewhat closely related to the concept of valence in chemistry. Cheminformatics is another emerging field in which quantitative structure-activity and Structure-property relationships predict the biological activities and properties of Nano-material. In these studies, some Physico-chemical properties and topological indices are used to predict bioactivity of the chemical compounds see [1-5]. Algebraic polynomials have also useful applications in chemistry such as Hosoya polynomial (also called Wiener polynomial) [6] which play a vital role in determining distance based topological indices. Among other algebraic polynomials, the *M*-polynomial [7] introduced in 2015, plays the same role in determining the closed form of many degree based topological indices [8-12]. The main advantage of the *M*-polynomial is the wealth of information that it contains about degree-based graph invariants.

Computational Procedure of M-Polynomial:

M-Polynomial of graph G is defined as If $G = (V, E)$ is a graph and $v \in V$, then $d_v(G)$ (or d_v for short if G is clear from the context) denotes the degree of v. Let G be a graph and let $m_{ij}(G), i, j \geq 1$, be the number of edges e = uv of G such that $\{d_u(G), d_v(G)\} = \{i, j\}$ .The M-polynomial of G as $M(G; x, y) = \sum_{i \leq j} m_{ij}(G) x^i y^j$ .For a graph $G = (V, E)$, a degree-based topological index is a graph invariant of the form $I(G) = \sum_{e=uv \in E} f(d_u, d_v)$ where $f = f(x, y)$ is a function appropriately selected for possible chemical applications.In this article, we compute closed form of some degree-based topological indices of the Banana tree graph by using the *M*-polynomial.B(p,q) banana tree, is defined by a graph obtained by connecting one leaf of each of p copies of q-star graphs with a single root vertex that is distinct from all the stars, $p \geq 2, q \geq 4$ .Here we considered the banana tree graphs for $p = 2, q \geq 4$, the results are given in case 1 to case 4 and for $p = 3, q \geq 4$, the results are given in case 5 to case 8 and for $p = 4, q \geq 4$, the results are given in case 9 to case 12 , for $p = 5, q \geq 4$, the results are given in case 13 to case 16, finally, the results are generalized for $p \geq 2, q \geq 4$.

Case (1):

For $B_{2,4}$, the number of edges with end degrees (2,2) is equal to 2, the number of edges with end degrees (2,3) is equal to 2, the number of edges with end degrees (1,3) is equal to 4, therefore the total number of edges is equal to 8.

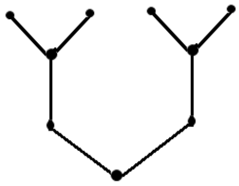

Figure 1: B(2,4)

Case (2):

For $B_{2,5}$, the number of edges with end degrees (2,2) is equal to 2, the number of edges with end degrees (2,3) is equal to 0, the number of edges with end degrees (2,4) is equal to 2, the number of edges with end degrees (1,3) is equal to 0,the number of edges with end degrees (1,4) is equal to 6, therefore the total number of edges is equal to 10.

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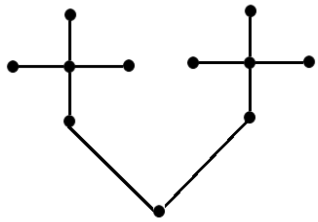

Figure 2: B(2,5)

Case (3):

For $B_{2,6}$, the number of edges with end degrees (2,2) is equal to 2, the number of edges with end degrees (2,5) is equal to 2, the number of edges with end degrees (1,3) is equal to 0, the number of edges with end degrees (1,4) is equal to 0, the number of edges with end degrees (1,5) is equal to 8, therefore the total number of edges is equal to 12.

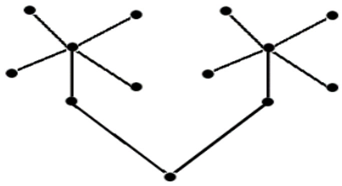

Figure 3: B(2,6)

Case (4):

For $B_{2,7}$, the number of edges with end degrees (2,2) is equal to 2, the number of edges with end degrees (2,3) is equal to 0, the number of edges with end degrees (1,3) is equal to 0, the number of edges with end degrees (1,4) is equal to 0, the number edges with end degrees (1,5) is equal to 0,the number of edges with end degrees (1,6) is equal to 10, the therefore total number of edges is equal to 14.

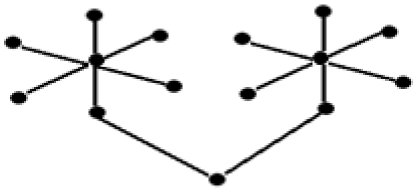

Figure 4: B(2,7)

Case (5):

For $B_{3,4}$, the number of edges is twice the pair of vertices with end degrees (2,3) is equal to (3,3), the number of edges with end degrees (1,3) is equal to 6, therefore the total number of edges is equal to 12.

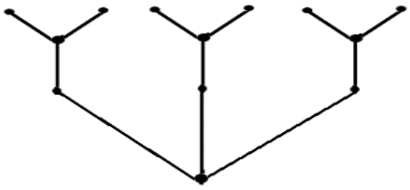

Figure 5: B(3,4)

Case (6):

For $B_{3,5}$ the number of edges with end degrees (2,3) is equal to 3, the number of edges with end degrees (2,4) is equal to 3,the number of edges with end degrees (1,3) is equal to 0, the number of edges with end degrees (1,4) is equal to 9, therefore the total number of edges is equal to 15.

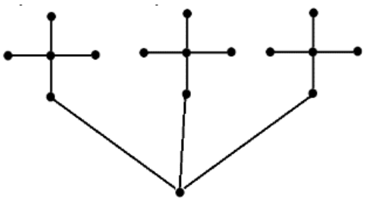

Figure 6: B(3,5)

Case (7):

For $B_{3,6}$, the number of edges with end degrees (2,3) is equal to3, the number of edges with end degrees (2,5) is equal to 3 ,the number of edges with end degrees (1,3) is equal to 0, the number of edges with end degrees (1,4) is equal to 0, the number of edges with end degrees (1,5) is equal to 12, therefore the total number of edges is equal to 18.

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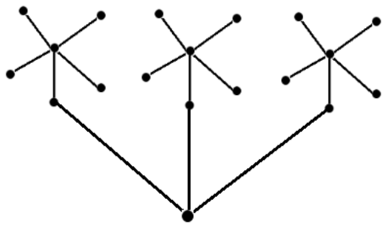

Figure 7: B(3,6)

Case (8):

For $B_{3,7}$, the number of edges with end degrees (2,3) is equal to 3 , the number of edges with end degrees (2,6) is equal to 3,the number of edges with end degrees (1,3) is equal to 0, the number of edges with end degrees (1,4) is equal to 0, the number of edges with end degrees (1,5) is equal to 0, the number of edges with end degrees (1,6) is equal to 15, therefore the total number of edges is equal to 21.

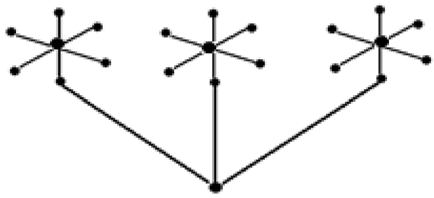

Figure 8: B(3,7)

Case (9):

For $B_{4,4}$, the number of edges with end degrees (2,4) is equal to 4, the number of edges with end degrees (2,3) is equal to 4, the number of edges with end degrees (1,3) is equal to 8, therefore the total number of edges is equal to 16.

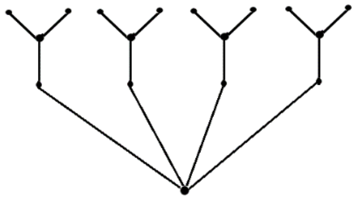

Figure 9: B(4,4)

Case (10):

For $B_{4,5}$, the number of edges with end degrees (2,4) is equal to 4, the number of edges with end degrees (4,2) is equal to 4, the number of edges with end degrees (1,3) is equal to 0, the number of edges with end degrees (1,4) is equal to12, therefore the total number of edges is equal to 20.

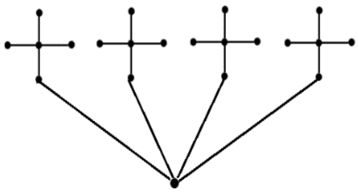

Figure 10: B(4,5)

Case (11):

For $B_{4,6}$, the number of edges with end degrees (2,4) is equal to 4, the number of edges with end degrees (2,5) is equal to 4,the number of edges with end degrees (1,3) is equal to 0, the number of edges with end degrees (1,4) is equal to 0, the number of edges with end degrees (1,5) is equal to16, therefore the total number of edges is equal to 24.

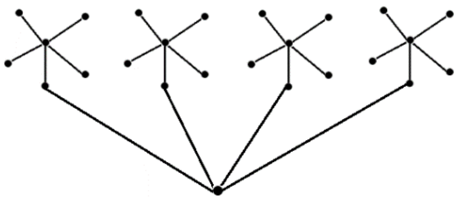

Figure 11: B(4,6)

Case (12):

For $J_{4,7}$, the number of edges with end degrees (2,4) is equal to 4, the number of edges with end degrees (2,6) is equal to 4, the number of edges with end degrees (1,3) is equal to 0, the number of edges with end degrees (1,4) is equal to 0, the number of edges with end degrees (1,5) is equal to 0, the number of edges with end degrees (1,6) is equal to 20, therefore the total number of edges is equal to 28.

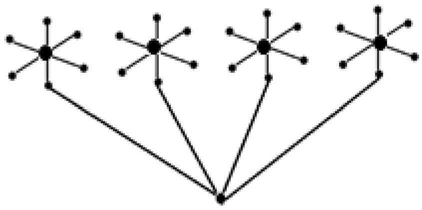

Figure 12: B(4,7)

Case (13):

For $B_{5,4}$, the number of edges with end degrees (2,5) is equal to 5, the number of edges with end degrees (2,3) is equal to 5, the number of edges with end degrees (1,3) is equal to 10, therefore the total number of edges is equal to 20.

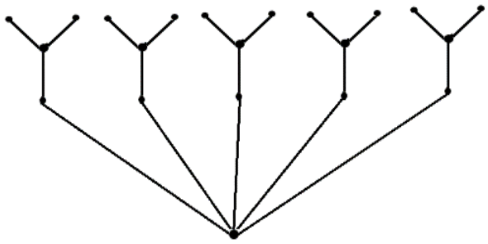

Figure 13: B(5,4)

Case (14):

For $B_{5,5}$, the number of edges with end degrees (2,5) is equal to 5, the number of edges with end degrees (2,3) is equal to 0, the number of edges with end degrees (2,4) is equal to 5, the number of edges with end degrees (1,3) is equal to 0,the number of edges with end degrees (1,4) is equal to 15, therefore the total number of edges is equal to 25.

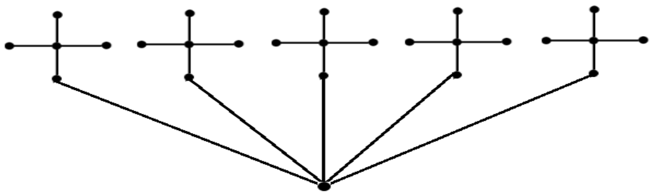

Figure 14: B(5,5)

Case (15):

For $B_{5,6}$, the number of edges is twice the pair of vertices with end degrees (2,5) is equal to 5, the number of edges with end degrees (1,3) is equal to 0,the number of edges with end degrees (1,4) is equal to 0, the number of edges with end degrees (1,5) is equal to 20, therefore the total number of edges is equal to 30.

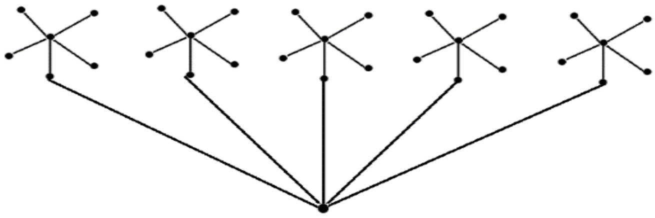

Figure 15: B(5,6)

Case (16):

For $B_{5,7}$, the number of edges with end degrees (2,5) is equal to 5, the number of edges with end degrees (2,6) is equal to 5, the number of edges with end degrees (1,3) is equal to 0, the number of edges with end degrees (1,4) is equal to 0, the number edges with end degrees (1,5) is equal to 0,the number of edges with end degrees (1,6) is equal to 25, the therefore total number of edges is equal to 35.

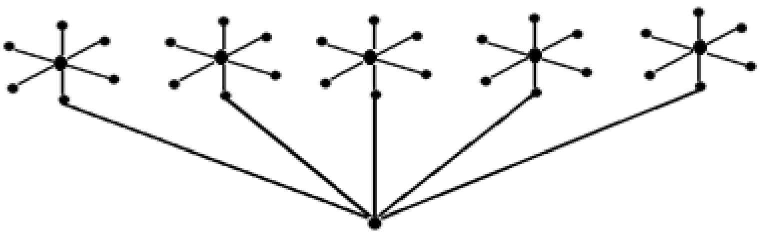

Figure 16: B(5,7)

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Generalized form for all $p \geq 2, q \geq 4$

B(p,q) will have edges whose end vertices with end degrees (2,p) is equal to p, the number of edges with end degrees (p,q-1) is equal to p, the number of edges with end degrees (1,q-1) is equal to p(q-2).

M-Polynomial of banana tree graph is developed and it is given by B(p,q)= $\{p\}x^2y^p + \{p\}x^py^{q-1} + \{p(q-1)\}x^1y^{q-1}$

Topological Index	$\{\varphi_{ij}\}$	Notation	Topological Index	$\{\varphi_{ij}\}$	Notation
Randic[13]	$\frac{1}{\sqrt{ij}}$	$\chi(G)$	Second modified Zagreb[19]	$\frac{1}{ij}$	$M_3(G)$
Geometric-Arithmetic[14]	$\frac{2\sqrt{ij}}{i+j}$	$GA(G)$	Inverse sum[20]	$\frac{ij}{i+j}$	$IS(G)$
Sum-Connectivity[15]	$\frac{1}{\sqrt{i+j}}$	$SCI(G)$	Albertson[21]	$\|i-j\|$	$Alb(G)$
Harmonic[16]	$\frac{2}{i+j}$	$HI(G)$	Atom-Bond connectivity[22]	$\sqrt{\frac{i+j-2}{ij}}$	$ABCG)$
First Zagreb[17]	$i+j$	$M_1(G)$	Symmetric Division Index[23]	$\frac{i^2+j^2}{ij}$	$SD(G)$
Second Zagreb[18]	ij	$M_2(G)$	Augmented Zagreb[24]	$\left(\frac{ij}{i+j-2}\right)^3$	$AZI(G)$

Table 1: Some important vertex degree based Topological indices

Theorem 1:

Let B (p,q) is a banana tree defined by connecting one leaf of each of p copies of q-star graphs with a single rooted vertex for $p \geq 2, q \geq 4$, then the Randic index is given by $\chi(B(p,q)) = \frac{\sqrt{p}}{\sqrt{2}} + \frac{p}{\sqrt{p(q-1)}} + \frac{p(q-2)}{\sqrt{q-1}}$

Proof:

Randic Index is given by $\chi(B(p,q)) = \sum_{e \in E(B)} \frac{1}{\sqrt{ij}}$

$$\Rightarrow \frac{1}{\sqrt{2p}}(P) + \frac{1}{\sqrt{p(q-1)}}(p) + \frac{1}{\sqrt{(1)(q-1)}}(p)(q-2)$$

$$\Rightarrow \frac{p}{\sqrt{2p}} + \frac{p}{\sqrt{p(q-1)}} + \frac{p(q-2)}{\sqrt{q-1}}$$

$$\Rightarrow \frac{\sqrt{p}\sqrt{p}}{\sqrt{2p}} + \frac{p}{\sqrt{p(q-1)}} + \frac{p(q-2)}{\sqrt{q-1}}$$

$$\Rightarrow \frac{\sqrt{p}}{\sqrt{2}} + \frac{p}{\sqrt{p(q-1)}} + \frac{p(q-2)}{\sqrt{q-1}}$$

Theorem 2:

Let B (p,q) is a banana tree defined by connecting one leaf of each of p copies of q-star graphs with a single rooted vertex for $p \geq 2, q \geq 4$, then the Geometric Arithmetic index is given by

$$GA(B(p,q)) = \left\{ \frac{2\sqrt{2}(p)^{3/2}}{2+p} + \frac{2p\sqrt{p(q-1)}}{p+q-1} + \frac{2[p(q-2)]\sqrt{q-1}}{q} \right\}$$

Proof:

Geometric – Arithmetic Index $GA(B(p,q)) = \sum_{e \in E(B)} \frac{2\sqrt{ij}}{i+j}$

$$\Rightarrow \frac{2\sqrt{ij}}{i+j}\{p\} + \frac{2\sqrt{ij}}{i+j}\{p\} + \frac{2\sqrt{ij}}{i+j}\{p(q-2)\}$$

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$$\Rightarrow \frac{2\sqrt{2p}}{2+p}(p)+\frac{2\sqrt{p(q-1)}}{p+q-1}(p)+\frac{2\sqrt{(1)(q-1)}}{1+q-1}[(p)(q-2)]$$

$$\Rightarrow \frac{2p\sqrt{2p}}{2+p}+\frac{2p\sqrt{p(q-1)}}{p+q-1}+\frac{2[p(q-2)\sqrt{q-1}]}{1+q-1}$$

$$\Rightarrow \frac{2p\sqrt{2p}}{2+p}+\frac{2p\sqrt{p(q-1)}}{p+q-1}+\frac{2[p(q-2)\sqrt{q-1}]}{q}$$

$$\Rightarrow \frac{2p\sqrt{2\sqrt{p}}}{2+p}+\frac{2p\sqrt{p(q-1)}}{p+q-1}+\frac{2[p(q-2)\sqrt{q-1}]}{q}$$

$$\Rightarrow \frac{2\sqrt{2}(p)(p)^{1/2}}{2+p}+\frac{2p\sqrt{p(q-1)}}{p+q-1}+\frac{2[p(q-2)]\sqrt{q-1}}{q}$$

$$\Rightarrow \left\{\frac{2\sqrt{2}(p)^{3/2}}{2+p}+\frac{2p\sqrt{p(q-1)}}{p+q-1}+\frac{2[p(q-2)]\sqrt{q-1}}{q}\right\}$$

Theorem 3:

Let B (p,q) is a banana tree defined by connecting one leaf of each of p copies of q-star graphs with a single rooted vertex for $p \geq 2, q \geq 4$, then the Sum-Connectivity index is given by

$$SCI(B(p,q)) = p\sqrt{q}-\frac{2p}{\sqrt{q}}+\frac{p}{\sqrt{2+p}}+\frac{p}{\sqrt{p+q-1}}$$

Proof:

Sum – Connectivity is given by $SCI(B(p,q)) = \sum_{e\in E(B)} \frac{1}{\sqrt{i+j}}$

$$\Rightarrow \frac{1}{\sqrt{i+j}}\{p\}+\frac{1}{\sqrt{i+j}}\{p\}+\frac{1}{\sqrt{i+j}}\{p(q-2)\}$$

$$\Rightarrow \frac{1}{\sqrt{2+p}}(p)+\frac{1}{\sqrt{p+q-1}}(p)+\frac{1}{\sqrt{1+q-1}}\{p(q-2)\}$$

$$\Rightarrow \frac{p}{\sqrt{2+p}}+\frac{p}{\sqrt{p+q-1}}+\frac{p(q-2)}{\sqrt{1+q-1}}$$

$$\Rightarrow p\left\{\frac{1}{\sqrt{2+p}}+\frac{1}{\sqrt{p+q-1}}+\frac{q-2}{\sqrt{q}}\right\}$$

$$\Rightarrow \frac{p}{\sqrt{2+p}}+\frac{p}{\sqrt{p+q-1}}+\frac{pq}{\sqrt{q}}-\frac{2p}{\sqrt{q}}$$

$$\Rightarrow \frac{p}{\sqrt{2+q}}+\frac{p}{\sqrt{p+q-1}}+\frac{pq}{\sqrt{q}}-\frac{2p}{\sqrt{q}}$$

$$\Rightarrow p\sqrt{q}-\frac{2p}{\sqrt{q}}+\frac{p}{\sqrt{2+p}}+\frac{p}{\sqrt{p+q-1}}$$

Theorem 4:

Let B (p,q) is a banana tree defined by connecting one leaf of each of p copies of q-star graphs with a single rooted vertex for $p \geq 2, q \geq 4$, then the Harmonic index is given by $HI(B(p,q))=\frac{-4p}{q}+\frac{2p}{p+q-1}+\frac{2(p+3)p}{p+2}$

Proof:

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Harmonic index is given by $HI(B(p,q)) = \sum_{e\in E(B)} \frac{2}{i+j}$

$$\Rightarrow \frac{2}{i+j}\{p\} + \frac{2}{i+j}\{p\} + \frac{2}{i+j}\{p(q-2)\}$$

$$\Rightarrow \frac{2p}{2+p} + \frac{2p}{p+q-1} + \frac{2\{p(q-2)\}}{1+q-1}$$

$$\Rightarrow \frac{2p}{2+q} + \frac{2p}{p+q-1} + \frac{2\{p(q-2)\}}{q}$$

$$\Rightarrow \frac{2p(p+q-1)+2p(2+p)}{(2+p)(p+q-1)} + \frac{2p(q-2)}{q}$$

$$\Rightarrow \frac{2p^2+2pq-2p+4p+2p^2}{(2+p)(p+q-1)} + \frac{2p(q-2)}{q}$$

$$\Rightarrow \frac{2p^2q+2pq^2+2pq+4pq+2p^2q+(2pq-4p)(2+p)(p+q-1)}{(2+p)(p+q-1)(q)}$$

$$\Rightarrow \frac{4p^2q+2pq+2pq^2+(2pq-4p)(2p+2q-2+p^2+pq-p)}{(p+q-1)(q)(p+2)}$$

$$\Rightarrow \frac{4p^2+2pq+2pq^2+(4p^2+4pq^2-4pq+2p^3q+2p^2q^2-2p^2q-8p^2-8pq+8p-4p^3-4p^2q+4p^2)}{(p+q-1)(q)(p+2)}$$

$$\Rightarrow \frac{2p^2q+6q^2p-10pq+2p^3q+2p^2q^2-4p^2-4p^3+8p}{(p+q-1)(q)(p+2)}$$

$$\Rightarrow \frac{2p\{p^2q+3q^2-5q+pq+pq^2-2p-2p^2+4\}}{(p+2)(q)(P+q-1)}$$

$$\Rightarrow \frac{2p}{2+p} + \frac{2p}{p+q-1} + \frac{2pq-4p}{q}$$

$$\Rightarrow \frac{-4P}{q} + \frac{2p}{p+q-1} + \frac{2pq}{q} + \frac{2p}{2+p}$$

$$\Rightarrow \frac{-4p}{q} + \frac{2p}{p+q-1} + \frac{2p(2+p)+2p}{2+p}$$

$$\Rightarrow \frac{-4p}{q} + \frac{2p}{p+q-1} + \frac{4p+2p^2+2p}{2+p}$$

$$\Rightarrow \frac{-4p}{q} + \frac{2p}{p+q-1} + \frac{6p+2p^2}{p+2}$$

$$\Rightarrow \frac{-4p}{q} + \frac{2p}{p+q-1} + \frac{2(p+3)p}{p+2}$$

Theorem 5:

Let B (p,q) is a banana tree defined by connecting one leaf of each of p copies of q-star graphs with a single rooted vertex for $p \geq 2, q \geq 4$, then the First Zagreb index is given by $M_1(B(p,q)) = p(2p-q+1+q^2)$

Proof:

First Zagreb Index is given by $M_1(B(p,q)) = \sum_{e\in E(B)} i+j$

$$\Rightarrow (2+p)p+(p+q-1)p+(1+q-1)(p)(q-2)$$

$$\Rightarrow 2p+p^2+p^2+qp-p+(qp)(q-2)$$

$$\Rightarrow 2p+2p^2+qp-p+pq^2-2pq$$

$\Rightarrow 2p^2 - pq + p + pq^2$

$\Rightarrow p(2p - q + 1 + q^2)$

Theorem 6:

Let B (p,q) is a banana tree defined by connecting one leaf of each of p copies of q-star graphs with a single rooted vertex for $p \geq 2, q \geq 4$, then the Second Zagreb index is given by $M_2(B(p,q)) = p(pq + p + q^2 - 3q + 2)$

Proof:

Second Zagreb index is given by $M_2(B(p,q)) = \sum_{e \in E(B)} ij$

$\Rightarrow (2p)(p) + (p)(q-1)(p) + (q-1)(p)(q-2)$

$\Rightarrow 2p^2 + p^2(q-1) + (p)(q-1)(q-2)$

$\Rightarrow 2p^2 + p^2(q-1) + (pq^2 - pq - 2pq + 2p)$

$\Rightarrow 2p^2 + p^2q - p^2 + pq^2 - pq + 2pq + 2p$

$\Rightarrow p(pq + p + q^2 - 3q + 2)$

Theorem 7:

Let B (p,q) is a banana tree defined by connecting one leaf of each of p copies of q-star graphs with a single rooted vertex for $p \geq 2, q \geq 4$, then the Second Modified Zagreb index is given by $M_3(B(p,q)) = \frac{2pq - 4p + q + 1}{2(q-1)}$

Proof:

Second Modified Zagreb index is given by $M_3(B(p,q)) = \sum_{e \in E(B)} \frac{1}{ij}$

$\Rightarrow \frac{1}{ij}\{p\} + \frac{1}{ij}\{p\} + \frac{1}{ij}\{p(q-2)\}$

$\Rightarrow \frac{p}{2p} + \frac{p}{(p)(q-1)} + \frac{(p)(q-2)}{(q-1)}$

$\Rightarrow \frac{1}{2} + \frac{1}{q-1} + \frac{pq}{q-1} - \frac{2p}{q-1}$

$\Rightarrow \frac{q - 1 + 2 + 2pq - 4p}{2(q-1)}$

$\Rightarrow \frac{2pq - 4p + q + 1}{2(q-1)}$

Theorem 8:

Let B (p,q) is a banana tree defined by connecting one leaf of each of p copies of q-star graphs with a single rooted vertex for $p \geq 2, q \geq 4$, then the Inverse sum index is given by

$$IS(B(p,q)) = \frac{p[2p^2q^2 - 2p^2q + 2p^2 + pq^3 + 2pq^2 - 5pq + 2p + 2q^3 - 8q^2 + 10q - 4]}{(p+q-1)(q)(p+2)}$$

Proof:

Inverse Sum Index is given by IS($B(p,q)$) = $\sum_{e \in E(B)} \frac{ij}{i+j}$

$\Rightarrow \frac{ij}{i+j}\{p\} + \frac{ij}{i+j}\{p\} + \frac{ij}{i+j}\{p(q-2)\}$

$\Rightarrow \frac{2p(p)}{2+p} + \frac{p(p)(q-1)}{p+q-1} + \frac{(q-1)(p)(q-2)}{1+q-1}$

$\Rightarrow \frac{2p^2}{2+p} + \frac{(q-1)p^2}{p+q-1} + \frac{(q-1)(p)(q-2)}{q}$

$$\Rightarrow \frac{2(p^2)(p+q-1)(q)+(q-1)(p)^2(2+p)(q)+(q-1)(p)(q-2)(2+p)(p+q-1)}{(p+q-1)(q)(2+p)}$$

$$\Rightarrow \frac{2p^2q(p+q-1)+(p^2q)(q-1)(2+p)+p(q^2-3q+2)(2+p)(p+q-1)}{(p+q-1)(q)(p+2)}$$

$$\Rightarrow \frac{2p^3q+2p^2q^2-2p^2q+[(p^2q)(2q+pq-2-p)]+[p(q^2-3q+2)(2q-2+p^2+pq+p]}{(p+q-1)(q)(p+2)}$$

$$\Rightarrow \frac{\begin{array}{l}2p^3q+2p^2q^2-2p^2q+[(2p^2q^2+p^3q^2-2p^2q-p^3q)]+[2pq^3-6pq^2+4pq-2pq^2+6pq-4p\\ +p^3q^2-3p^3q+2p^3+p^2q^3-3p^2q^2+2p^2q+p^2q^2-3p^2q+2p^2]\end{array}}{(p+q-1)(q)(p+2)}$$

$$\Rightarrow \frac{p[2p^2q^2-2p^2q+2p^2+pq^3+2pq^2-5pq+2p+2q^3-8q^2+10q-4]}{(p+q-1)(q)(p+2)}$$

Theorem 9:

Let B (p,q) is a banana tree defined by connecting one leaf of each of p copies of q-star graphs with a single rooted vertex for $p \geq 2, q \geq 4$, then the Albertson index is given by

$$Alb(B(p,q)) = p[|2-p|+|p-q+1|(p)+|2-p|(pq)+|2-q|(-2p)]$$

Proof:

Albertson index Alb($B(p,q)$) = $\sum_{e\in E(B)} |i-j|$

$$\Rightarrow |2-p|(p)+|p-q+1|(p)+|1-q+1|[p(q-2)]$$

$$\Rightarrow p[|2-p|+|p-q+1|(p)+|2-q|(q-2)]$$

$$\Rightarrow p[|2-p|+|p-q+1|(p)+|2-p|(pq)+|2-q|(-2p)]$$

Theorem 10:

Let B (p,q) is a banana tree defined by connecting one leaf of each of p copies of q-star graphs with a single rooted vertex for $p \geq 2, q \geq 4$, then the Atom-Bond connectivity index is given by

$$ABC(B(p,q)) = \frac{p}{2}\left\{\sqrt{2}+2q\sqrt{\frac{q-2}{q-1}}-4\sqrt{\frac{q-2}{q-1}}+p\sqrt{\frac{p+q-1}{p(q-1)}}\right\}$$

Proof:

Atom Bomb Connectivity is given by ABC ($B(p,q)$) = $\sum_{e\in E(G)} \sqrt{\frac{i+j-2}{ij}}$

$$\Rightarrow \sqrt{\frac{i+j-2}{ij}}\{p\}+\sqrt{\frac{i+j-2}{ij}}\{p\}+\sqrt{\frac{i+j-2}{ij}}\{p(q-2)\}$$

$$\Rightarrow \sqrt{\frac{2+p-2}{2p}}(p)+\sqrt{\frac{p+q-1-2}{p(q-1)}}(p)+\sqrt{\frac{1+q-1-2}{q-1}}[p(q-2)]$$

$$\Rightarrow \sqrt{\frac{p}{2p}}(p)+\sqrt{\frac{p+q-1}{p(q-1)}}(p)+\sqrt{\frac{q-2}{q-1}}[p(q-2)]$$

$$\Rightarrow p\left\{\sqrt{\frac{1}{2}}+\sqrt{\frac{q-2}{(q-1)}}(q-2)+\sqrt{\frac{p+q-1}{p(q-1)}}\right\}$$

when 2 is multiplied and divided in above condition

$$\Rightarrow p\left\{\frac{2}{2\sqrt{2}}+\frac{2}{2}\sqrt{\frac{q-2}{q-1}}(q-2)+\frac{2}{2}\sqrt{\frac{p+q-1}{p(q-1)}}\right\}$$

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$$\Rightarrow p\left\{\frac{1}{\sqrt{2}}+\sqrt{\frac{q-2}{q-1}}(q-2)+\sqrt{\frac{p+q-1}{p(q-1)}}\right\}$$

$$\Rightarrow \frac{p}{2}\left\{\sqrt{2}+2q\sqrt{\frac{q-2}{q-1}}-4\sqrt{\frac{q-2}{q-1}}+p\sqrt{\frac{p+q-1}{p(q-1)}}\right\}$$

Theorem 11:

Let B (p,q) is a banana tree defined by connecting one leaf of each of p copies of q-star graphs with a single rooted vertex for $p \geq 2, q \geq 4$, then the Symmetric Division Index is given by

$$SD(B(p,q)) = \frac{p^2(q+1)}{2(q-1)}+\frac{pq^3-4pq^2+6pq-4p}{q-1}+(q+1)$$

Proof:

Symmetric Division Index SD ($B(p,q)$) = $\sum_{e\in E(B)}\frac{i^2+j^2}{ij}$

$$\Rightarrow \frac{i^2+j^2}{ij}\{p\}+\frac{i^2+j^2}{ij}\{p\}+\frac{i^2+j^2}{ij}\{p(q-2)\}$$

$$\Rightarrow \frac{[(2)^2+p^2]}{2p}(p)+\frac{[(p)^2+(q-1)^2]}{p(q-1)}(p)+\frac{[(1)^2+(q-1)^2]}{q-1}\{p(q-2)\}$$

$$\Rightarrow \frac{\left[4+p^2\right]}{2}+\frac{\left[p^2+(q-1)2\right]}{q-1}+\frac{1^2+(q-1)^2}{q-21}\{p(q-2)\}$$

$$\Rightarrow \frac{\left[4+p^2\right][q-1]+2\left[p^2+(q-1)^2\right]+(2)(1+(q-1)^2)(p(q-2))}{2(q-1)}$$

$$\Rightarrow \frac{[4q-4+p^2q-p^2]+[2p^2+2(q-1)^2]+[(2+2(q-1)^2)(pq-2p)]}{(2)(q-1)}$$

$$\Rightarrow \frac{[-p^2+p^2q+4a-4]+[2p^2+2(q^2+1-2q)+2pq-4p+2pq(q^2+1-2q)-4p(q^2+1-2q)]}{2(q-1)}$$

$$\Rightarrow \frac{p^2+p^2q+2q^2+2pq^3-8pq^2+12pq-8p-2}{2(q-1)}$$

$$\Rightarrow \frac{p^2(q+1)}{2(q-1)}+\frac{pq^3-4pq^2+6pq-4p}{q-1}+(q+1)$$

Theorem 12:

Let B (p,q) is a banana tree defined by connecting one leaf of each of p copies of q-star graphs with a single rooted vertex for $p \geq 2, q \geq 4$, then the Augmented Zagreb index is given by

$$AZI(B(p,q)) = 8p+\frac{p^4q^3}{(p+q-3)^3}-\frac{3p^4q^2}{(p+q-3)^3}+\frac{3p^4q}{(p+q-3)^3}-\frac{p^4}{(p+q-3)^3}+\frac{pq^3}{(q-2)^2}-\frac{3pq^2}{(q-2)^2}+\frac{3pq}{(q-2)^2}-\frac{p}{(q-2)^2}$$

Proof:

Augmented Zagreb index is given by AZI($B(p,q)$) = $\sum_{e\in E(B)}\left(\frac{ij}{i+j-2}\right)^3$

$$\Rightarrow \left(\frac{ij}{i+j-2}\right)^3\{p\}+\left(\frac{ij}{i+j-2}\right)^3\{p\}+\left(\frac{ij}{i+j-2}\right)^3\{p(q-2)\}$$

$$\Rightarrow \left(\frac{2p}{2+p-2}\right)^3(p)+\left(\frac{p(q-1)}{p+q-1-2}\right)^3(p)+\left(\frac{q-1}{1+q-1-2}\right)^3\{p(q-2)\}$$

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$$\Rightarrow \left(\frac{2p}{p}\right)^3 (p)+\left(\frac{p(q-1)}{p+q-3}\right)^3 (p)+\left(\frac{q-1}{q-2}\right)^3 \{p(q-2)\}$$

$$\Rightarrow 8p+\left(\frac{pq-p}{p+q-3}\right)^3 (p)+\left(\frac{p(q-1)^3}{(q-2)^3}\right)$$

$$\Rightarrow 8p+\frac{(pq-p)^3}{p+q-3}(p)+(p)\left[\frac{(q-1)^3}{(q-2)^3}\right]$$

$$\Rightarrow \frac{[(8p)(p+q-3)^3(q-2)^2]+[(pq-p)^3(p)(q-2)^2]+[(p)(q-1)^3(p+q-3)^3]}{(p+q-3)^3(q-2)^3}$$

$$\Rightarrow 8p+\frac{p^4q^3}{(p+q-3)^3}-\frac{3p^4q^2}{(p+q-3)^3}+\frac{3p^4q}{(p+q-3)^3}-\frac{p^4}{(p+q-3)^3}+\frac{pq^3}{(q-2)^2}-\frac{3pq^2}{(q-2)^2}+\frac{3pq}{(q-2)^2}-\frac{p}{(q-2)^2}$$

Table 2: Results of Topological Indices of Banana Tree

Topological Index	Formulas Obtained
Randic	$\chi(B(p,q))=\frac{\sqrt{p}}{\sqrt{2}}+\frac{p}{\sqrt{p(q-1)}}+\frac{p(q-2)}{\sqrt{q-1}}$
Geometric-Arithmetic	$GA(B(p,q)) =\left\{\frac{2\sqrt{2}(p)^{3/2}}{2+p}+\frac{2p\sqrt{p(q-1)}}{p+q-1}+\frac{2[p(q-2)]\sqrt{q-1}}{q}\right\}$
Sum Connectivity	$SCI(B(p,q)) = p\sqrt{q}-\frac{2p}{\sqrt{q}}+\frac{p}{\sqrt{2+p}}+\frac{p}{\sqrt{p+q-1}}$
Harmonic	$HI(B(p,q))=\frac{-4p}{q}+\frac{2p}{p+q-1}+\frac{2(p+3)p}{p+2}$
First Zagreb	$M_1(B(p,q))=p(2p-q+1+q^2)$
Second Zagreb	$M_2(B(p,q))=p(pq+p+q^2-3q+2)$
Second modified Zagreb	$M_3(B(p,q))=\frac{2pq-4p+q+1}{2(q-1)}$
Inverse sum	$IS(B(p,q))=\frac{p[2p^2q^2-2p^2q+2p^2+pq^3+2pq^2-5pq+2p+2q^3-8q^2+10q-4]}{(p+q-1)(q)(p+2)}$
Albertson	$Alb(B(p,q))=p[\lvert 2-p\rvert+\lvert p-q+1\rvert(p)+\lvert 2-p\rvert(pq)+\lvert 2-q\rvert(-2p)]$
Atom-bond-connectivity	$ABC(B(p,q))=\frac{p}{2}\left\{\sqrt{2}+2q\sqrt{\frac{q-2}{q-1}}-4\sqrt{\frac{q-2}{q-1}}+p\sqrt{\frac{p+q-1}{p(q-1)}}\right\}$
Symmetric Division Index	$SD(B(p,q)) = \frac{p^2(q+1)}{2(q-1)}+\frac{pq^3-4pq^2+6pq-4p}{q-1}+(q+1)$
Augmented Zagreb	$AZI(B(p,q))=8p+\frac{p^4q^3}{(p+q-3)^3}-\frac{3p^4q^2}{(p+q-3)^3}+\frac{3p^4q}{(p+q-3)^3}-\frac{p^4}{(p+q-3)^3}+\frac{pq^3}{(q-2)^2}-\frac{3pq^2}{(q-2)^2}+\frac{3pq}{(q-2)^2}-\frac{p}{(q-2)^2}$

Conclusion:

In this article, we computed the degree based topological indices Randic Index, Geometric-Arithmetic Index, Sum-Connectivity Index, Harmonic Index, First Zagreb, Second Zagreb, Second Modified Zagreb, Inverse sum, Albertson, Atom Bond connectivity, Symmetric Division Index and Augmented Zagreb of Banana Tree B(p,q) as a closed formula which are very much useful for the chemist for their future prediction about the compounds Properties for their research analysis.

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