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SQUARE DIMENSIONAL MEMBERSHIP FUNCTIONS OF SOFT STRUCTURES

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Abstract:

A new dimensional membership functions called square membership functions [SDMFSS] under soft set theory are defined in this article. Also we have studied arbitrary intersections, level membership functions and its subgroup structures with suitable examples.

Key Words: Membership Functions, Index Set, Arbitrary Set, Family of Square Functions & Subgroup Structures

1. Introduction:

The idea of bipolar valued fuzzy set was introduced by K.M.Lee [8, 9], as a generalization of the notion of fuzzy set. Since then, the theory of bipolar valued fuzzy sets has become a vigorous area of research in different disciplines such as algebraic structure, medical science, graph theory, decision making, machine theory and so on [2, 4, 6, 16,7]. Convexity plays a most useful role in the theory and applications of fuzzy sets. In the basic and classical paper [15], Zadeh paid special attention to the investigation of the convex fuzzy sets. Following the seminal work of Zadeh on the definition of a convex fuzzy set, Ammar and Metz defined another type of convex fuzzy sets in [1]. From then on, Zadeh's convex fuzzy sets were called quasi-convex fuzzy sets in order to avoid misunderstanding. Soft set theory was introduced by Molodtsov [13] for modeling vagueness and uncertainty and it has been received much attention since Maji et al [12], Ali et al [4] and Sezgin and Atagun [14] introduced and studied operations of soft sets. This theory has started to progress in the mean of algebraic structures, since Aktas¸ and Cagman [3] defined and studied soft groups. A new dimensional membership functions called square membership functions [SDMFSS] under soft set theory are defined in this article. Also we have studied arbitrary intersections, level membership functions and its subgroup structures with suitable examples.

2. Basic and Previous Concepts:

2.1 Definition [D. Molodtsov]: A pair (δ,A) is called a soft set over U, where δ is a mapping given by δ : A→P(U). In other words, a soft set over U is a parameterized family of subsets of the universe U. Note that a soft set (δ, A) can be denoted by δ_A. In this case, when we define more than one soft set in some subsets A, B, C of parameters E, the soft sets will be denoted by δ_A, δ_B, δ_C, respectively. On the other case, when we define more than one soft set in a subset A of the set of parameters E, the soft sets will be denoted by δ_A,δ_A, λ_A, respectively.

2.2 Definition [M.I. Ali, F. Feng]: Let δ_A and Δ_B be two soft sets over U such that $A \cap B \neq \emptyset$,. The restricted intersection of δ_A and Δ_B is denoted by $\delta_A \Cap \Delta_B$, and is defined as $\delta_A \Cap \Delta_B = (\lambda, C)$, where C = A∩B and for all c ∈ C, λ(c) = δ(c)∩Δ(c).

2.3 Definition [M.I. Ali, F. Feng]: Let δ_A and Δ_B be two soft sets over U such that $A \cap B \neq \emptyset$,. The restricted union of δ_A and Δ_B is denoted by $\delta_A \cup_R \Delta_B$, and is defined as $\delta_A \cup_R \Delta_B = (\lambda, C)$,where C = A∩B and for all c ∈ C, λ(c) = δ(c)∪Δ(c).

2.4Definition: Let δ_A be a SDMFSS over U. Then (M,N)-level of SDMFSS δ_A, denoted by , is defined as follows $\delta_A^{(M,N)} = \left\{ x \in A / \delta_A^{P}(x) \geq M \text{ and } \delta_A^{N}(x) \leq N \right\}$ for $M \cap N = \phi$.

Assume at if $M = \phi$ or $N = U$, then $\delta_A^{(\phi,U)} = \left\{ x \in A / \delta_A^{P}(x) \neq \phi \text{ and } \delta_A^{N}(x) = U \right\}$ is called Support of δ_A, and denoted by Supp(δ_A).

2.5 Example: Let D = {d1, d2, d3, d4 } be an initial universe and f = {f1,f 2, f3} be a parameter set. If we define SDMFSS as follows

Then the positive membership functions are defined {{f1, f3}, {f1, f2, f,3} and the negative membership functions are {f1,f2,f3,f1f2, f2f3}.

Let $M = \{ f2, f3 \}$ and $N = \{ f2, f3, f1f2 \}$. Then $\delta_A^{(M,N)} = \{ f1f2 \}$.

3. Some Standard Results (Propositions):

The following propositions are proved based on the definitions

3.1Proposition:

Let δ_A and δ_B be two SDMFSS over U. $A,B \subseteq E$. Then following assertions hold**(*)** $\delta_A \subseteq \delta_B \Rightarrow \delta_A{}^{(M,N)} \subseteq \delta_B{}^{(M,N)}$, for all $M,N \subseteq U$ such that $M \cap N = \phi$.

(**)If $M_1 \subseteq M_2$ and $N_1 \subseteq N_2$, then $\delta_A{}^{(M,N_2)} \subseteq \delta_A{}^{(M,N_1)}$ for all $M_1, M_2, N_1, N_2 \in U$ such that M1 ∩ N1 = ⊗ or M2 ∩ N2 = ⊗.

(***)Some places are denoted M, N for alpha and beta notations.

(****) $\delta_A = \delta_B \Rightarrow \delta_A{}^{(\alpha,\beta)} = \delta_B{}^{(\alpha,\beta)}$, for all $\alpha, \beta \subseteq U$ such that $\alpha \cap \beta = \phi$.

Proof:

Let us assume that δ_A and δ_B be two SDMFSS over U.

*Let $x \in \delta_A{}^{(\alpha,\beta)}$, then $\delta_A{}^P(x) > \alpha$ and $\delta_A{}^N(x) \leq \beta$.

Since $\delta_A \subseteq \delta_B$, $\alpha \subseteq \delta_A{}^P(x) \subseteq \delta_B{}^P(x)$ and $\delta_A{}^P(x) \supseteq \delta_B{}^P(x) \supseteq \beta$ for all $x \in G$.

$\Rightarrow x \in \delta_B{}^{(\alpha,\beta)}$. Hence $\delta_A{}^{(\alpha,\beta)} \subseteq \delta_B{}^{(\alpha,\beta)}$.

**Let $\alpha_1 \subseteq \alpha_2$ and $\beta_1 \subseteq \beta_2$ and $x \in \delta_A{}^{(\alpha_2,\beta_2)}$, then $\delta_A{}^P(x) \geq \alpha_2$ and $\delta_A{}^N(x) \leq \beta_2$.

But we have $\alpha_1 \subseteq \alpha_2$ and $\beta_1 \subseteq \beta_2$, $\delta_A{}^P(x) \geq \alpha_1$ and $\delta_A{}^N(x) \leq \beta_1 \Rightarrow \delta_A{}^{(\alpha_2,\beta_2)} \subseteq \delta_A{}^{(\alpha_1,\beta_1)}$.

****This case proved in a simple manner.

3.2Proposition:

Assume that δ_A and δ_B be two SDMFSS over U. $A,B \subseteq E$ and $\alpha, \beta \subseteq E$ such that $\alpha \cap \beta = \phi$. Then

(i) $\delta_A{}^{(\alpha,\beta)} \cup \delta_B{}^{(\alpha,\beta)} \subseteq (\delta_A \cup \delta_B)^{(\alpha,\beta)}$

(ii) $\delta_A{}^{(\alpha,\beta)} \cap \delta_B{}^{(\alpha,\beta)} \subseteq (\delta_A \cap \delta_B)^{(\alpha,\beta)}$

Proof:

(i) For all $x \in E$, let $x \in \delta_A{}^{(\alpha,\beta)} \cup \delta_B{}^{(\alpha,\beta)}$

$\Rightarrow$ ($\delta_A{}^P(x) \geq \alpha$ and $\delta_A{}^N(x) \leq \beta$) $\vee$ ($\delta_B{}^P(x) \geq \alpha$ and $\delta_B{}^N(x) \leq \beta$)

$\Rightarrow$ ($\delta_A{}^P(x) \cup \delta_B{}^P(x) \geq \alpha$) or ($\delta_A{}^N(x) \cap \delta_B{}^N(x) \leq \beta$) $\Rightarrow x \in (\delta_A \cup \delta_B)^{(\alpha,\beta)}$

Hence $\delta_A{}^{(\alpha,\beta)} \cup \delta_B{}^{(\alpha,\beta)} \subseteq (\delta_A \cup \delta_B)^{(\alpha,\beta)}$.

(ii) For all $x \in E$, let $x \in \delta_A{}^{(\alpha,\beta)} \cup \delta_B{}^{(\alpha,\beta)}$

$\Rightarrow$ ($\delta_A{}^P(x) < \alpha$ and $\delta_A{}^N(x) > \beta$) $\vee$ ($\delta_B{}^P(x) \geq \alpha$ and $\delta_B{}^N(x) \leq \beta$)

$\Rightarrow$ ($\delta_A{}^P(x) \cup \delta_B{}^P(x) \geq \alpha$) or ($\delta_A{}^N(x) \cap \delta_B{}^N(x) \leq \beta$) $\Rightarrow x \in (\delta_A \cup \delta_B)^{(\alpha,\beta)}$

Hence $\delta_A{}^{(\alpha,\beta)} \cup \delta_B{}^{(\alpha,\beta)} \subseteq (\delta_A \cup \delta_B)^{(\alpha,\beta)}$.

3.3 Propsotion:

Let I be an index set and δ_A be a family of SDMFSS over U. Then, for any $\alpha, \beta \subseteq U$ such that $\alpha \cap \beta = \phi$,

(i) $\bigcup_{i \in I} (\delta_{A_i}{}^{(\alpha,\beta)}) \subseteq (\bigcup_{i \in I} \delta_{A_i})^{(\alpha,\beta)}$

(ii) $\bigcap_{i \in I} (\delta_{A_i}{}^{(\alpha,\beta)}) = (\bigcap_{i \in I} \delta_{A_i})^{(\alpha,\beta)}$

3.4 Proposition:

Let δ_A be a BSFS-sets over U and $\{\alpha_i / i \in I\}$ and $\{\beta_j / i \in I\}$ be two non-empty family of subsets of U. If $\alpha = \bigcap \{\alpha_i / i \in I\}$, $\overline{\alpha} = \bigcup \{\alpha_i / i \in I\}$, $\beta = \bigcap \{\beta_j / i \in I\}$ and $\overline{\beta} = \bigcup \{\beta_j / i \in I\}$, then the following assertions hold,

$$\bigcup_{i \in I} \delta_A{}^{(\alpha_i,\beta_j)} \subseteq \delta_A{}^{(\underline{\alpha},\overline{\beta})} \quad \bigcap_{i \in I} \delta_A{}^{(\alpha_i,\beta_j)} = \delta_A{}^{(\overline{\alpha},\underline{\beta})}$$

3.5 Proposition:

Let δ_G be a SDMFS subgroup over U and $\alpha, \beta \subseteq U$ such that $\alpha \cap \beta = \phi$. Then $\delta_G^{(\alpha,\beta)}$ is also SDMFS-subgroup of G whenever it is non empty.

Proof:

It is clear that $\delta_G^{(\alpha,\beta)} \neq \phi$.

Suppose that $x, y \in \delta_G^{(\alpha,\beta)}$, then $\delta_G{}^P(x) \geq \alpha$, $\delta_G{}^P(y) \geq \alpha$ and $\delta_G{}^N(x) \leq \beta$, $\delta_G{}^N(y) \leq \beta$.

$\inf \delta_G{}^P(xy^{-1}) \geq \min\{\inf \delta_G{}^P(x), \inf \delta_G{}^P(y^{-1})\} = \min\{\inf \delta_G{}^P(x), \inf \delta_G{}^P(y)\}$
$\geq \min(\alpha,\alpha) \geq \alpha$

$\sup\delta_G{}^P(xy^{-1}) \geq \min\{\sup\delta_G{}^P(x), \sup\delta_G{}^P(y^{-1})\} = \min\{\sup\delta_G{}^P(x), \sup\delta_G{}^P(y)\}$
$\geq \min(\alpha,\alpha) \geq \alpha$

and

$\inf \delta_G{}^N(xy^{-1}) \leq \max\{\inf \delta_G{}^N(x), \inf \delta_G{}^N(y^{-1})\} = \max\{\inf \delta_G{}^N(x), \inf \delta_G{}^N(y)\}$
$\leq \max(\beta,\beta) \leq \beta$

$\sup\delta_G{}^N(xy^{-1}) \leq \max\{\sup\delta_G{}^N(x), \sup\delta_G{}^N(y^{-1})\} = \max\{\sup\delta_G{}^N(x), \sup\delta_G{}^N(y)\}$
$\leq \max(\beta,\beta) \leq \beta$. *Therefore, we have* $xy^{-1} \in \delta_G^{(\alpha,\beta)}$ and $\delta_G^{(\alpha,\beta)}$ is a SDMFSS subgroup of G.

3.6 Proposition:

Let δ_{G_i} be a family of SDMFS-subgroup over U for all $i \in I$. Then $\bigcap_{i\in I} \delta_{G_i}$ is a SDMFS -subgroup over U.

Proof:

Let $x, y \in G$. Since δ_{G_i} be a SDMFS-subgroup over U for all $i \in I$. This shows that $\delta_{G_i}{}^P(xy^{-1}) \geq \min\{\delta_{G_i}{}^P(x), \delta_{G_i}{}^P(y)\}$ for all $i \in I$. Then

$\inf \delta_{G_i}{}^P(xy^{-1}) \geq \min\{\inf \delta_{G_i}{}^P(x), \inf \delta_{G_i}{}^P(y)\}$

$\inf\left(\bigcap_{i\in I} \delta_{G_i}{}^P(xy^{-1})\right) \geq \bigcap_{i\in I} \min\{\inf \delta_{G_i}{}^P(x), \inf \delta_{G_i}{}^P(y)\}$

$= \min\left\{\inf\left(\bigcap_{i\in I} \delta_{G_i}{}^P(x)\right), \inf\left(\bigcap_{i\in I} \delta_{G_i}{}^P(y)\right)\right\}$ and

$\sup\delta_{G_i}{}^P(xy^{-1}) \geq \min\{\sup\delta_{G_i}{}^P(x), \sup\delta_{G_i}{}^P(y)\}$

$\sup\left(\bigcap_{i\in I} \delta_{G_i}{}^P(xy^{-1})\right) \geq \bigcap_{i\in I} \min\{\sup\delta_{G_i}{}^P(x), \sup\delta_{G_i}{}^P(y)\}$

$= \min\left\{\sup\left(\bigcap_{i\in I} \delta_{G_i}{}^P(x)\right), \sup\left(\bigcap_{i\in I} \delta_{G_i}{}^P(y)\right)\right\} = \min\{\sup\delta_{G_i}{}^P(x), \sup\delta_{G_i}{}^P(y)\}$

and

$\inf \delta_{G_i}{}^N(xy^{-1}) \leq \max\{\inf \delta_{G_i}{}^N(x), \inf \delta_{G_i}{}^N(y)\}$

$\inf\left(\bigcup_{i\in I} \delta_{G_i}{}^N(xy^{-1})\right) \leq \bigcup_{i\in I} \max\{\inf \delta_{G_i}{}^N(x), \inf \delta_{G_i}{}^N(y)\}$

$= \max\left\{\inf\left(\bigcup_{i\in I} \delta_{G_i}{}^N(x)\right), \inf\left(\bigcup_{i\in I} \delta_{G_i}{}^N(y)\right)\right\}$

and $\sup\delta_{G_i}{}^N(xy^{-1}) \leq \max\{\sup\delta_{G_i}{}^N(x), \sup\delta_{G_i}{}^N(y)\}$

$\sup\left(\bigcup_{i\in I} \delta_{G_i}{}^N(xy^{-1})\right) \leq \bigcup_{i\in I} \max\{\sup\delta_{G_i}{}^N(x), \sup\delta_{G_i}{}^N(y)\}$

$= \max\left\{\sup\left(\bigcup_{i\in I} \delta_{G_i}{}^N(x)\right), \sup\left(\bigcup_{i\in I} \delta_{G_i}{}^N(y)\right)\right\}$. Thus $\bigcap_{i\in I} \delta_{G_i}$ is a SDMFS -subgroup over U.

3.7 Proposition:

Let δ_G be a SDMFS-subgroup over U. Then $\delta_G(x^n) \geq \delta_G(x)$ for all $x \in G$ where $n \in N$.

Proof:

Suppose that δ_G is a SDMFS-subgroup over U. Then

$$\delta_G{}^P(x^n) \geq \delta_G{}^P(x) \cap \delta_G{}^P(x) \cap \cdots \cap \delta_G{}^P(x)$$

$$\therefore \inf\left(\delta_G{}^P(x^n)\right) \geq \min\left\{\inf\left(\delta_G{}^P(x)\right), \inf\left(\delta_G{}^P(x)\right), \cdots \inf\left(\delta_G{}^P(x)\right)\right\}$$

and

$$\delta_G{}^N(x^n) \leq \delta_G{}^N(x) \cup \delta_G{}^N(x) \cup \cdots \cup \delta_G{}^N(x)$$

$$\therefore \inf\left(\delta_G{}^N(x^n)\right) \leq \max\left\{\inf\left(\delta_G{}^N(x)\right), \inf\left(\delta_G{}^N(x)\right), \cdots \inf\left(\delta_G{}^N(x)\right)\right\}$$

Similarly

$$\sup\left(\delta_G{}^P(x^n)\right) \geq \min\left\{\sup\left(\delta_G{}^P(x)\right), \sup\left(\delta_G{}^P(x)\right), \cdots \sup\left(\delta_G{}^P(x)\right)\right\}$$

$$\sup\left(\delta_G{}^N(x^n)\right) \leq \max\left\{\sup\left(\delta_G{}^N(x)\right), \sup\left(\delta_G{}^N(x)\right), \cdots \sup\left(\delta_G{}^N(x)\right)\right\}$$ and for all $x \in G$.

Thus $\delta_G(x^n) \geq \delta_G(x)$.

3.8 Proposition:

Let δ_G be a SDMFS-subgroup over U. If for all $x, y \in G$,

$\inf\left(\delta_G{}^P(xy^{-1})\right) = U$ and $\inf\left(\delta_G{}^N(xy^{-1})\right) = \phi$ and $\sup\left(\delta_G{}^P(xy^{-1})\right) = U$ and $\sup\left(\delta_G{}^N(xy^{-1})\right) = \phi$

Then $\inf \delta_G{}^P(x) = \inf \delta_G{}^P(y)$ and $\sup \delta_G{}^N(x) = \sup \delta_G{}^N(y)$.

Proof:

For any $x, y \in G$

$$\inf\left(\delta_G{}^P(x)\right) = \inf\left(\delta_G{}^P(xy^{-1})y\right) \geq \min\left\{\inf \delta_G{}^P(xy^{-1}), \inf \delta_G{}^P(y)\right\} = \min\left\{U, \inf \delta_G{}^P(y)\right\}$$

$$= \inf \delta_G{}^P(y)$$

And $\inf\left(\delta_G{}^P(y)\right) = \inf\left(\delta_G{}^P(y^{-1})\right) = \inf\left(\delta_G{}^P(x^{-1}(xy^{-1}))\right) \geq \min\left\{\inf \delta_G{}^P(x^{-1}), \inf \delta_G{}^P(xy^{-1})\right\}$

$= \min\left\{\inf \delta_G{}^P(x^{-1}), U\right\} = \inf \delta_G{}^P(x)$. Thus $\inf \delta_G{}^P(x) = \inf \delta_G{}^P(y)$. Also

$$\sup\left(\delta_G{}^N(x)\right) = \sup\left(\delta_G{}^N(xy^{-1})y\right) \leq \max\left\{\sup \delta_G{}^N(xy^{-1}), \sup \delta_G{}^N(y)\right\} = \max\left\{\phi, \sup \delta_G{}^N(y)\right\}$$

$= \sup \delta_G{}^N(y)$ and

$$\sup\left(\delta_G{}^N(y)\right) = \sup\left(\delta_G{}^N(y^{-1})\right) = \sup\left(\delta_G{}^N(x^{-1}(xy^{-1}))\right) \leq \max\left\{\sup \delta_G{}^N(x^{-1}), \sup \delta_G{}^N(xy^{-1})\right\}$$

$= \max\left\{\sup \delta_G{}^N(x^{-1}), \phi\right\} = \sup \delta_G{}^N(x)$. Thus $\sup \delta_G{}^N(x) = \sup \delta_G{}^N(y)$.

Conclusion:

In this paper, we have to characterize the new membership function which is very useful for artificial intelligence and traffic signal problems.

References:

1. E. Ammar, J. Metz, On fuzzy convexity and parametric fuzzy optimization, Fuzzy Sets and Systems, 49 (1992) 135–141.
2. M. Akram Bipolar fuzzy graphs Information Sciences, Vol. 181, No.24, pp. 5548-5564, 2011.
3. H. Aktas and N. Cagman, Soft sets and soft groups, Inform. Sci. 177, 2726-2735 (2007).
4. M.I. Ali, F. Feng, X. Liu, W.K. Min and M. Shabir, on somenew operations in soft set theory, Comput. Math. Appl. 57, 1547-1553 (2009).
5. Y. B. Jun and S. Z. Song, Subalgebras and closed ideals of BCH-algebras based on bipolar-valued fuzzy sets, Sci. Math. Jpn, Vol.68, No.2, pp.287-297, 2008.
6. Y. B. Jun and C. H. Park, Filters of BCH-Algebras based on bi-polar-valued fuzzy sets, Int. Math. Forum, Vol 14, No. 13, pp. 631-643, 2009.
7. Y. B. Jun and J. Kavikumar, Bipolar fuzzy finite state machines, Bull. Malays. Math. Sci. Soc. Vol, 34, No.1, pp.181-188, 2011.
8. K. J. Lee, Bipolar fuzzy sub algebras and bipolar fuzzy ideals of BCK/BCI-algebras, Bull. Malays. Math. Sci. Soc., Vol.32, No.3, pp.361-373, 2009.

9. K. M. Lee, Bipolar-valued fuzzy sets and their operations, Proc. Int. Conf. on Intelligent Technologies, Bangkok, Thailand, pp.307-312, 2000.
10. R. Lowen, Convex fuzzy sets, Fuzzy Sets and Systems 3(1980)291–310.
11. K. M. Lee, Comparison of interval-valued fuzzy sets, intuitionistic fuzzy sets, and bipolar-valued fuzzy sets, J. Fuzzy Logic Intelligent Systems, Vol.14, No.2, pp.125-129, 2004.
12. P. K. Maji, A.R. Roy and R. Biswas, An application of softsets in a decision making problem, Comput. Math. Appl. 44, 1077-1083 (2002).
13. D. Molodtsov, Soft set theory-first results, Comput. Math.Appl. 37, 19-31 (1999).
14. A. Sezgin and A. O. Atag¨un, Soft groups and normalisticsoft groups, Comput. Math. Appl. 62 (2), 685-698 (2011).
15. L. A. Zadeh, Fuzzy sets, Inform. And Control 8 (1965)338–353.