International Journal of Current Research and Modern Education (IJCRME)
Impact Factor: 6.925, ISSN (Online): 2455 - 5428
(www.rdmodernresearch.com) Volume 3, Issue 1, 2018

INTERVAL VALUED BI-FUZZY MEMBERSHIP FUNCTIONS ON GROUP STRUCTURES

V. Ramadas* & P. Shanmugam**
* Professor, Department of Mathematics, PRIST University, Tanjore, Tamilnadu
** Research Scholar, Department of Mathematics, PRIST University, Tanjore, Tamilnadu

Cite This Article: V. Ramadas & P. Shanmugam, "Interval Valued Bi-Fuzzy Membership Functions on Group Structures", International Journal of Current Research and Modern Education, Volume 3, Issue 1, Page Number 218-220, 2018.

Abstract:

In this paper, we study the characterizations of interval valued bi-fuzzy soft membership functions, level set, homomorphic image and its arbitrary intersections with group nature with suitable examples.

Index Terms: Soft Set, Bi-Fuzzy Set, Interval Valued Fuzzy Soft Set, Homomorphic Image, Level Set, Intersection, Group & Interval Number

Introduction:

The idea of bipolar valued fuzzy set was introduced by K. M. Lee [2004], as a generalization of the notion of fuzzy set. Since then, the theory of bipolar valued fuzzy sets has become a vigorous area of research in different disciplines such as algebraic structure, medical science, graph theory, decision making, machine theory and so on An interval number on [0,1] say ℓ is a closed subinterval of [0,1], $\ell = [\ell_P, \ell_N]$ where $0 \le \ell_N \le \ell_P \le 1$. For any interval numbers $\ell = [\ell_P, \ell_N]$ on [0, 1], we define

(i) $\ell \le m$ if and only if $\ell_N \le m_N$ and $\ell_P \le m_P$.

(ii) $\ell = m$ if and only if $\ell_N = m_N$ and $\ell_P = m_P$

(iii) $\ell + m = [\ \ell_N + m_N\ ,\ \ell_P + m_N\ \}$, whenever $\ell_N + m_N \le 1$ and $\ell_P + m_N \le 1$. Zadeh [1965] introduced the concept of fuzzy set as a new mathematical tool for dealing with uncertainties. There are several kinds of fuzzy set extensions in the fuzzy set theory, for example, intuitionistic fuzzy sets, interval-valued fuzzy sets, rough fuzzy set, soft fuzzy set, vague sets, etc [1971]. Bipolar-valued fuzzy set is another extension of fuzzy set whose membership degree range is extended from the interval [0; 1] to the interval [-1; 1]. The soft set theory Moldtsov [1999] can be used as a newly mathematical tool to handle uncertainty. However, the classical soft sets are not appropriate to deal with imprecise and fuzzy parameters. In this paper, we discuss the characterizations of interval valued bi-fuzzy soft membership functions, level set, homomorphic image and its arbitrary intersections with group nature with suitable examples.

Preliminaries:

In this section, we see the basic concepts of interval-valued sets, lower level cut, interval number and so on. Let X be a set. A mapping $\delta : X \rightarrow [\ 0,1]$ is called a fuzzy set in X. Let 'δ' be a fuzzy set in X and α € [0, 1]. Define $L(\delta\ ;\ \alpha)$ as follows $L(\delta\ ;\ \alpha) = \{\ x \in X\ /\ \delta(x) \le \alpha\ \}$. Then $L(\delta\ ;\ \alpha)$ is called the lower level cut of δ. Let X be a set. A mapping $[\delta] : X \rightarrow D[0,1]$ is called an interval valued fuzzy set (briefly IVFS) of X, where D [0,1] denotes the family of all closed subintervals of [0,1] and $[\ \delta]\ (x) = \{\ \delta_P(x)\ ,\ \delta_N(x)\ \}$, for all x € X, where δ_P and δ_N are fuzzy sets in X.

Definition 2.1: A pair (δ,A) is called a soft set over U, where δ is a mapping given by $\delta : A \rightarrow P(U)$.

Definition 2.2: Let U denote an universe of discourse. A bipolar fuzzy soft set $\overline{A}$ is an object having the form $\overline{A} = \left\{(x, \delta_{\overline{A}}{}^{P}(x), \delta_{\overline{A}}{}^{N}(x)) / x \in U\right\}$ where $\delta_{\overline{A}}{}^{P} : U \rightarrow [0,1]$ and $\delta_{\overline{A}}{}^{N} : U \rightarrow [-1,0]$ satisfy $-1 \le \delta_A{}^P + \delta_A{}^N \le 1$ for all $x \in U$, $\delta_{\overline{A}}{}^{P}$ and $\delta_{\overline{A}}{}^{N}$ are called the degree of positive membership function and degree of negative membership functions respectively.

Definition 2.3: An interval valued bi-fuzzy soft set [IVBFSS] in X is defined as an object of the form $[\delta\ ] = \{\ x\ ,\ [\delta]_P(x),\ [\delta]_N(x) > /\ x \in X\}$ where $[\delta_P] : X \rightarrow D\ [0,1]$ and $[\delta_N] : X \rightarrow D\ [-1,0]$ is called positive and negative interval valued bi-fuzzy membership functions respectively.

Example 2.4: consider [δ] = {< u_1, [0.2, 0.4], [-0.5,-0.6]>, <u_2, [0.3, 0.9], [-0.7, -0.6] >, <u_3, [0.7, 0.9], [-0.4, -0.6]>} is IVBFSS of X= { u_1, u_2, u_3}.

Definition 2.5: Let G be a group. An IVBFSS [δ] of G is said o be IVBFS- subgroup of G if the following conditions are satisfied

$$(IVBFSG1)\ \delta_P(x * y) \ge rmin\ \{\ \delta_P(x)\ ,\ \delta_P(y)\ \} \delta_P(x^{-1}) \ge \delta_P(x)$$

$$(IVBFSG2)\ \delta_N(x * y) \le rmax\ \{\ \delta_N(x)\ ,\ \delta_N(y)\ \},\ \delta_N(x-1) \le \delta_N(x) \text{ for all } x, y \in G.$$

Example: Let G = { e,a,b,c } be a group with respect to usual multiplications.

Example 2.6:

Consider [δ] = {< e, [0.3, 0.1] ,[-0.7 ,-0.5]> , <a, [0.2, 0.5],[-0.3, -0.9] >, <b, [0.1,0.9], [-0.2, -0.7]> <c, [0.6, 0.8], [-0.1, -0.5] > }is IVBFSG of G.

Properties of Bi-Fuzzy Soft Membership Functions:

Proposition 3.1:

Let [δ] = {x, $[\delta]_P(x)$, $[\delta]_N(x)$ > / x € X} be a IVBFSG 'δ' in a group G. Then $\delta_P(x^{-1}) = \delta_P(x)$ and $\delta_N(x\text{-}1) = \delta_N(x)$ for all x € G.

Proof:

For all x € G, we have $\delta_P(x) = \delta_P(x\text{-}1)^{-1} \geq \delta_P(x)$ and $\delta_N(x) = \delta_N(x^{-1})^{-1} \leq \delta_N(x)$.

Hence $\delta_P(^{x-1}) = \delta_P(x)$ and $\delta_N(x^{-1}) = \delta_N(x)$.

Proposition 3.2:

An IVBFS-set [δ] = { x , $[\delta]_P(x)$, $[\delta]_N(x)$ > / x € X} is IVBFSG of G if and only if $\delta_P(x*y^{-1}) \geq r \min \{\delta_P(x), \delta_P(y)\}$ and $\delta_N(x*y^{-1}) \leq r \max \{\delta_N(x), \delta_N(y)\}$, for all x € G.

Proof:

Assume that [δ] = { x , $[\delta]_P(x)$, $[\delta]_N(x)$ > / x € X} is a IVBFSG of G and x, y € G.

Then $\delta_P(x*y^{-1}) \geq r \min \{\delta_P(x), \delta_P(y^{-1})\}$, by definition

$= r \min \{\delta_P(x), \delta_P(y)\}$by proposition (1)

Also $\delta_N(x*y\text{-}1) \leq r \max \{\delta_N(x), \delta_N(y^{-1})\}$, by definition

$= r \max\{\delta_P(x), \delta_P(y)\}$by proposition (1).

Conversely, suppose (i) and (ii) are valid. If we take $y = x^{-1}$ in (i) and (ii), then

$\delta_P(e) = \delta_P(xx^{-1}) \geq r \min \{\delta_P(x), \delta_P(x^{-1})\}$

$= r \min \{\delta_P(x), \delta_P(y)\}$by proposition (1)

$\delta_P(e) \geq \delta_P(x)$ and

$\delta_N(e) = \delta_N(xx^{-1}) \geq r \min \{\delta_N(x), \delta_N(x^{-1})\}$

$= r \min \{\delta_N(x), \delta_N(y)\}$by proposition (1)

$\delta_N(e) \leq \delta_N(x)$. It follows that from (i) and (ii), that

$\delta_P(y^{-1}) = \delta_P(ey^{-1}) \geq r \min \{\delta_P(e), \delta_P(y^{-1})\}$

$= r \min \{\delta_P(e), \delta_P(y)\}$by proposition (1)

$\delta_P(y^{-1}) \geq \delta_P(y)$. Also $\delta_N(y^{-1}) = \delta_N(ey^{-1}) \leq r \max \{\delta_N(e), \delta_N(y^{-1})\}$

$= r \max\{\delta_N(e), \delta_N(y)\}$by proposition (1)

$\delta_N(y^{-1}) \leq \delta_N(y)$.

Then $\delta_P(x*y) = \delta_P(x*(y^{-1})^{-1}) \geq r \min \{\delta_P(x), \delta_P(y^{-1})\}$, by definition

$= r \min \{\delta_P(x), \delta_P(y)\}$

$\delta_N(x*y) = \delta_N(x*(y^{-1})^{-1}) \leq r \max\{\delta_N(x), \delta_N(y^{-1})\}$, by definition

$= r \max \{\delta_N(x), \delta_N(y)\}$. Therefore [δ] = {x, $[\delta]_P(x)$, $[\delta]_N(x)$ > / x € X} is IVBFSG of G.

Proposition 3.3:

Let δ_G be a IVBSF-subgroup over U. If for all $x, y \in G$, $\left(\delta_G{}^P(x*y^{-1})\right) = U$ and $\left(\delta_G{}^N(x*y^{-1})\right) = \phi$ $\left(\delta_G{}^P(x*y^{-1})\right) = U$ and $\left(\delta_G{}^N(x*y^{-1})\right) = \phi$.Then $\delta_G{}^P(x) = \delta_G{}^P(y)$ and $\delta_G{}^N(x) = \delta_G{}^N(y)$.

Proof:

For any $x, y \in G$

$$\left(\delta_G{}^P(x)\right) = \left(\delta_G{}^P(x*y^{-1})y\right) \geq r\min\left\{\delta_G{}^P(x*y^{-1}),\ \delta_G{}^P(y)\right\} = r\min\left\{U,\ \delta_G{}^P(y)\right\} = \delta_G{}^P(y)$$

And $$\left(\delta_G{}^P(y)\right) = \left(\delta_G{}^P(y^{-1})\right) = \left(\delta_G{}^P(x^{-1}(x*y^{-1}))\right) \geq r\min\left\{\delta_G{}^P(x^{-1}),\ \delta_G{}^P(x*y^{-1})\right\} = r\min\left\{\delta_G{}^P(x^{-1}), U\right\} = \delta_G{}^P(x)$$

.Thus $\delta_G{}^P(x) = \delta_G{}^P(y)$.

Also $$\left(\delta_G{}^N(x)\right) = \left(\delta_G{}^N(x*y^{-1})y\right) \leq r\max\left\{\delta_G{}^N(x*y^{-1}),\ \delta_G{}^N(y)\right\} = r\max\left\{\phi, \delta_G{}^N(y)\right\}$$

And $$\left(\delta_G{}^N(y)\right) = \left(\delta_G{}^N(y^{-1})\right) = \left(\delta_G{}^N(x^{-1}(x*y^{-1}))\right) \leq r\max\left\{\delta_G{}^N(x^{-1}),\ \delta_G{}^N(xy^{-1})\right\} = r\max\left\{\delta_G{}^N(x^{-1}), \phi\right\} = \delta_G{}^N(x)$$

.Thus $\delta_G{}^N(x) = \delta_G{}^N(y)$.

Definition 3.4: Let [δ] = { x , $[\delta]_P(x)$, $[\delta]_N(x)$ > / x € X} be a IVBFS-set 'δ' in a group G. Let U {[δ] / [α, β], [γ . δ]}= { x € G / $\delta_P(x) \geq$ [α, β], $\delta_N(y) \leq$ [γ . δ] is called bi-level subset of [δ].

Proposition 3.5:

Let [δ] = {x, $[\delta]_P(x)$, $[\delta]_N(x)$ > / x € X} be a IVBFSG 'δ' in a group G. Then the following conditions are equivalent;

(i) [δ] = {x, $[\delta]_P(x)$, $[\delta]_N(x)$ > / x € X} be a IVBFSG of G.
(ii) The non-empty bi-level set of Let [δ] = { x , $[\delta]_P(x)$, $[\delta]_N(x)$ > / x € X} is a subgroup of G.

Proof:

Assume that [δ] = {x, $[\delta]_P(x)$, $[\delta]_N(x)$ > / x € X} be a IVBFSG of group G.

Let x,y € U {[δ] / [α, β], [γ . δ]}, for all [α, β] € D [0,1] and [γ . δ] € D [-1,0].
Then $\delta_P(x) \geq$ [α, β], $\delta_N(x) \leq$ [γ . δ]. It follows that, Then $\delta_P(x*y\text{-}1) \geq$ r min {$\delta_P(x)$, $_{\delta P}(y^{-1})$} = r min {$\delta_P(x)$, $_{\delta P}(y)$} ≥ [α, β], $\delta_P(x*y^{-1}) \leq$ r max {$\delta_P(x)$, $\delta_P(y^{-1})$} = r max {$\delta_P(x)$, $\delta_P(y)$} ≤ [γ . δ].
So that xy-1 € U {[δ] / [α, β], [γ. δ]}. The non-empty bi-level set [δ] = {x, $[\delta]_P(x)$, $[\delta]_N(x)$ > / x € X} is IVBFSG of G.
Conversely, [α, β] and [γ. δ] € D [-1, 1], such that U {[δ] / [α, β], [γ . δ]} ≠ Φ and U {[δ] / [α, β], [γ . δ]} is a subgroup of G.
Suppose that Proposition – 3.1 (i) is not true and Proposition – 3.1 (ii) is valid. Then there exists [α_0, β_0] € D [-1, 1] and a,b € G such that, Then $\delta_P(a*b^{-1}) \leq$ [α_0, β_0] ≤ r min {$\delta_P(a)$, $\delta_P(b)$} ,$\delta_P(a*b^{-1}) \geq$ [α_0, $_{\beta 0}$] ≥ r max {$\delta_P(a)$, $\delta_P(b)$}.

Proposition 3.6:

Let h: G→ G1 is a homomorpism of groups. If [δ] = { x , $[\delta]_P(x)$, $[\delta]_N(x)$ > / x € X} be a IVBFSG 'δ' in a group G1 , then $[\delta^h]$ = { x , $[\delta^h]_{P(}x)$, $[\delta^h]_N(x)$ > / x € X} be a IVBFSG 'δ' in a group G.

Proof:

Since h: G→ G^1 is a homomorpism of groups.
Now, (IVBFSG1) $\delta_P{}^h(x*y) = \delta_P(h(x*y)) = \delta_P(h(x)\ h(y)) \geq$ r min { $\delta_P(h(x))$, $\delta_P(h(y))$}= r min { $\delta_P{}^h(x))$, $\delta_P{}^h(y))$} and $\delta_P{}^h(x^{-1}) = \delta_P(h(x)^{-1}) \geq \delta_P(h(x)) = \delta_P{}^h(x)$,
(IVBFSG2) $\delta_N{}^h(x*y) = \delta_N(h(x*y)) = \delta_N(h(x)\ h(y)) \leq$ r max { $\delta_N(h(x))$, $\delta_N(h(y))$}= r max { $\delta_N{}^h(x))$, $\delta_N{}^h(y))$} and $\delta_N{}^h(x^{-1}) = \delta_N(h(x)^{-1}) \leq \delta_N(h(x)) = \delta_N{}^h(x)$.
Therefore $[\delta^h]$ = {x, $[\delta^h]_P(x)$, $[\delta^h]_N(x)$ > / x € X} be a IVBFSG 'δ' in a group G.

Proposition 3.7:

If {δi}, I € δ is a family of IVBFSG's of G, then ∩δi is IVBFSG's of G where ∩δi = {{x, $[\delta i]_P(x)$, $[\delta i]_N(x)$ > / x € G} be a IVBFSG 'δ' in a group G.

Proof:

Let x, y € G.
(IVBFSG1) $(\cap\delta_{Pi})(x * y) = \prod \cap\delta_{Pi}\ (x * y)$
$\geq$ r min {$\delta_{Pi}(x)$, $\delta_{Pi}(y)$} = r min { $\prod \delta_{Pi}(x)$, $\prod\delta_{Pi}(y)$}= r min {$(\cap\delta_{Pi})(x)$, $(\cap\delta_{Pi}(y))$}
And $(\cap\delta_{Pi})(x^{-1}) = \prod \delta_{Pi}\ (x^{-1}) \geq \prod \delta_{Pi}(x) = (\cap\delta_{Pi})(x)$.
(IVBFSG2) $(\cap\delta_{Ni})(x * y) = \prod \cap\delta_{Ni}\ (x * y)$
$\leq$ r max {$\delta_{Ni}(x)$, $\delta_{Ni}(y)$} = r max{ $\prod \delta_{Ni}(x)$, $\prod\delta_{Ni}(y)$}= r max {$(\cap\delta_{Ni})(x)$, $(\cap\delta_{Ni}(y))$}
And $(\cap\delta_{Ni})(x^{-1}) = \prod \delta_{Ni}\ (x^{-1}) \leq \prod \delta_{Ni}\ (x) = (\cap\delta_{Ni})(x)$.

Conclusion:

In this paper, we propose the concept of IVBFSG which are a combination of interval-valued fuzzy sets and soft sets. The homomorphic image and its arbitrary intersections are discussed with level set.

References:

1. M. Akram Bipolar fuzzy graphs Information Sciences, Vol. 181, No.24, pp. 5548-5564, 2011.
2. N. Chen, Z. S. Xu and M. M. Xia, Interval-valued hesitant preference relations and their applications to group decision making, Knowledge-Based Systems, 37(2013), 528–540.
3. M. B. Gorzalzany, A method of inference in approximate reasoning based on interval-valued fuzzy sets, Fuzzy Sets and Systems 21 (1987), 1–17.
4. K. M. Lee, Comparison of interval-valued fuzzy sets, intuitionistic fuzzy sets, and bipolar-valued fuzzy sets, J. Fuzzy Logic Intelligent Systems, Vol.14,No.2, pp.125,129, 2004.
5. D. A. Molodtsov, Soft set theory-_first results, Comput. Math. Appl., 37 (1999), 19-31.
6. P. K. Maji, R. Biswas, and A. R. Roy, "Soft set theory," Computers and Mathematics with Applications, vol. 45, no. 4-5, pp. 555–562, 2003.
7. P. K. Maji, R. Biswas, and A. R. Roy, "Fuzzy soft sets," The Journal of Fuzzy Mathematics, vol. 9, no. 3, pp. 589–602, 2001.
8. X. B. Yang, T. Y. Lin and J. Y. Yang, Combination of interval-valued fuzzy set and soft set, Computers & Mathematics Applications 58 (2009), 521–527
9. L. A. Zadeh, Fuzzy sets, Inform. And Control 8 (1965)338–353.
10. L. A. Zadeh, Shadows of fuzzy sets, Problems Inform. Transmission 2(1966), 37–44.