International Journal of Current Research and Modern Education (IJCRME)
Impact Factor: 6.925, ISSN (Online): 2455 - 5428
(www.rdmodernresearch.com) Volume 3, Issue 1, 2018

VERTEX COLORING OF (α-CUT) COMPLEMENT FUZZY GRAPH

R. Parimaleswari*, S. Anbalagan & Subiramaniyan***
* Department of Mathematics, PRIST University, Thanjavur, Tamilnadu
** Department of Mathematics, Rajah Serfoji Government College (Autonomous), Thanjavur, Tamilnadu

Cite This Article: R. Parimaleswari, S. Anbalagan & Subiramaniyan, "Vertex Coloring of (α-Cut) Complement Fuzzy Graph", International Journal of Current Research and Modern Education, Volume 3, Issue 1, Page Number 574-578, 2018.

Abstract:

Let G=(V_F , E_F) be a simple connected undirected fuzzy graph where V_F is a fuzzy set of vertices where each vertices has membership value μ and E_F is a fuzzy set of edges where each edge has a membership value σ. Vertex coloring is a function which assign colors to the vertices so that adjacent vertices receive different colors. We have examined the vertex coloring of complement fuzzy graph through the α-cuts and found the chromatic number of that fuzzy graph.

Key Words: Fuzzy Graph, Compliment Graph, Graph Coloring & Chromatic Number

1. Introduction:

We know that graphs are simply model of relation. A graph is a convenient way of representing information involving relationship between objects. In 1736 Euler and Leohard came out with the solution in terms of graph theory [1]. A graph coloring is one of the most studied problems of combinatorial optimization. The basic graph coloring problem is to group items in as few groups as possible, subject to the constraint that no incompatible items end up in the same group. Which was initialized by ex-student in 1852 Augest us Demorgon, Francis Guthrie, noticed that the countries in England could be colored using four colors so that no adjacent countries were assigned the same color. A coloring function is a mapping $C: V \rightarrow N$ in a graph $G = (V, E)$ and it is identify $C(i)$ as the color of node $i \epsilon V$ in such a way that two adjacent nodes cannot share the same color i.e., $C(i) \neq C(j)$ if i≠ j. Application of graph coloring are job scheduling, aircraft scheduling, computer network securing, map coloring and GSM mobile phone networks, automatic channel allocation for small wireless local area network. The complement of a graph G is a graph H on the same vertices such that two distinct vertices of H are adjacent if and only if they are not adjacent in G. That is, to generate the complement of a graph, one fill in all the missing edges required to form a complete graph, and removes all the edges that were previously these. The complement of Graph was introduced by Corneil, in 1981.

In 1975 Rosenfield [6] discussed the concept of fuzzy graph by using the fuzzy relation whose basic idea was introduced by kauflmann in 1973[2]. Coloring of fuzzy graph play a vital role in solving complication in network. A large number of variation in coloring of fuzzy graphs are available in literature coloring of fuzzy graphs, were introduced by monoz et. al [5]. Anjali and Sunitha introduced chromatic number of fuzzy graph and developed algorithms to the same [3]. Samanta and Pal introduced fuzzy coloring of fuzzy graph [7]. It is mainly studied in combinatorial optimization like control, exam scheduling, register allocation etc. Moderson [4] introduced the concept of complement of fuzzy graph. Later Suita and Vijayakumar re-defined the complement of fuzzy graph [8]. In this paper we studied about the vertex coloring of α complement fuzzy graph for given fuzzy graph. We given to color the vertex independent for each α belongs to [0, 1].

2. Preliminaries:

Definition 2.1:

[Blue et al] Fuzzy graph $G = G_{F2} \cup G_{F4}$. We can define this fuzzy graph using their membership value of vertices and edges. Let V be a finite nonempty set. The triple $G = (V, \sigma, \mu)$ is called a fuzzy graph on V where μ and σ are fuzzy sets on V and $E(V \times V)$, respectively such that $\mu(u, v) \leq \min\{\sigma(u), \sigma(v)\}$ for all $u, v \in V$. The relation between crisp graph and fuzzy graph is all the crisp graphs are fuzzy graphs but not the converse holds.

Definition 2.2:

The α cut of fuzzy graph defined as $G_\alpha = (V_\alpha, E_\alpha)$ where $V_\alpha = \{v \in V | \sigma \geq \alpha\}$ and $E_\alpha = \{e \in E | \mu \geq \alpha\}$.

Definition 2.3:

The vertex coloring of a graph is an assignment of labels or colors to each vertex of a graph such that no edge connects two identically colored vertices.

Definition 2.4:

A proper vertex coloring of a simple graph $G = (V, E)$ is defined as a vertex coloring from a set of colors such that no two adjacent vertices share a same color".

Definition 2.5:

The complement of a fuzzy graph G :(σ, μ) Is G^C= (σ^c, μ^c), the advantage of this defined was that, for every fuzzy graph G. $(G^C)^C$ = G, where σ^c=σ and $\mu^c(x,y) = (\sigma(x)\Lambda\sigma(y)) - \mu(x,y)$

Definition 2.6:

The minimum number of colors required to color the vertices of the given fuzzy graph is known as chromatic number. The chromatic number is denoted by, $\chi(G)$. $\chi(G) = \{(x_\alpha, \alpha)\}$ where x_α is the chromatic number of G_α and α values are the same different membership value of vertices and edges of graph $G = (V_F, E_F)$.

3. Vertex Coloring of Complement Fuzzy Graph:

We have computed the complement of the fuzzy graph by using [2]. We took α value from different membership value of vertices and edges in the complement fuzzy graph. For solving this problem we have done the calculation into three steps. In 1st step we take a fuzzy graph (G) which has five vertices and five edges. All the vertices and edges have fuzzy membership value. In 2nd step we find the complement of this fuzzy graph (G^c). In 3rd step we define the vertex coloring function to color the complement fuzzy graph. We have taken a fuzzy graph between which have five vertices. To color the vertices in G we follow,

Step 1: We consider a fuzzy graph G which have five vertices V_1, V_2, V_3, V_4, V_5 and the corresponding membership value are 0.9, 0.7, 0.6, 0.8, 0.5. Fuzzy Graph G consist of five edges e_1, e_2, e_3, e_4, e_5 with their corresponding membership value 0.7, 0.5, 0.6, 0.4, 0.5.

$$\mu_1 = \begin{array}{c|ccccc} & V_1 & V_2 & V_3 & V_4 & V_5 \\ \hline V_1 & 0.0 & 0.7 & 0.0 & 0.0 & 0.5 \\ V_2 & 0.7 & 0.0 & 0.5 & 0.0 & 0.0 \\ V_3 & 0.0 & 0.5 & 0.0 & 0.6 & 0.0 \\ V_4 & 0.0 & 0.0 & 0.6 & 00 & 0.4 \\ V_5 & 0.5 & 0.0 & 0.0 & 0.4 & 0.4 \end{array}$$

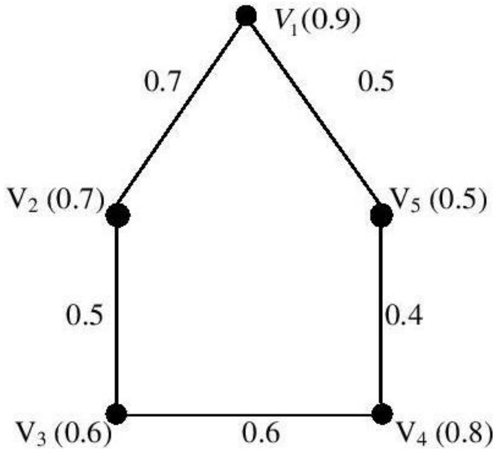


Figure 1: Fuzzy Graph G

Let $G = (V_F, E_F)$ be a fuzzy graph where and $V_F = \{(V_1,0.9),(V_2,0.7)(V_3,0.6),(V_4,0.8)(V_5,0.5)\}$

$E_F = \{(e_1,0.7),(e_2,0.5),(e_3,0.6),(e_4,0.4),(e_5,0.5)\}$

In second step to find the complement graph of the fuzzy graph (G^C) by Definition (2.5)

Step 2:

$$\mu_2 = \begin{array}{c|ccccc} & V_1 & V_2 & V_3 & V_4 & V_5 \\ \hline V_1 & 0.0 & 0.0 & 0.6 & 0.8 & 0.0 \\ V_2 & 0.0 & 0.0 & 0.1 & 0.7 & 0.5 \\ V_3 & 0.6 & 0.1 & 0.0 & 0.0 & 0.5 \\ V_4 & 0.8 & 0.7 & 0.0 & 0.0 & 0.1 \\ V_5 & 0.0 & 0.5 & 0.6 & 0.1 & 0.0 \end{array}$$

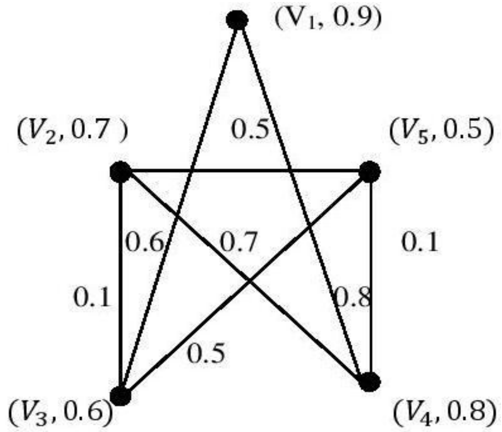


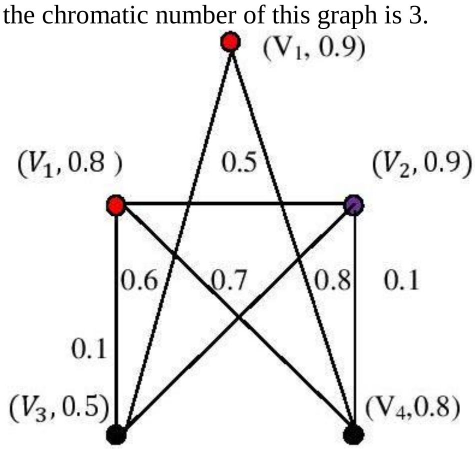
Figure 2: G^C(complement of a fuzzy graph)

Step 3: Graph G^C we have the α values {0.1, 0.5, 0.6, 0.7, 0.8, 0.9}. By using Definition (2.3) we have colored the vertices of G_α^C for each α belongs to the above set and also find its chromatic number by the Definition (2.6)

For α = 0.1 , the fuzzy graph G_α^C where $\sigma = \{0.5,0.6,0.7,0.8,0.9\}$ and the Adjacent Matrix is

$$\mu_3 = \begin{array}{c} \\ V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{array} \begin{array}{c} \begin{array}{ccccc} V_1 & V_2 & V_3 & V_4 & V_5 \end{array} \\ \begin{bmatrix} 0.0 & 0.0 & 0.6 & 0.8 & 0.0 \\ 0.0 & 0.0 & 0.1 & 0.7 & 0.5 \\ 0.6 & 0.1 & 0.0 & 0.0 & 0.5 \\ 0.8 & 0.7 & 0.0 & 0.0 & 0.1 \\ 0.0 & 0.5 & 0.6 & 0.1 & 0.0 \end{bmatrix} \end{array}$$

For α = 0.1, we found the fuzzy graph $G_{0.1}^C$ (figure-3). Then we use the proper coloring to color the vertices of fuzzy graph G_α^C and the chromatic number of this graph is 3.

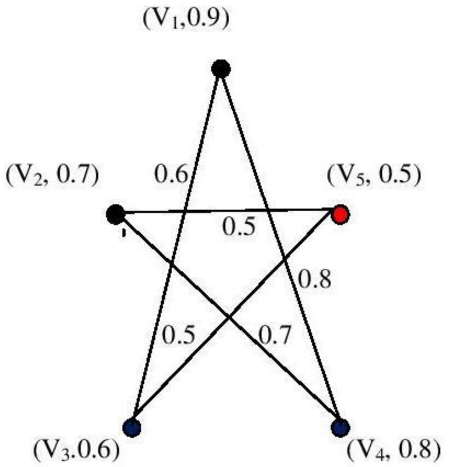


$$\therefore \chi_{(0.1)} = 3$$

Figure 3

For α=0.5, the fuzzy graph $G_{0.5}^C$ where $\sigma = \{0.5,0.6,0.7,0.8,0.9\}$ and the Adjacent Matrix is

$$\mu_3 = \begin{array}{c} \\ V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{array} \begin{array}{c} \begin{array}{ccccc} V_1 & V_2 & V_3 & V_4 & V_5 \end{array} \\ \begin{bmatrix} 0.0 & 0.0 & 0.6 & 0.8 & 0.0 \\ 0.0 & 0.0 & 0.1 & 0.7 & 0.5 \\ 0.6 & 0.1 & 0.0 & 0.0 & 0.5 \\ 0.8 & 0.7 & 0.0 & 0.0 & 0.1 \\ 0.0 & 0.5 & 0.6 & 0.1 & 0.0 \end{bmatrix} \end{array}$$



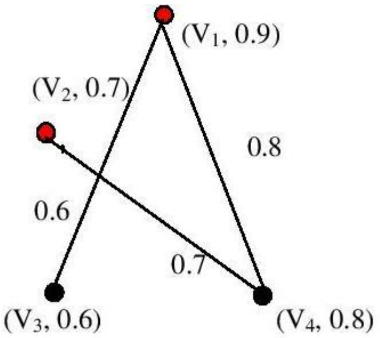
$\therefore \chi_{(0.5)} = \mathbf{3}$

Figure 4

For α=0.5, we find the graph $G_{0.5}^{C}$ (Figure 4). Then we proper color all the vertex of this graph and the chromatic number of this graph is 3.

For α=0.6, the fuzzy graph $G_{0.6}^{C}$ where $\sigma = \{0.6, 0.7, 0.8, 0.9\}$and the Adjacent Matrix is

$$\mu_4 = \begin{array}{c} \\ V_1 \\ V_2 \\ V_3 \\ V_4 \end{array} \begin{array}{c} \begin{array}{cccc} V_1 & V_2 & V_3 & V_4 \end{array} \\ \begin{bmatrix} 0.0 & 0.0 & 0.6 & 0.8 \\ 0.0 & 0.0 & 0.0 & 0.7 \\ 0.6 & 0.0 & 0.0 & 0.0 \\ 0.8 & 0.7 & 0.0 & 0.0 \end{bmatrix} \end{array}$$



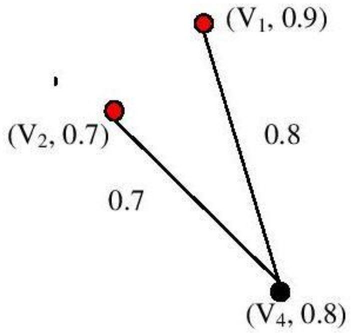
$\therefore \chi_{(0.6)} = \mathbf{2}$

Figure 5

For $\alpha = 0.6$ we find the graph $G_{0.6}^{C}$ **(**Figure-5). Then we use the proper coloring to color the vertex of graph G and the chromatic number of this graph is 2.

For $\alpha = 0.7$, the fuzzy graph $G_{0.7}^{C}$ where $\sigma = \{0.7, 0.8, 0.9\}$. Now for $\alpha = 0.7$, we find the graph $G_{0.7}^{C}$ (Figure-6).Then we proper color all the vertex of this graph and the chromatic number of this graph is 2.

$$\mu_5 = \begin{array}{c} \\ V_1 \\ V_2 \\ V_4 \end{array} \begin{array}{c} \begin{array}{ccc} V_1 & V_2 & V_4 \end{array} \\ \begin{bmatrix} 0.0 & 0.0 & 0.8 \\ 0.0 & 0.0 & 0.8 \\ 0.8 & 0.7 & 0.0 \end{bmatrix} \end{array}$$

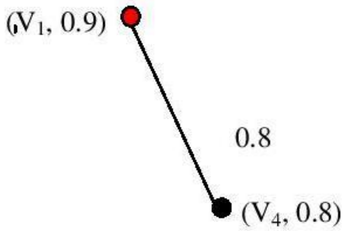


$\therefore \chi_{(0.7)} = 2$

Figure 6

For $\alpha = 0.8$, the fuzzy graph $G_{0.8}^{C}$ (Figure-7) where $\sigma = \{0.8, 0.9\}$ and use the proper coloring to color the vertex and we have $\chi_{(0.8)} = 2$.

$$\mu_6 = \begin{array}{c} \\ V_1 \\ V4 \end{array} \begin{array}{c} \begin{matrix} V_1 & V_4 \end{matrix} \\ \begin{bmatrix} 0.0 & 0.8 \\ 0.8 & 0.0 \end{bmatrix} \end{array}$$

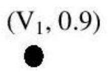


$\therefore \chi_{(0.8)} = 2$

Figure 7

For $\alpha = 0.9$, the fuzzy graph $G_{0.9}^{C}$ (Figure-8) where $\sigma = \{0.9\}$ and use the proper coloring to color the vertex and we have $\chi_{(0.9)} = 1$.

$$\mu_7 = \begin{array}{c} \\ V_1 \\ V_4 \end{array} \begin{array}{c} \begin{matrix} V_1 & V_4 \end{matrix} \\ \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \end{array}$$



$\therefore \chi_{(0.9)} = 1$

Conclusion:

In our study, we computed chromatic number for the vertex coloring of a complement fuzzy graph using α cut. We took α value from the different membership values of vertices and edges in G. We found the chromatic number (vertex coloring) for the different α value of complement fuzzy graph G as follows

$$\chi(G) = \{(3,0.1),(3,0.5),(2,0.6),(2,0.7),(2,0.8),(1,0.9)\}$$

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